Moderations Lecture Synopses 2001–2002
for examination in 2002

Contents

1 Foreword

These synopses have been approved by the Subfaculty Teaching Committee as indicating the intended content of the lecture courses.

Note that the syllabus for the examination is defined in the Examination Decrees.

These synopses are revised annually. Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Secretary in the Mathematical Institute.

July 2001

2 Moderations

2.1 Paper 1: Algebra

2.1.1 Algebra — Dr Lennox — 16 MT

Aims

In recognition of the fact that first year students will have differing mathematical backgrounds, the introductory lectures are intended to give an unhurried introduction to naive set theory and to the nature of a formal proof. This section is designed, so far as it is possible, to be self-contained.

The second aim of the course is to develop the algebra of matrices. Algorithms for manipulated matrices are described (informally) so that students are able to solve systems of linear equations. The emphasis in this part is on understanding the underlying principles, leaving practical work to be done with computers.

The last part of the course is designed to introduce students to formal axiomatic methods through the development of finite dimensional vector spaces. One of the aims will be to bring students to the point where they can use formal methods with confidence. The crowning objective is to develop the theory to the point where earlier results on linear equations and matrices are shown to be applications of the theory in a manner which provides both insight and understanding.

Synopsis

1–5 Sets: introduction to the standard notation of set theory, examples of sets, especially $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{C}$, real polynomials; inclusion, unions, intersections, power sets, product sets, ordered pairs (all naive).

Maps, composition, 1–1ness, ontoness, bijections (including characterisation as invertible maps) [images and pre-images delayed to HT]. Finite and countable sets, Cantor's argument (all naively).

Relations. Order relations, with examples, especially inclusion and divisibility, as well as the natural orders on $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, dictionary orders. Introduction to equivalence relations, with examples, especially arithmetic congruence.

6–8 The sets $M_{m\times n}(\mathbb{R})$. Addition and multiplication of matrices. Derivation of common matrix algebra identities. Row and column vectors as matrices. Matrix representation of a system of real-valued simultaneous linear equations (homogeneous or inhomogeneous); the fact that solutions are unchanged by elementary row operations. Matrices in row-echelon form and solution of associated systems of linear equations; illustrative example(s) showing how any given matrix can be reduced to echelon form by elementary row operations and presentation of algorithmic form of the process used [statement that it works, but no formal proof]; application to deduction of consistency results for systems of linear equations and to systematic procedure for obtaining a complete set of solutions (in parametric form). Illustrative examples. Elementary matrices and relation to elementary row and column operations. Invertible matrices; use of row operations to diagnose invertibility and to calculate inverse.

9–12 Introduction to the axioms for vector spaces (over $\mathbb{R}$) by reference to familiar examples; formal definition of a vector space over $\mathbb{R}$. (Brief mention of how the definition also extends to $\mathbb{C}$.) Simple consequences of the axioms. Subspaces, with examples including subspaces of $\mathbb{R}^2$ and $\mathbb{R}^3$ in geometrical terms; solution spaces of systems of linear homogeneous equations. The span $\langle S \rangle$ of a set S (finite or arbitrary) of vectors; characterisation of $\langle S \rangle$ as the smallest subspace containing S; examples.

Finite dimensionality (as existence of finite spanning set). Linear dependence and linear independence. Definition of a basis. Examples of bases (including bases for the solution spaces of homogeneous linear equations). The co-ordinate system associated with a basis: co-ordinate vectors. The isomorphism between a k-dimensional vector space and $\mathbb{R}^k$ given by a basis. In finite dimensions: reduction of a spanning set and extension of a linearly independent set to a basis; proof that any two bases for a finite dimensional vector space contain the same number of elements (either by means of the 'Steinitz Exchange Lemma' or using the impossibility of having an $m \times n$ matrix A and an $n \times m$ matrix B with $AB = I_m$ when $n < m$). Definition of dimension (warning that goodness of definition not self-evident); the result that, for a subspace U of a finite dimensional space V, $\dim U \leq \dim V$, with equality if and only if $U = V$.

13–16 Sums and intersections of subspaces; formula for $\dim(U + W)$.

The row space and row rank of a matrix and the invariance of both under transformations by elementary row operations; column space and column rank, analogously. Canonical form for matrices of row (column) rank r; application to equality of row rank and column rank. Introduction to linear transformations from one (real) vector space to another. Examples (to include the transformation $\underline{x} \mapsto A\underline{x}$ from $\mathbb{R}^n$ to $\mathbb{R}^m$ induced by an $m \times n$ matrix A). The image and kernel as subspaces. The rank-nullity theorem. Illustrative examples. Applications (such as the result rank $(ST) \leq \min\{\text{rank } S, \text{rank } T\}$).

