Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part B 2005–6
for examination in 2006

Contents

1 Foreword

Important Information

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Assistant in the Mathematical Institute.

1.1 Honour School of Mathematics

[See the current edition of the *Examination Regulations* for the full regulations governing these examinations.]

In Part B each candidate shall offer a total of four units from the schedule of units and half-units.

(a) A total of at least three units offered should be from the schedule of 'Mathematics Department units and half-units'

(b) At most one unit may be offered from those designated as M-level.

(c) Candidates may offer at most one unit which is designated as an Extended Essay[1]

Details of units for Part C will be published in 2006. Students staying on to take the four-year course will take 3 units from Part C in their fourth year.

In the classification awarded at the end of the degree, marks in Part A will be given a 'weighting' of 2, and marks in Part B will be given a 'weighting' of 3.

For those students staying on to do the fourth year, the Part C marks will also be taken into account with a 'weighting' of 4. Students wishing to take the four-year course should register to do so at the beginning of their third year. Details will be sent to tutors and students.

YOU MUST REGISTER BY THE END OF TRINITY TERM 2005 FOR CLASS ATTENDANCE FOR ALL COURSES YOU WISH TO TAKE IN 2005-06. A REGISTRATION FORM IS ATTACHED TO THESE SYNOPSES.

SOME COMBINATIONS OF SUBJECTS ARE NOT ADVISED AND LECTURES IN THESE SUBJECTS MAY CLASH. STUDENTS WHO NEVERTHELESS WISH TO TAKE ANY OF THESE COMBINATIONS OF LECTURES SHOULD NOTIFY THE ACADEMIC ADMINISTRATOR, MATHEMATICAL INSTITUTE, BY FRIDAY OF WEEK 6 IN THE TERM PRECEDING THE LECTURES. SEE SECTION 6 "REGISTRATION FOR PART B COURSES 2005-06" FOR DETAILS.

Language Classes Students taking Mathematics are also invited to apply to take classes in a foreign language. In 2005-06 classes are offered in French. Students' performance in these classes will not contribute to the degree classification in Mathematics. However, successful completion of the course may be recorded on student transcripts. See section 6 for more details.

[1] Units which may be offered under this heading are indicated in the synopses.

1.2 Honour School of Mathematics and Philosophy

In Part B each candidate shall offer

(i) a total of at least two units in *Mathematics* from the schedule of 'Mathematics Department units and half-units' for the Honour School of Mathematics of which one must be *Foundations* (B1: Logic and Set Theory) as specified for the Honour School of Mathematics, and

(ii) at least three subjects in *Philosophy* from subjects 101-18, 120-2 as prescribed in the Regulations of Philosophy in all Honour Schools including Philosophy, of which two must be subjects 102 and 122, such that

(iii) the total number of units and subjects together is six.

At most one unit in *Mathematics* at M-level is permitted in Part B.

Each subject in *Philosophy* shall be examined in one 3-hour paper.

Foundations (B1) is normally taken in the second year. [This is the same as the course B1: Logic and Set Theory given in 2004-05. Students in Mathematics and Philosophy will usually take this course in their second year.]

YOU MUST REGISTER BY THE END OF TRINITY TERM 2005 FOR CLASS ATTENDANCE FOR MATHEMATICS COURSES YOU WISH TO TAKE IN 2005-06. A REGISTRATION FORM IS ATTACHED TO THESE SYNOPSES.

SOME COMBINATIONS OF SUBJECTS ARE NOT ADVISED AND LECTURES IN THESE SUBJECTS MAY CLASH. STUDENTS WHO NEVERTHELESS WISH TO TAKE ANY OF THESE COMBINATIONS OF LECTURES SHOULD NOTIFY THE ACADEMIC ADMINISTRATOR, MATHEMATICAL INSTITUTE, BY FRIDAY OF WEEK 6 IN THE TERM PRECEDING THE LECTURES. SEE SECTION 6 "REGISTRATION FOR PART B COURSES 2005-06" FOR DETAILS.

1.3 "Units" and "half-units" and methods of examination

Most subjects offered have a 'weight' of one unit, and will be examined in a 3-hour examination paper. In many of these subjects it will also be possible to take the first half, or either half, of the subject as a 'half-unit'. Where this is the case, a half-unit will usually be examined in an examination paper of $1\frac{3}{4}$-hours.

Rubrics on 3-hour examination papers

The rubric on 3-hour examination papers will usually be: "Submit answers to a maximum of five questions. Submit at least one answer in each section (half-unit) of the paper. The best answer in each section will count, along with the two best of the remaining answers".

Rubrics on 1 $\frac{3}{4}$-hour examination papers

The rubric on 1 $\frac{3}{4}$-hour examination papers will usually be: "Submit answers to a maximum of three questions. The best two answers will be counted".

2 Mathematics Department units and half-units

2.1 B1: Logic and Set Theory

NB: This course is 'Foundations' in the Honour School of Mathematics and Philosophy, where it should usually be taken in the second year.

Level: H-level **Method of Assessment:** 3-hour or $1\frac{3}{4}$-hour examination

Weight: One unit, or can be taken as either a half-unit in Logic or a half-unit in Set Theory.

Quota: Not imposed this year.

2.1.1 Part I: Logic — Prof Wilkie — 16 MT

[Option **B1a** if taken as a half-unit.]

Aims & Objectives To give a rigorous mathematical treatment of the fundamental ideas and results of logic that is suitable for the non-specialist mathematicians and will provide a sound basis for more advanced study. Cohesion is achieved by focusing on the Completeness Theorems and the relationship between probability and truth. Consideration of some implications of the Compactness Theorem gives a flavour of the further development of model theory. To give a concrete deductive system for predicate calculus and prove the Completeness Theorem, including easy applications in basic model theory.

Synopsis The notation, meaning and use of propositional and predicate calculus. The formal language of propositional calculus: truth functions; conjunctive and disjunctive normal form; tautologies and logical consequence. The formal language of predicate calculus: satisfaction, truth, validity, logical consequence.

Deductive system for propositional calculus: proofs and theorems, proofs from hypotheses, the Deduction Theorem; Soundness Theorem. Maximal consistent sets of formulae; completeness; constructive proof of completeness.

Statement of Soundness and Completeness Theorems for a deductive system for predicate calculus; derivation of the Compactness Theorem; simple applications of the Compactness Theorem.

A deductive system for predicate calculus; proofs and theorems; prenex form. Proof of Completeness Theorem. Existence of countable models, the downward Löwenheim-Skolem Theorem.

Reading

- R Cori and D Lascar, *Mathematical Logic: A Course with Exercises (Part I)* Oxford University Press (2001), sections 1, 3, 4.

- A G Hamilton, *Logic for Mathematicians*, 2nd edition, CUP (1988), pp.1-69, pp.73-76 (for statement of Completeness (Adequacy)Theorem), pp.99-103 (for the Compactness Theorem).
- W B Enderton, *A Mathematical Introduction to Logic*, Academic Press (1972), pp.101-144
- D Goldrei, *Propositional and Predicate Calculus: A model of argument*, Springer (2005)

Further Reading

- R Cori and D Lascar, *Mathematical Logic: A Course with Exercises (Part II)* Oxford University Press (2001), section 8.

2.1.2 Part II: Set Theory — Dr Priestley — 16 HT

[Option **B1b** if taken as a half-unit.]

Aims & Objectives To introduce sets and their properties as a unified way of treating mathematical structures, including encoding of basic mathematical objects using set theoretic language. To emphasize the difference between intuitive collections and formal sets. To introduce and discuss the notion of the infinite, the ordinals and cardinality. To consider the Axiom of Choice and its implications.

Synopsis What is a set? Introduction to the basic axioms of set theory. Ordered pairs, cartesian products, relations and functions. Axiom of Infinity and the construction of the natural numbers; induction and the recursion theorem.

Cardinality; the notions of finite and countable and uncountable sets; Cantor's Theorem on power sets. The Tarski Fixed Point theorem. The Schröder-Bernstein Theorem.

Isomorphism of ordered sets; well-orders. Transfinite induction; transfinite recursion [informal treatment only].

Comparability of well-orders.

The Axiom of Choice, Zorn's Lemma, the Well-ordering Principle; comparability of cardinals. [The proofs that (WO) implies (AC); (ZL) implies (AC); (ZL) implies (CC) are included; brief discussion will also be given of the method of proof of additional implications, sufficient to establish equivalence.] Ordinals. Arithmetic of cardinals and ordinals [the proof that, in [ZFC], $\kappa.\kappa = \kappa$ for an infinite cardinal κ is excluded].

Reading

- D Goldrei, *Classic Set Theory*, Chapman and Hall (1996)
- W B Enderton, Elements of Set Theory, Academic Press (1978)

Further Reading

- R Cori and D Lascar, *Mathematical Logic: A Course with Exercises (Part II)* Oxford University Press (2001), section 7.1–7.5.

2.2 B2 Algebra

Level: H-level **Method of Assessment:** 3-hour examination

Prerequisites: All second-year algebra.

Weight: One Unit

Quota: Not imposed this year.

Aims & Objectives The intention is to present an integrated unit. The first half extends some of the general ideas studied separately before for groups and particular commutative rings to a more unified setting. This leads on to basic study of representation theory as a generalisation of the permutation representations of groups studied in the second year; modules are introduced, and they are studied in the context of representations of semisimple algebras and of modules over Euclidean domains.

The second half specialises to the study of finite groups and their characters, using the results of the first half, and culminating in the one classical application of character theory, Frobenius' theorem, that to this day has not yielded to a purely group theoretic proof.

2.2.1 Part I: Algebras — Dr Erdmann — 16 MT

Synopses Noncommutative rings, one- and two-sided ideals. Associative algebras (over fields). Main examples matrix algebras, polynomial rings and quotients of polynomial rings. Group algebras, representations of groups, with examples coming from permutation actions.