Reading

M B Powell, *Linear Algebra*, Mathematical Institute Notes (1995)

G Smith, *Introductory Mathematics: Algebra and Analysis*, Springer-Verlag (1998), Chs 1-2

T S Blyth and E F Robertson, *Basic Linear Algebra*, Springer-Verlag (1998)

A O Morris, *Linear Algebra*, 2nd edition, Chapman and Hall (1982) - out of print

D T Finkbeiner, *Introduction to Matrices and Linear Transformations* - out of print

Further reading

C Plumpton, E Shipton, R L Perry, *Proof*, MacMillan (1984)

D A Towers, *Guide to Linear Algebra*, Cambridge (1989)

A G Hamilton, *Linear Algebra*, Cambridge (1989)

G Strang, *Linear Algebra and its Applications*, Harcourt Brace Jovanovich (1988)

2.1.2 Abstract Algebra — Prof Heath-Brown — 12 HT

Aims

The overall aim is to develop the beginnings of the theory of groups in a way which will serve as a model when more complicated algebraic structures must be investigated.

As a preliminary basic properties of the integers are investigated. An understanding of primes, and of greatest common divisors, is important later in the course.

One specific group (the set of permutations of a finite set) is studied in some detail at an elementary level. The aim is to develop confidence in working with a new and moderately complicated system, and to develop a preference for clear and efficient notation, and to provide a source of examples.

Other examples of groups (already known informally to the students) are then introduced, both arithmetic and geometric ones: and from these the group axioms are abstracted. The aim here is to develop the philosophy that the group axioms are the axioms we choose because they are satisfied by the "groups" we want to study (and not conversely!).

It is then natural to study subgroups and how their properties are related to those of the group. Lagrange's Theorem is proved; and the applications are given of this result in a selection of the familiar examples such as permutation groups or cyclic groups. The aim is to show the value of abstraction by showing how one abstract theorem can have a variety of useful and interesting applications.

Finally, we study homomorphisms, those maps between groups which "preserve" the operations, the group theory analogues of the linear maps of vector space theory. The Isomorphism Theorem (the analogue of the Rank-Nullity Theorem), which gives a complete description of what a homomorphism looks like, is proved. The aim is to introduce the idea that homomorphisms are the natural maps to use, and that the homomorphisms on a group are already built into the structure of the group.

Synopsis

1–3 Introduction. Equivalence relations revisited, the set of equivalence classes, correspondence between equivalence relations and partitions. Maps revisited: domain and codomain, images and pre-images (with examples from MT course); Isomorphism Theorem for sets.

Division algorithm for the integers; Euclid's algorithm for gcd. Elementary properties of primes. Division algorithm for polynomials; the remainder theorem.

4–6 Permutations. Orbits, cycles, good notation, the standard cycle-product form. Multiplication, inverses, conjugation (all calculated using the standard form); examples. The order of a permutation; reference back to orders on $\mathbb{N}$; how to calculate it from cycle-structure. The parity of a permutation; the goodness of the definition; how to calculate it from cycle-structure. Examples. Some enumerations.

7–10 Examples to motivate the study of groups, both arithmetic and geometric, and to suggest the form of the group axioms. Axioms for a group (in terms of identity, inverse, product). Subgroups. Intersections of subgroups. Cyclic subgroups. Orders of subgroups and orders of elements; definitions and examples. Cosets and Lagrange's Theorem. Straightforward examples. The structure of cyclic groups; generators for cyclic groups; orders of elements in cyclic groups; numbers of elements of each order.

11–12 Motivating examples of homomorphisms; definition of homomorphisms; examples, and comparison with linear transformations. Kernels, normal subgroups and quotient groups. The Isomorphism Theorem. Examples.

Reading

W B Stewart, *Abstract Algebra*, Mathematical Institute Notes (1994).

I N Herstein, *Abstract Algebra*, 3rd edition, Prentice-Hall (1996), Ch 1, section 5; Chs 4, 5

Further reading

J A Green, *Sets and Groups*, 2nd edition, Routledge (1988)

F M Hall, *An Introduction to Abstract Algebra*, Vol. 1, 2nd edition, CUP (1972)

I N Herstein, *Topics in Algebra*, 2nd edition, Wiley (1975)

P M Cohn, *Algebra*, 2nd edition, Wiley (1982)

2.1.3 Linear Algebra — Lecturer t.b.a. — 12 TT

Aims

This course picks up linear algebra where it was left after the Michaelmas Term algebra course, and you learn to switch flexibly between abstract algebra descriptions and matrix algebra. The second half of the course will cover concepts such as the characteristic equation, eigenvectors and eigenvalues, and geometrical applications.