Modules, relationship with representations. Finite dimensional modules over algebras; irreducible (simple) and indecomposable modules, composition series, Jordan-Hölder theorem. Semisimple algebras and their representations; the Wedderburn theorem, Maschke's theorem and the 'classification' of all representations in terms of simple modules.

Modules over Euclidean domains; cyclic decomposition for finitely generated modules. Application to finitely generated abelian groups and canonical forms for linear transformations (Jordan form, rational canonical form).

[Lectures split 4+6 +6]

Reading P M Cohn, *Algebra.* 2nd edition, Chichester : Wiley (1982-1991). Parts I and II contain much of the course.

For the last part:

B Hartley and T O Hawkes, *Rings, Modules and Linear Algebra* London : Chapman & Hall (1970).

M E Keating, *A first course in module theory* London : Imperial College Press (c1998).

2.2.2 Part II: Finite Groups and Characters— Dr Neumann — 16 HT

Synopses Some classic theorems of finite group theory: Cauchy's Theorem, Sylow's Theorems, the Jordan–Hölder Theorem.

Simplicity of alternating groups for degree not 4. Soluble groups and their characterisation by composition factors. Semidirect products. The groups of small orders.

Complex representations of a finite group, Maschke's Theorem, simple modules and Schur's Lemma, analysis of the regular representation module. Characters of a finite group. Orthogonality of irreducible characters. Character tables. Characters of permutation representations. Examples.

Induced representations and characters; the Frobenius Reciprocity Theorem. Frobenius groups; easy examples as semidirect products. Frobenius' theorem that all Frobenius groups are semidirect products.

Reading Almost everything is covered by:

J I Alperin and Rowen B Bell, *Groups and representations*, Springer-Verlag (GTM 162) 1995; ISBN 0-387-94526-1.

Alternative reading:

Joseph J Rotman, *An introduction to the theory of groups* (4th edition), Springer-Verlag (GTM 148) 1995; ISBN 3-540-94285-8; Ch 4, and parts of Chapters 3, 5, 7.

Gordon James and Martin Liebeck, *Representations and characters of finite groups*, (2nd edition), CUP 2001; ISBN 0-521-00392-X.

W Ledermann, *Introduction to group theory*, Longman (Oliver & Boyd) 1973; ISBN 0-582-44180-3.

W Ledermann, *Introduction to group characters*, CUP 1977; ISBN 0-521-29170-4.

2.3 B3 Geometry

Level: H-level **Method of Assessment:** 3-hour or $1\frac{3}{4}$-hour examination

Weight: One unit, or can be taken as either a half-unit in Geometry of Surfaces or a half-unit in Algebraic Curves (but see "Prerequisites").

Prerequisites: 2nd year core algebra and analysis, 2nd year topology. Multivariable calculus and groups in action would be useful but not essential. Also, Part I is helpful, but not essential, for Part II.

Quota: Not imposed this year.

2.3.1 Part I: Geometry of Surfaces — Prof Hitchin — 16 MT

[Option **B3a** if taken as a half-unit.]

Aims and Objectives Different ways of thinking about surfaces (also called two-dimensional manifolds) are introduced in this course: first topological surfaces and then surfaces with extra structures which allow us to make sense of differentiable functions ('smooth surfaces'), holomorphic functions ('Riemann surfaces') and the measurement of lengths and areas ('Riemannian 2-manifolds').

These geometric structures interact in a fundamental way with the topology of the surfaces. A striking example of this is given by the Euler number, which is a manifestly topological quantity, but can be related to the total curvature, which at first glance depends on the geometry of the surface.

The course ends with an introduction to hyperbolic surfaces modelled on the hyperbolic plane, which gives us an example of a non-Euclidean geometry (that is, a geometry which meets all Euclid's axioms except the axioms of parallels).

Synopsis The concept of a topological surface (or 2-manifold); examples, including polygons with pairs of sides identified. Orientation and the Euler characteristic. Classification theorem for compact surfaces (the proof will not be examined).

Riemann surfaces; examples, including the Riemann sphere, $\mathbb{C}/\Lambda$ where Λ is a lattice in $\mathbb{C}$, complex algebraic curves in $\mathbb{C}^2$. Informal discussion of multivalued holomorphic functions, branched covers and the Riemann-Hurwitz formula.

Smooth surfaces in $\mathbb{R}^3$ and their first fundamental forms. The concept of a Riemannian 2-manifold; isometries; Gaussian curvature.

Geodesics. The Gauss-Bonnet Theorem (statement of local version and deduction of global version). Critical points of real-valued functions on compact surfaces.

The hyperbolic plane, its isometries and geodesics. Compact hyperbolic surfaces as Riemann surfaces and as surfaces of constant negative curvature.

Reading A Pressley, *Elementary Differential Geometry, Springer Undergraduate Mathematics Series*, 2001 (Chapters 4-8 and 10-11)

GB Segal, *Geometry of Surfaces*, Mathematical Institute Notes (1989)

R Earl, *The Local Theory of Curves and Surfaces*, Mathematical Institute Notes (1999)

J McCleary, *Geometry from a Differentiable Viewpoint*, Cambridge 1997.