Synopsis

1–3 Review of bases, independence, subspaces and concept of direct sum. Transformations of $\mathbb{R}^2$ and $\mathbb{R}^3$ and their matrix representations; rotations and orthogonal transformations. The matrix of a linear transformation $T : U \to V$ relative to a pair of bases, in general. The significance of a diagonal representation, eigenvectors and eigenvalues. Examples. The relationship between the co-ordinates of a vector with respect to the co-ordinate system of one basis and its co-ordinates with respect to another; change of bases (or basis) in matrix form..

4–6 Determinants of $n \times n$ matrices for $n = 1, 2, 3$ and then more general definitions. Properties of the determinant function [Proofs to be limited to easier results with as much explanation of the others as time permits.] Computation of determinant by reduction to row echelon form. Worked example(s). The formula $|AB| = |A|\,|B|$. The result that A is invertible if and only if $|A| \neq 0$.

7–9 The result that a scalar λ is an eigenvalue for T if and only if rank $(A - \lambda I) < \dim V$, where A is any matrix representing T. The characteristic equation of a square matrix A; result that any two matrices representing a linear transformation $T : V \to V$ have the same characteristic equations. Examples of finding a matrix P such that $P^{-1}AP$ is diagonal; non-diagonalisable matrices. The linear independence of a set of eigenvectors associated with distinct eigenvalues and the corollary that if $T : V \to V$ has dim V distinct eigenvalues then it admits a diagonal representation. Examples, including the solution of a suitable set of first-order differential equations. Brief mention of the extension of diagonalisation results to matrices over $\mathbb{C}$.

10–12 General form of conic and quadric; quadratic forms and their representation by real symmetric matrices. Other applications. Orthogonal reduction of real symmetric matrices (2×2 and 3×3) and extension to $n \times n$. Orthogonal reduction of real quadratic forms; geometrical interpretation. Non-singular reduction: rank and signature [uniqueness stated but not proved].

Reading

The texts given under the Michaelmas Term Algebra course are again appropriate, to which should be added:

J Roe, *Elementary Geometry*, OUP (1992), Chapter 10

S Lang, *Linear Algebra*, Addison-Wesley (1980)

P J Braam, *Linear Algebra 2*, Mathematical Institute Notes, (1995)

S M Salamon, *Exploring Mathematics with Maple*, Students' Guide HT 99 (project 3)

2.2 Paper 2: Analysis

2.2.1 Differentiability and Convergence — Dr Stewart — 20 MT

Aims

Briefly, the aims are to understand differentiability and convergence, and learn how to apply them and how to make watertight proofs about them.

The intention is to build on knowledge about functions and skills in calculus, developing more general methods and expanding the range of tools available for applications. At the same time, by the use of precise definitions and construction of careful proofs, the theory will be put on a sound footing; this is the aspect of the course where the difference from school calculus is most evident. The course aims to show why this rigorous approach is necessary as well as interesting. The concept of a limit is examined in detail and will be applied to the precise formulation of a definition of differentiation.

The course deals also with convergence of sequences and series. One aim is again to put the theory on a sound footing, using a precise definition of limit just as in the study of differentiation. Another is to show how the 'elementary' functions are defined.

Synopsis

1–4 Real numbers. Sequences of real numbers, definition of limit, simple examples. Limits and inequalities, algebra of limits.

5–6 The idea of a (real-valued) function of a real variable. Recall the idea of differentiation, and the need for limits. Definition of limit of a function. Idea of continuity.

7–11 Differentiable functions, statement that differentiablility implies continuity. Differentiation of $f + g$, αf, fg, f/g, function of a function.

Proof that derivative vanishes at interior local maximum or minimum.

Rolle's theorem. Mean Value Theorem. Applications of Mean Value Theorem: increasing, decreasing and constant functions; Taylor's theorem with remainder; L'Hôpital's rule.

12–13 Subsequences. Proof that every subsequence of a convergent sequence converges to the same limit.

Statement that bounded monotone sequences converge, statement of Bolzano-Weierstrass Theorem (proofs deferred to Hilary Term).

Cauchy's Convergence Principle (C.C.P.). Standard proofs that convergent sequences satisfy the Cauchy condition and that sequences that satisfy the Cauchy condition are bounded. Proof that the convergence of a sequence satisfying Cauchy condition is derivable from the Bolzano-Weierstrass Theorem.