Further Reading PA Firby and CE Gardiner, *Surface Topology*, Ellis Horwood 1991 (Chapters 1-4 and 7)

F Kirwan, *Complex Algebraic Curves*, London Mathematical Society Student Texts 23, Cambridge 1992 (Chapter 5.2 only)

B O'Neill, *Elementary Differential Geometry,* Academic Press 1997.

2.3.2 Part II: Algebraic Curves — Dr Dancer — 16 HT

[Option **B3b** if taken as a half-unit.]

Aims and Objectives A real algebraic curve in the plane $\mathbb{R}^2$ is a subset of $\mathbb{R}^2$ defined by a polynomial equation $P(x, y) = 0$; a complex algebraic curve is defined similarly with $\mathbb{C}$ instead of $\mathbb{R}$. Real algebraic curves have been studied for more than two thousand years, although it was not until the introduction of the systematic use of coordinates into geometry in the seventeenth century that they could be described in the form $\{(x, y) \in \mathbb{R}^2 : P(x, y) = 0\}$. Once complex numbers were recognised as acceptable mathematical objects it quickly became clear that complex algebraic curves have at once simpler and more interesting properties than real algebraic curves. In this course algebraic curves over $\mathbb{C}$ (or more generally over any algebraically closed field of charactistic 0) are studied, using ideas from algebra, from topology and from complex analysis.

Synopsis Projective spaces, homogeneous and inhomogeneous coordinates, projective transformations (all over an algebraically closed field k of characteristic 0).

Algebraic curves in the affine plane. Projective curves in PP^2. Points at infinity. Euler's relation. Irreducible components. Singular and nonsingular points, tangent lines.

Bezout's theorem for curves which intersect transversely (the proof will not be examined), with informal discussion of intersection multiplicities. Pascal's mystic hexagon. Points of inflection.

Branched covers of PP^1. The degree-genus formula.

Holomorphic differentials on a nonsingular curve. Divisors and degrees. Canonical divisors. Statement of the Riemann-Roch Theorem. Geometric genus equals topological genus.

Elliptic curves. The Weierstrass $\mathfrak{p}$-function and the cubic curve associated to a lattice. The group law on a nonsingular cubic.

Reading F Kirwan, *Complex Algebraic Curves*, LMS Student Texts 23, Cambridge 1992, Chapters 2-6.

2.4 B4 Analysis

Level: H-level **Method of Assessment:** 3-hour or $1\frac{3}{4}$-hour examination

Prerequisites: For the first half of B4, Topology is desirable and Integration is useful.

For the whole unit, Topology is desirable and Integration is desirable.

Weight: One unit, or the Michaelmas Term course may be taken alone as a half-unit.

Quota: Not imposed this year.

Aims and Objectives Many problems in analysis are best discussed and solved using infinite-dimensional vector spaces of functions. For example, the set of smooth functions on $\mathbb{R}$ is an infinite-dimensional vector space, and differentiation is a linear map upon it. So, for a deeper understanding of analysis we need a good theory of infinite-dimensional vector spaces.

The two most important kinds of infinite-dimensional vector space are Banach spaces and Hilbert spaces; they provide the theoretical underpinnings for much of differential equations, and also for quantum theory in physics. This course provides an introduction to Banach spaces and Hilbert spaces. It combines familiar ideas from topology and linear algebra. It would be useful background for further work in analysis, differential equations, and so on.

2.4.1 Analysis I — Prof Haydon — 16 MT

[Option **B4a** if taken as a half-unit.]

Synopses Real and complex normed vector spaces, their geometry and topology. Completeness. Banach spaces, examples (ℓ^p, ℓ^∞, L^p, $C(K)$, spaces of differentiable functions).

Finite-dimensional normed spaces; equivalence of norms and completeness. Separable spaces; separability of subspaces.

Continuous linear functionals. Dual spaces. Hahn-Banach Theorem (proof for real separable spaces only) and applications including density of subspaces.

Bounded linear operators, examples (including integral operators). Adjoint operators. Spectrum and resolvent. Spectral mapping theorem for polynomials.

Reading *Essential*

E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley (revised edition, 1989), Chs 2, 4.2-4.3, 4.5, 7.1-7.4

Further

G.F. Vincent-Smith, *B4: Analysis, Mathematical Institute Notes (1991)*, Chs 1, 2, 5.2

2.4.2 Analysis II — Dr Edwards — 16 HT

Synopses Hilbert spaces; examples including L^2 spaces. Orthogonality, orthogonal complement, closed subspaces, projection theorem. Riesz Representation Theorem.

Orthonormal sets, Pythagoras, Bessel's inequality. Complete orthonormal sets, Parseval.

Orthogonal expansions, examples (Legendre, Laguerre, Hermite etc.) Classical Fourier series: Riemann-Lebesgue lemma; Dirichlet kernel, pointwise convergence of Fourier series, Dini's test, Fejér's theorem, Weierstrass' approximation theorem. Completeness of trigonometric system.