14–18 Convergence of real series, simple examples (including sum of geometric progression).

Tests for series of *positive* terms: comparison test, ratio test, integral test.

Alternating series. Absolute convergence. State that all rearrangements of absolutely convergent series are convergent to the same sum; remark that once one moves beyond absolutely convergent series, this is not the case.

Power series, derivation of the radius of convergence of a power series.. State termwise differentiation and integration within the radius of convergence (proof deferred to a2 Complex Analysis). Brief mention of termwise multiplication of series. Brief survey of elementary functions obtained from power series, including the Binomial Theorem for arbitrary index.

19–20 Asymptotics: basic ideas of O(n) and Ω(n), and a estimate for n!.

Reading

M Hart, *Guide to Analysis*, Macmillan (1988), Chs. 2, 3, 4.1, 5

K G Binmore *Mathematical Analysis*, 2nd edition, CUP (1982), Chs 4, 5, 6, 7, 8, 10, 11, 12 15

J C Burkill, *A First Course in Mathematical Analysis*, CUP (1962), Chs. 2,4,5

M Spivak, *Calculus*, 3rd edition, Publish or Perish Inc (1994), Part II: 3, 5; Part III: 9, 11, 15, 16

2.2.2 Continuity — Dr Sutherland — 20 HT

Aims

Analysis lies at the heart of much mathematics and a mathematician should understand its foundations. The aim of this course will be to understand the concepts and the way proofs work. Central is the completeness axiom. Many of the familiar theorems are a consequence of this axiom. Using metric space topology we will see that many of the results that hold in $\mathbb{R}$ can be put in a more abstract context and extended to other metric spaces.

Synopsis

1–4 Introduction to the idea of continuity. Suprema, infima, $\mathbb{R}$ as a complete ordered field. Revision on limits, definition of continuous function of a real-valued function of a single real variable, algebra of continuous functions. Examples.

5–9 Bolzano-Weierstrass theorem. Boundedness/max-min and intermediate value theorems for functions defined on a closed bounded interval. Inverse function theorem for strictly monotonic functions. Applications.

10–15 Step functions and their integrals. The integral of a continuous function on a closed bounded interval, defined as the supremum of the integrals of the step functions it dominates. Properties of this integral. The Fundamental Theorem of Calculus; integration by parts and integration by substitution; the logarithm function as an integral. Taylor's Theorem with integral remainder. The Mean Value Theorem for integrals of continuous functions. The trapezium rule.

16–20 Continuity extended to functions of several real variables. Metric spaces (stressing $\mathbb{R}$ and $\mathbb{R}^2$): open and closed subsets, limit points, continuity in terms of open sets, definitions of connectedness and compactness (using open sets).

Proof of connexion of (0,1) and compactness of [0,1]. Statement of characterisations of connected and compact subsets of $\mathbb{R}$. The continuous image of a compact (respectively, connected) set is compact (respectively, connected). Simple applications and mention of extensions to $\mathbb{R}^2$.

Reading

J C Burkill, *A First Course in Mathematical Analysis*, CUP (1962), Chs 1, 3, 8.1, 8.2

P J Collins, *Elements of Euclidean Topology*, Mathematical Institute Notes (1994)

M Hart, *Guide to Analysis*, Macmillan (1988), Chs. 1, 4.2, 4.3

H A Priestley, *Introduction to Integration.* OUP (1997), Chs 1–7.

J Roe, *Integration*, Mathematical Institute Notes (1994)

M Spivak, *Calculus*, Benjamin (1967), Part I, Part II: 6–8, Part IV: 21–23

2.3 Paper 3: Non-Physical Applied Mathematics

2.3.1 Differential Equations — Dr Lackenby — 8 MT

Aims

To provide some familiarity with differential equations and a number of common techniques used to solve them. This may be regarded as an extension of the problem of integrating a given function. Difference equations, such as that used to define the Fibonacci numbers, can be solved in a similar way.

Synopsis

1–4 Standard integrals, integration by parts.

Definition of order of an ODE—example of separation of variables. General linear homogenous ODEs: integrating factor for first order linear ODEs, second solution when one solution known for second order linear ODEs. First and second order linear differential equations with constant coefficients. General solution of linear inhomogeneous ODE as particular solution plus homogeneous solution. Examples of finding particular integrals by guesswork. Euler's homogeneous equation.

5–8 Euler's method for solving differential equations leading to difference equations. Linear difference equations with constant coefficients, and in particular those of first and second order. General solution of first and second-order homogeneous linear difference equation with constant coefficients. General solution of linear inhomogeneous difference equation as particular solution plus homogeneous solution. Examples of finding particular solutions by guesswork. Comparison between Euler's method applied to simple initial value problems and exact solution.