Operators on Hilbert space, adjoint operators. Self-adjoint operators, orthogonal projections, unitary operators, and their spectra.

Reading

Essential

E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley (revised edition, 1989), Ch 3

N. Young, *An Introduction to Hibert Space*, CUP (1988), Chs 1-7

Further

G.F. Vincent-Smith, *B4 analysis*, Mathematical Institute Notes (1991), Chs 3, 4

H.A. Priestley, *Introduction to Integration*, OUP (1997), Chs 28-32

A. Vretblad, *Fourier Analysis and its Applications*, Springer (2003).

2.5 B5 Applied Analysis

Level: H-level **Method of Assessment:** 3-hour or $1\frac{3}{4}$-hour examination

Prerequisites: Calculus of Variations and Fluid Mechanics from Part A are desirable but not essential.

Weight: One unit, or can be taken as either a half-unit in Techniques of Applied Mathematics or a half-unit in Applied Partial Differential Equations.

Quota: Not imposed this year.

2.5.1 Part I: Techniques of Applied Mathematics — Dr Fowler — 16 MT

[Option **B5a** if taken as a half-unit.]

Aims & Objectives This course develops mathematical techniques which are useful in solving 'real-world' problems involving differential equations, and is a development of ideas which arise in the second year differential equations course. The course aims to show in a practical way how equations 'work', what kinds of solution behaviours can occur, and some techniques which are useful in their solution.

Synopsis Introduction to mathematical modelling.

Perturbation methods: Poincaré-Lindstedt, simple boundary layers.

Ordinary differential equations: stability, oscillations.

Sturm-Liouville systems, comparison methods. Integral equations and eigenfunctions.

Partial differential equations: shocks, similarity solutions.

Reading A.C. Fowler 2005 *Techniques of Applied Mathematics.* Mathematical Institute lecture notes.

J.P. Keener 2000 *Principles of Applied Mathematics: Transformation and Approximation,* revised edition. Perseus Books, Cambridge, Mass.

E.J. Hinch 1991 *Perturbation Methods.* CUP, Cambridge.

J.R. Ockendon, S. D. Howison, A.A. Lacey and A.B. Movchan 2003 *Applied Partial Differential Equations,* revised edition. OUP, Oxford.

R. Haberman 1998 *Mathematical Models.* SIAM, Philadelphia.

S.D. Howison 2005 *Practical Applied Mathematics: Modelling, Analysis, Approximation.* CUP, Cambridge (UK).

2.5.2 Part II: Applied Partial Differential Equations — Dr Norbury — 16 HT

[Option **B5b** if taken as a half-unit.]

Aims & Objectives This course continues the Part A Differential Equations course, and extends some of the techniques of B5a, to partial differential equations. In particular, general nonlinear first order partial differential equations are solved, the classification of second order partial differential equations is extended to systems, with hyperbolic systems being solved by characteristic variables. Then Green's function, maximum principle and similarity variable methods are demonstrated for partial differential equations, together with eigenfunction expansions.

Synopsis Charpit's equations; eikonal equation.

Systems of partial differential equations, characteristics. Weak solutions. Riemann's function.

Maximum principles, comparison methods, well-posed problems, and Green's functions for the heat equation and for Laplace's equation.

Delta functions. Eigenfunction expansions.

Reading Web notes are available(JN).

Institute lecture notes will be available (JN).

M. Renardy and R.C. Rogers *An introduction to partial differential equations*, 2004, Springer-Verlag, New York.

J.P. Keener 2000 *Principles of Applied Mathematics: Transformation and Approximation*, revised edition. Perseus Books, Cambridge, Mass.

J.R. Ockendon, S. D. Howison, A.A. Lacey and A.B. Movchan 2003 *Applied Partial Differential Equations*, revised edition. OUP, Oxford.

2.6 B6 Theoretical Mechanics

Level: H-level **Method of Assessment:** 3-hour or $1\frac{3}{4}$-hour examination

Weight: One unit, or can be taken as either a half-unit in Viscous Flow, or a half-unit in Waves and Compressible Flow.

Prerequisites: The Part A (second-year) course 'Fluid Dynamics and Waves'. Though each half-unit is intended to stand alone, they will complement each other. This course combines well with B5 Applied Analysis.

Quota: Not imposed this year.

2.6.1 Viscous Flow — Dr H Ockendon — 16 MT

[Option **B6a** if taken as a half-unit.]

Aims Viscous fluids are important in so many facets of everyday life that everyone has some intuition about the diverse flow phenomena that occur in practice. This course is distinctive in that it shows how quite advanced mathematical ideas such as asymptotics, partial differential equation theory and group theory can be used to analyse the underlying differential equations and hence give scientific understanding about flows of great practical importance. These include air flow round wings and flow in oil reservoirs.