Elementary introduction to partial derivatives including the chain rule (without formal proof).

Reading

E Kreyszig, *Advanced Engineering Mathematics*, John Wiley and Sons (1993), parts of Chs. 1,2,20,22.

S Salamon, *Differential Equations and Discrete Mathematics*, Mathematical Institute Lecture Notes 1996

Further Reading

G Birkhoff and G-C Rota, *Ordinary Differential Equations*, 4th edition, Wiley (1989), Chs. 1 (§1–4), 3 (§1–2)

2.3.2 Discrete Mathematics and Probability — Teaching Responsibility of the Statistics Department — Prof Welsh and Dr A D Lunn — 12 MT and 8 HT

Aims

Discrete Mathematics

Use of mathematics in planning efficient solutions to everyday problems in business and commerce has greatly expanded. Discrete Mathematics is a rapidly developing field which sets out to provide the methods for delivering quantitative answers.

Probability

Understanding of random phenomena is becoming increasingly important in today's world, within social and political sciences, finance, life sciences and many other fields. The aim of this introduction to probability is to develop the concept of chance in a mathematical framework. Both discrete and continuous random variables are introduced, with examples involving most of the common distributions.

Discrete Mathematics and Probability — Prof Welsh — 12MT)

1–4 Basic concepts of counting, permutations and combinations, ordered selections, the binomial theorem (for positive integer exponents). Recurrence relations; elementary treatment of generating functions and their use in solving recurrence relations. Inclusion-exclusion principle, derangements.

5–7 Motivating probabilistic models, relative frequency, chance (what do we mean by a 1 in 4 chance?). Sample space as set of all possible outcomes - examples. Events and probability function. Examples using counting methods, sampling with and without replacement. Algebra of events. Conditional probability, partitions of sample space, theorem of total probability, Bayes' theorem, independence. Examples with statistical implications.

8–12 Random variables. Probability mass function. Discrete distributions: Bernoulli, binomial, Poisson, geometric, situations in which these distributions arise. Expectation: mean and variance. Probability generating functions, use in calculating expectations. Bivariate discrete distribution, conditional and marginal distributions. Extension to many random variables. Independence for discrete random variables. One-dimensional random walks (finite state space only), examples using recurrence relations.

Further Probability — Dr A D Lunn — 8 HT)

1–3 Expectations of functions of more than one random variable. Random sample. Conditional expectation, application of theorem of total probability to expectation of a random variable. Sums of independent random variables. Probability generating function for (fixed length) sum of independent, identically distributed random variables. Indicator random variables. Examples from well-known distributions.

4–8 Continuous random variables, motivation. Cumulative distribution function for both discrete and continuous random variables. Probability density function - analogy with mass and density of matter. Examples: uniform, exponential, gamma, normal. Practical examples. Expectation. Cdf and pdf for function of a single continuous random variable. Simple examples of joint distributions of 2 or more continuous random variables - independence - expectation (mean and variance of sums of independent, identically distributed random variables). [Excludes change of variable and moment generating functions]

Reading

D Stirzaker, *Probability and Random Variables: A Beginner's Guide*, CUP (1999)

D Stirzaker, *Elementary Probability*, CUP (1994), Chs 1–4, 5.1–5.6, 6.1–6.3, 7.1–7.2, 7.4, 8.1, 8.3, 8.5 (exclude joint generating function)

Additional Reading

J Pitman, *Probability*, Springer-Verlag (1993)

S Ross, *A First Course in Probability*, Prentice-Hall (1994 or later)

G R Grimmett and D J A Welsh, *Probability: An Introduction*, OUP (1986), Chs 1–4, 5.1–5.4, 5.6, 6.1, 6.2, 6.3 (parts of), 7.1–7.3, 10.4

2.3.3 Statistics — Teaching Responsibility of the Statistics Department — Dr Laws — 8 HT

Aims

The theme is the investigation of real data using the method of maximum likelihood to provide point estimation, given unknown parameters in the models. Maximum likelihood will be the central unifying approach. Examples will involve a distribution with a single unknown parameter, for which confidence intervals may be found by using the central limit theorem (stated only). The culmination of the course will be the link of maximum likelihood technique to the simple straight line fit with normal errors.