Synopsis Derivation of Navier–Stokes equations for an incompressible Newtonian fluid. Vorticity. Energy equation and dissipation. Exact solutions for unidirectional flows; shear flow, Poiseuille flow, Rayleigh flow. Dimensional analysis, Reynolds number. Derivation of equations for high and low Reynolds number flows.

Derivation of Prandtl's boundary-layer equations. Similarity solutions for flow past a semi-infinite flat plate and for jets. Discussion of separation and application to the theory of flight. Jeffery–Hamel flow.

Slow flow past a circular cylinder and a sphere. Non-uniformity of the two dimensional approximation; Oseen's equation. Lubrication theory: bearings, thin films and Hele-Shaw cell. Flow in a porous medium. Stability and the transition to turbulaence.

Reading D.J. Acheson, *Elementary Fluid Dynamics*, OUP (1990), Chs 2, 6, 7, 8

H. Ockendon and J.R. Ockendon, *Viscous Flow*, Cambridge Texts in Applied Mathematics (1995)

M.E. O'Neill and F. Chorlton, *Viscous and Compressible Fluid Dynamics*, Ellis Horwood (1989), Chs 2, 3, 4.1–4.3, 4.19–4.20, 4.22–4.24, 5.1–5.2, 5.6

2.6.2 Waves and Compressible Flow — Dr Howell — 16 HT

[Option **B6b** if taken as a half-unit.]

Aims Propagating disturbances, or waves, occur frequently in applied mathematics, especially in applied mechanics. This course will be centred on some prototypical examples from fluid dynamics, the two most familiar being surface gravity waves and waves in gases. The models for compressible flow will be derived and then analysed for small amplitude motion. This will shed light on the important phenomena of dispersion, group velocity and resonance, and the differences between supersonic and subsonic flow, as well as revealing the crucial dependence of the waves on the number of space dimensions.

Larger amplitude motion of liquids and gases will be described by incorporating nonlinear effects, and the theory of characteristics for partial differential equations will be applied to understand the shock waves associated with supersonic flight.

Synopsis 1–2 Equations of inviscid compressible flow including flow relative to rotating axes.

3–6 Models for linear wave propagation including Stokes waves, Inertial waves, Rossby waves and simple solutions.

7–10 Theories for Linear waves: Fourier Series, Fourier integrals, method of stationary phase, dispersion and group velocity. Flow past thin wings, Huyghens principle.

11–12 Nonlinear Waves: method of characteristics, simple wave flows applied to one-dimensional unsteady gas flow and shallow water theory.

13–16 Shock Waves: weak solutions, Rankine-Hugoniot relations, oblique shocks, bores and hydraulic jumps.

Reading H. Ockendon and J.R. Ockendon, *Waves and Compressible Flow*, Springer (2004).

J.R. Ockendon, S. D. Howison, A.A. Lacey and A.B. Movchan *Applied Partial Differential Equations*, revised edition, 2003, OUP, Oxford, Chs 2.5, 4.5–7.

D.J. Acheson, *Elementary Fluid Dynamics*, OUP (1990), Ch 3

J. Billingham and A.C. King, *Wave Motion*, CUP (2000) Ch 1-4, 7,8

Background Reading M.J. Lighthill, *Waves in Fluids*, CUP (1978)

G.B. Whitham, *Linear and Nonlinear Waves*, Wiley (1973)

2.7 B7 Electromagnetism, Quantum Mechanics, and Special Relativity

Level: H-level **Method of Assessment:** 3-hour or $1\frac{3}{4}$-hour examination

Prerequisites: Part A Quantum Mechanics is necessary. Calculus of Variations is helpful with part of Electromagnetism. Familiarity with the terminology of Classical Mechanics would also be helpful.

Weight: One unit. Part I could be taken as a free-standing half-unit but Part II could not.

Quota: Not imposed this year.

2.7.1 Part I: Quantum Mechanics and Electromagnetism — Dr De la Ossa — 16 MT

[Option **B7a** if taken as a half-unit.]

Aims & Objectives Experiments in the late 19th and early 20th centuries, revealed a breakdown of classical mechanics and classical particle physics. Modern physics is built on three interrelated theories: Maxwell's electrodynamics, quantum theory and Einstein's relativity. They have brought about a revolution in our understanding of space, time and the structure of matter. They underpin many recent technological developments (e.g. lasers and transistors) and have stimulated important 20th century advances in pure mathematics in, for example, differential geometry and functional analysis. In spite of their revolutionary impact and central importance, the basic mathematical ideas are easily accessible and provide fresh and surprising applications of the mathematical techniques met with in other branches of mathematics. Part I builds further on the ideas in the Part A course on Quantum Mechanics and provides a short introduction to Maxwell's theory of electromagnetism.

Synopsis Maxwell's equations in vacuum and with sources. Lorentz force law. Plane waves and polarization. Energy density and the Poynting vector.