Synopsis

Random sample, concept of a statistic and its distribution, sample mean as a measure of location and sample variance as a measure of spread. Concept of likelihood - examples of likelihood for simple distributions. Estimation for a single unknown parameter by maximising likelihood. Examples drawn from: Bernoulli, Binomial, Geometric, Poisson, Exponential (parameterised by mean), Normal (mean only, variance known). Data to include simple surveys, opinion polls, archaeological studies etc. Properties of estimators - unbiasedness, Mean Squared Error (=(bias)^2 + variance). Statement of Central Limit Theorem (excluding proof). Confidence intervals using CLT. Simple straight line fit, Y_t = a + bx_t + ε_t, with ε_t normal independent errors of zero mean and common known variance.

Estimators for a, b by maximising likelihood using partial differentiation, unbiasedness and calculation of variance as linear sums of Y_t. (No confidence intervals). Examples (use of scatterplots to show suitability of linear regression).

Reading

Material can be found in:

F Daly, D J Hand, M C Jones, A D Lunn and K J McConway, *Elements of Statistics*, Addison Wesley, 1995 (chs 1–5 give background including plots and summary statistics, ch. 6 and parts of ch 7 are directly relevant).

Additional reading

J A Rice, *Mathematical Statistics and Data Analysis*, Wadsworth and Brooks Cole, 1988.

2.4 Paper 4: Physical Applied Mathematics

2.4.1 Geometry — Dr Day — 10 MT

Aims

The aim of the lectures in Geometry is to provide a geometrical underpinning for the Mechanics part of the course and to solve a variety of purely geometrical problems, often with the aid of vectors. We introduce the *vector product*, a way of multiplying vectors, which is indispensable in problems involving rotation. We discuss *conics* or *conic sections*, a class of curves studied by the Greeks for their geometric interest but which turn out to be the orbits of planets or comets moving under gravity.

Synopsis

1–3 Euclidean geometry of two and three dimensions ($\mathbb{R}^2$, $\mathbb{R}^3$), approached by vectors and coordinates. Vector addition and scalar multiplication. The scalar product, equations of lines and circles.

3–5 The vector product in three dimensions. Scalar triple products and vector triple products, vector algebra.

6–8 Conics in $\mathbb{R}^2$, defined using focus and directrix. Equations of conics in Cartesian and polar coordinates

8–10 Isometries in two and three dimensions. Orthogonal matrices, rotations and reflections. Rotating frames and angular velocity.

Reading

J. Roe, *Elementary Geometry*, OUP (1992). Chs. 1, 2.2, 3.4, 4, 7.1, 7.2, 8.1–8.3

2.4.2 Mechanics — Dr Acheson — 14 MT

Aims

Classical mechanics lies at the heart of much of applied mathematics, and this course aims to introduce some of the key ideas. A major theme is the modelling of a physical system by a set of differential equations, and one of the highlights involves using the law of gravitation to account for the motion of the planets.

Synopsis

1–4 Motion of a particle; forces, acceleration, Newton's laws of motion. Elementary examples, including the harmonic oscillator and the simple pendulum. Stable and unstable equilibria; examples using linearized equations.

5–9 Motion under a central force, including the inverse square law of gravitation. Conservation of angular momentum. Differential equation for the particle path. Planetary orbits, path of a comet.

10–12 Kinetic energy, conservative forces and potential energy. Application to central force problems and to problems involving 3D motion.

13-14 Nonlinear 1D first-order dynamical systems in discrete time. The discrete logistic equation. Stable and unstable fixed points, periodic solutions and chaos.

Reading

D Acheson, *From Calculus to Chaos; an Introduction to Dynamics*, OUP (1997), Chs 1, 5, 6, 10, 11

M Lunn, *A First Course in Mechanics*, OUP (1991), Chs 1-4

D W Jordan and P Smith, *Mathematical Techniques*, 2nd edition, OUP (1997), Ch 37

2.4.3 Waves and Diffusion — Dr Acheson — 12 HT

Aims

Aims to introduce students to the use of Fourier Series and separation of variables in finding simple solutions of partial differential equations such as the wave equation, the heat equation and Laplace's equation. Introduces basic concepts of modelling in continuum mechanics which lead to these partial differential equation problems, and the ideas of existence and uniqueness of solutions to such problems.

Synopsis

1–4 Formulation of the wave equation for a stretched string. Finite string, standing wave. Separation of variables. Fourier series on $(-\pi, \pi)$, statement of main theorem, odd and even functions, sine and cosine series. Normal modes for finite string. Superposition of normal modes and solution of the initial value problem.

5–8 General solution of the wave equation. Wave profile and wave velocity for a travelling wave. D'Alembert's solution for the infinite string. Uniqueness of solution of the initial value problem. Examples illustrating wave-fronts and characteristic diagrams (excluding reflection and transmission). Energy and uniqueness of solution.