The mathematical structure and the postulates of quantum mechanics. Commutation relations, Poisson's brackets and Dirac's quantization scheme. Heisenberg's uncertainty principle. Creation and annihilation operators for the harmonic oscillator. Measurements and the interpretation of quantum mechanics. Schrödinger's cat. Spin-$\frac{1}{2}$ particles. Angular momentum in quantum mechanics. Particle in a central potential. The hydrogen atom.

Reading • K C Hannabuss, *Introduction to quantum mechanics,* OUP (1997).

• N W J Woodhouse,*Special Relativity,* Springer (2002). Chapters 2 & 3.

Good alternative textbooks for quantum mechanics are:

• I P Grant, *Classical and Quantum Mechanics,* Mathematical Institute Notes (1991).

• A I M Rae, *Quantum Mechanics,* 4th edition, Institute of Physics (1993)

Further Reading • L I Schiff, *Quantum Mechanics,* 3rd edition, McGraw Hill (1968).

• W J Duffin, *Advanced electricity and Magnetism,* McGraw-Hill (1968).

• An advanced textbook on electrodynamics is: J D Jackson, *Classical Electromagnetism,* Wiley (1962).

• R Penrose, *The Emperor's New Mind,* OUP (1999). Chapter 6.

2.7.2 Part II: Special Relativity and Electromagnetism — Dr Woodhouse — 16 HT

Aims & Objectives Maxwell's electromagnetic theory revealed light to be an electromagnetic phenomenon whose speed of propagation proved to be observer-independent. This discovery led to the overthrow of classical Newtonian mechanics, in which space and time were absolute, and its replacement by Special Relativity and space-time. The aim of this course is to study Einstein's special theory of relativity, to understand space-time, and to incorporate into it Maxwell's electrodynamics. These theories together with quantum theory are essential for an understanding of modern physics.

Synopsis Constancy of the speed of light; Lorentz transformations and the invariance of the wave operator; time dilation, length contraction and the relativistic Doppler effect; the resolution of the simple 'paradoxes' of relativity. Four-vectors; four-velocity and four-momentum; equivalence of mass and energy; particle collisions and four-momentum conservation; four-acceleration and four-force; the example of the constant-acceleration world-line. Contravariant and covariant vectors and tensors; index notation.

Solving Poisson's equation and the wave-equation with sources. Derivation of Maxwell's equations with sources from a variational principle.

Electromagnetism in four-dimensional form; the electromagnetic field tensor; the transformation law for the electric and magnetic fields; the Lorentz four-force law; the electromagnetic four-potential and the energy-momentum tensor.

Reading The preferred text is:

- N W J Woodhouse, *Special Relativity,* Springer (2002).

An alternative is:

- W Rindler, *Introduction to Special Relativity,* 2nd edition, OUP (1991).

Further Reading For the experimental background to special relativity, and in many libraries:

- A P French, *Special Relativity,* MIT Introductory Physics Series, Nelson Thornes (1971)

For advanced texts on electromagnetism, see:

- W J Duffin, *Advanced Electricity and Magnetism,* McGraw-Hill (1968).
- J D Jackson, *Classical Electromagnetism,* Wiley (1962).

2.8 B8 Topics in Applied Mathematics

Level: H-level **Method of Assessment:** 3-hour or $1\frac{3}{4}$-hour examination

Prerequisites: Part A core material (especially differential equations)

Weight: One unit, or can be taken as either a half-unit in Nonlinear Systems or a half-unit in Mathematical Ecology and Biology.

Quota: Not imposed this year.

2.8.1 Part I: Mathematical Ecology and Biology — Prof Maini — 16 MT

[Option **B8a** if taken as a half-unit.]

Aims & Objectives Mathematical Ecology and Biology introduces the applied mathematician to practical applications in an area that is growing very rapidly. The course mainly focusses on situations where continuous models are appropriate and where these may be modelled by deterministic ordinary and partial differential equations. By using particular modelling examples in ecology, chemistry, biology, physiology and epidemiology, the course demonstrates how various applied mathematical techniques, such as those describing linear stability, phase planes, singular perturbation and travelling waves, can yield important information about the behaviour of complex models.

Synopsis Continuous and discrete population models for a single species, including Ludwig's 1978 insect outbreak model for spruce budworm. Harvesting and strategies for sustainable fishing. Modelling interacting populations, including the Lotka-Volterra model for predator-prey (with application to hare-lynx interactions), and Okubo's 1989 model for red-grey squirrel competition. Principle of competitive exclusion.

Michaelis-Menten model for enzyme-substrate kinetics.

The Belousov-Zhabotinskii reaction: threshold phenomena, excitable systems, wave propagation.

Biological pattern formation. Turing's model for animal coat markings. Chemotaxis models for reptilian coat patterns.

Simple models for the spread of disease. Travelling waves in a model for rabies infection.

Reading J.D. Murray *Mathematical Biology, 3rd edition, Springer-Verlag. Volume I: An Introduction (2002); Volume II: Spatial Models and Biomedical Applications (2003).*

Volume I: 1.1, 1.2, 1.6, 2.1-2.4, 3.1, 3.3-3.6, 3.8, 6.1-6.3, 6.5, 6.6, 8.1, 8.2, 8.4, 8.5, 10.1, 10.2, 11.1-11.5, 13.1-13.5, Appendix A.