9–12 Applications of separation of variables technique to other models involving partial differential equations. 1-dimensional unsteady heat flow - including steady state solution. Separation of variables technique applied to Laplace's equation in 2-dimensions for circular and rectangular regions.

Reading

W E Boyce and R C Prima, *Elementary Differential Equations and Boundary Value Problems*, 6th edition, Wiley (1997), Ch 10

E Kreyszig, *Advanced Engineering Mathematics*, 8th edition, Wiley (1988), Chs 10.1–10.5, 11.1–11.5

2.5 Paper 5: Mathematical Methods and Models

2.5.1 Mathematical Methods — Dr Dyson — 20 HT

Aims

The course deals with calculus in more than one dimension. It aims to give students a grounding in some of the basic tools needed to study applied mathematics both later in the first year and in subsequent years. The emphasis is on applying techniques and the approach is not intended to be rigorous.

Synopsis

1–5 Partial differentiation: Taylor's theorem, critical points, Lagrange multipliers. The Jacobian.

6–10 Multiple integrals. Line, volume and surface integrals. Change of variable using Jacobian. Divergence theorem in 2D.

11–15 Vector operators. Grad, div and curl and related formulae. Examples in Cartesians and with radial symmetry in 2 or 3D but excluding general curvilinear coordinates.

16–20 Divergence theorem and Green's formulae. Uniqueness theorem for Laplace's equation. Stokes' theorem etc. Line integral of a gradient and path independence.

Reading

D E Bourne and P C Kendall, *Vector Analysis and Cartesian Tensors*, 3rd edition, Chapman and Hall (1992), Chs 4, 5, 6, 7.1–7.2

E Kreyszig, *Advanced Engineering Mathematics*, 8th Edition, Wiley (1999), Chs 8, 9, 11.5, 11.11

H M Schey, *div grad curl and all that: an informal text on vector calculus*, W W Norton and Company (1992)

Further Reading

A S Ramsey, *Newtonian Attraction*, CUP (1940), Chs 2.1-2.5, 3.1-3.3, 4.1-4.5 — out of print (scarce)

2.5.2 Mathematical Models — Dr Day — 16 TT

Aims

In this course we aim to show how to model continuous physical systems and how, having constructed a model, the powerful tools of mathematics can be applied to predict and explain physical phenomena. As most models are constructed in the same way we look in some detail at the construction of models for heat and for gravitation. We show that global heat conservation can be expressed very neatly in terms of local properties, the continuity equation. All physical systems have global conservation laws: mass, charge, momentum, angular momentum, energy, in quantum mechanics probability etc. and for all of these there will be corresponding continuity equations.

Synopsis

1–4 Diffusion. Derivation of continuity equation for any flow and specialization to heat and chemical concentrations (including unsteady, 3D and sources). Examples.

5–8 Steady diffusion for 1D problems (including 2 and 3D with radial symmetry). Point and line sources. Superposition. Boundary conditions.

9–12 Gravitation/Potential Theory. Definition of gravitational field. Proof of existence of a potential for a conservative field. Field/potential due to point masses, and simple bodies using $\phi = \int \frac{\gamma\rho}{r}\,\mathrm{d}T$. Mention of electrostatics and potential flow in fluids.

13–16 Gauss' Theorem and Poisson's Equation. Solutions for spheres, cylinders etc. Equivalence of Poisson and $\phi = \int \frac{\gamma\rho}{r}\,\mathrm{d}T$. [Gravitational potential energy is excluded.]

Reading

D E Bourne and P C Kendall, *Vector Analysis and Cartesian Tensors*, 3rd edition, Chapman and Hall, Chs 4, 5, 6, 7.1-7.2

I S Sokolnikov and R M Redheffer, *Mathematics of Physics and Modern Engineering*, McGraw Hill (1980), Chs. 5, 6.1-6.17 – out of print

E Kreyszig, *Advanced Engineering Mathematics,* 8th Edition, Wiley (1993), Chs 8, 9, 11.5, 11.6, 11.9, 11.11

Further reading

W A Strauss, *Partial Differential Equations*, Wiley (1992), Chs. 1.1-1.5

A S Ramsey, *Newtonian Attraction*, CUP (1940), Chs. 2.1-2.5, 3.1-3.3, 4.1-4.5 – out of print (scarce)

2.5.3 Exploring Mathematics with Maple — Dr Stewart and Dr Lackenby — 16 MT and 16 HT

Background

Mathematicians (like other professionals) use a wide range of generic computer packages: email, word-processors, web-browsers, spreadsheets, database managers and so on. Many, if not most, of the students on the Oxford Mathematics courses will have already used some of these packages; and are encouraged to use the facilities available centrally and in colleges to continue to develop their skills with these during their course.

The use by mathematicians of software developed for handling specific sorts of mathematical problems, especially numerical ones, is well-established; lecture courses in later years will, where appropriate, introduce students to these applications.

Increasingly professional mathematicians use general purpose mathematical packages; sometimes these are called symbolic calculators, or algebraic manipulation packages. Such a package can be used as a super graphics calculator, as a scratchpad, or as a handbook of mathematical functions; its virtue is flexibility. Maple, used in this introductory course, is a good example of such a package.

Aims and Objectives

The aim of the course is to demonstrate the potential of general purpose mathematical packages; to allow students to gain familiarity with one of them (Maple V, Release 5); to provide a tool which can be used in the later years of the course.

By the end of Michaelmas term students should be able to

(i) use the Unix system;

(ii) edit, save, and use Maple worksheets;

(iii) manipulate expressions in Maple, and plot simple graphs using Maple;

(iv) write simple programs in Maple for solving problems in algebra, calculus, and applied mathematics.

By the end of Hilary term students should be able to

(i) use the linalg Maple package;

(ii) complete two or three small projects exploring some mathematical problem using Maple;

(iii) provide reports on the projects in the form of commented Maple worksheets.

Synopsis

The Michaelmas term work consists of:

Using the workstation: accounts, passwords, logging in/out; keyboard layout; X-windows environment. Files and directories. Introduction to computer algebra systems: Maple V, Release 5; worksheets. Using Maple as a calculator. Manipulation of algebraic formulae. Sets, arrays, tables, and lists. Solution of algebraic equations. Linear Algebra in Maple. Calculus in Maple. Simple graphics. Elementary programming in Maple.

The Hilary term work is based on a menu of mathematical projects; the current list is printed in the second part of the Maple Course Manual.

Access to the system

Undergraduates use the workstations in the Mathematical Institute Computer Laboratory (G17). You must register as a user of the Mathematical Institute network; arrangements will be made to ensure that, as far as possible, you are allocated an account before MT lectures begin.

Students may also access the system through college or individual computers; for details of how to do this they should consult the computing support at their own college. The Maple package may also be installed and used on personally owned computers under the University's site licence; the projects, however, must be submitted through the Maple system on the Mathematical Institute network.

Teaching and Assessment

The course deliberately relies heavily on self-teaching through practical exercises, supported by demonstrations. A manual for the course and a series of worksheets with examples to be worked will be provided.

You will be timetabled for 4 practicals of 2 hours each in Michaelmas term. You will work alone on the projects in Hilary term.

The Moderators in Mathematics are required, when assessing the performance of candidates, to take into account your work on the Maple course. The projects will count as the equivalent of two questions.

The logging system of the Mathematics Institute network is used to monitor the work of students on their worksheets.

Reading

Exploring Mathematics with Maple: Students' Guide (Mathematical Institute notes — available from reception).

W Burkhardt, *First Steps in Maple* (pbk.) (Springer-Verlag, 1994)

E Kreyszig and E J Norminton, *Maple Computing Manual for Advanced Engineering Mathematics* (Wiley, 1994) (For college reference.)

3 APPENDIX - Mathematical Sciences (3-year course) and Mathematics (4-year course)

Undergraduates are urged to consult their tutors as to whether the three-or four-year option would be better for their chosen path.

Excerpt from the Mathematics Course Committee Reports, as approved by the Subfaculty of Mathematics

Selective admission to the four-year course

Undergraduates will be instructed to consult their tutors early in the Michaelmas Term following Moderations and to apply by Wednesday of Week 2 of that term, if they wish to take the four-year course. The Adjudication Committee will then determine, taking resources into consideration, the number that can be admitted on to the four-year course and hence whether a selection procedure has to be put fully in place for that year. The selection procedure, if it were required, would be carried out by the Adjudication Committee, which is chaired by the Vice-Chairman of the Sub-faculty of Mathematics; the other members of the committee would be two Moderators nominated by the Chairman of the Moderators (one in Pure Mathematics and one in Applied Mathematics) and two other members (again one in Pure Mathematics and one in Applied Mathematics). The procedure would be as follows: one or two college references (at the candidate's choice) would be requested for the end of Week 4 of Michaelmas Term; decisions (no appeals) would be communicated to candidates and colleges by the end of Week 7 of Michaelmas Term.