Volume II: 1.6, 2, 3.1, 3.2, 5.1, 5.2, 13.1-13.4.

Further Reading J. Keener and J. Sneyd 1998 *Mathematical Physiology.* Springer, Berlin: 1.1, 1.2, 9.1, 9.2.

H. Meinhardt 2000 *The Algorithmic Beauty of Sea Shells*, 2nd enlarged edition, Springer, Berlin.

2.8.2 Part II: Nonlinear Systems — Dr Moroz — 16 HT

[Option **B8b** if taken as a half-unit.]

Aims & Objectives This course aims to provide an introduction to the tools of dynamical systems theory, which are essential in the realistic modelling and study of many disciplines, including Mathematical Ecology and Biology, Fluid mechanics, Economics, Mechanics and Celestial Mechanics. The course will include the study of both deterministic ordinary differential equations, as well as nonlinear difference equations, drawing examples from the various areas of application, whenever possible and appropriate. The course will include the use of numerical software involving Matlab in the homework exercises.

Synopsis BIFURCATIONS FOR ORDINARY DIFFERENTIAL EQUATIONS

Bifurcations for simple ordinary differential equations: saddle-node, transcritical, pitchfork, Hopf. Centre stable and unstable manifolds. Normal forms. The Hopf Bifurcation Theorem. Lyapunov functions. [6 lectures]

BIFURCATIONS FOR MAPS

Poincaré section and first-return maps. Brief review of multipliers, stability and periodic cycles. Elementary bifurcations of one-dimensional maps: saddle-node, transcritical, pitchfork, period-doubling. Two-dimensional maps. Hénon and Standard map. [6 lectures]

CHAOS

Logistic map. Bernoulli shift map and symbolic dynamics. Smale Horseshoes. Lorenz equations. [4 lectures]

Reading G.L. Baker and J.P. Gollub, *Chaotic Dynamics: An Introduction*, 2nd ed. C.U.P., Cambridge (1996).

P.G. Drazin, *Nonlinear Systems.* C.U.P., Cambridge (1992).

2.9 B9 Number Theory

Level: H-level **Method of Assessment:** 3-hour or $1\frac{3}{4}$-hour examination

Prerequisites: All second-year algebra and arithmetic. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

Weight: One unit, or Polynomial Rings and Galois Theory can be taken as half-unit but Algebraic Number Theory cannot.

Quota: Not imposed this year.

2.9.1 Part I: Polynomial Rings and Galois Theory —Dr Szendroi — 16 MT

[Option **B9a** if taken as a half-unit.]

Aims and Objectives The course starts with a review of second-year ring theory with a particular emphasis on polynomial rings. We also discuss general integral domains and fields of fractions. This is followed by the classical theory of Galois field extensions, culminating in a discussion of some of the classical theorems in the subject: the insolubility of the general quintic and impossibility of certain ruler and compass constructions considered by the Ancient Greeks.

Synopsis Review of polynomial rings, factorisation, integral domains. Any nonzero homomorphism of fields is injective. Fields of fractions.

Review of group actions on sets, Gauss' lemma and Eisenstein's criterion for irreducibility of polynomials, field extensions, degrees, the tower law. Symmetric polynomials.

Separable extensions. Splitting fields. The theorem of the primitive element. The existence and uniqueness of algebraic closure.

Groups of automorphisms, fixed fields. The fundamental theorem of Galois theory.

Examples: Kummer extensions, cyclotomic extensions, finite fields and the Frobenius automorphism. Techniques for calculating Galois groups.

Soluble groups. Solubility by radicals, solubility of polynomials of degree at most 4, insolubility of the general quintic, impossibility of some ruler and compass constructions.

Reading J. Rotman, *Galois Theory*, Springer-Verlag NY Inc (2001/1990)

I. Stewart, *Galois Theory*, Chapman and Hall (2003/1989)

D.J.H. Garling, *A Course in Galois Theory*, Cambridge University Press I.N. (1987)

Herstein, *Topics in Algebra*, Wiley (1975)

2.9.2 Part II: Algebraic Number Theory — Dr Lauder — 16 HT

Aims and Objectives An introduction to algebraic number theory. The aim is to describe the properties of number fields, but particular emphasis in examples will be placed on quadratic fields, where it is possible to really calculate the properties of some of the objects being considered. In such fields the familiar unique factorisation enjoyed by the integers may fail, and a key objective of the course is to introduce the class group which measures the failure of this property.

Synopsis

- field extensions, minimum polynomial, algebraic numbers, conjugates, -discriminants, Gaussian integers, algebraic integers, integral basis
- examples: quadratic fields, cyclotomic fields
- norm of an algebraic number
- existence of factorisation
- factorisation in $Q(\sqrt{d})$
- ideals, Z-basis, maximal ideals, prime ideals
- unique factorisation theorem of ideals
- relationship between factorisation of number and of ideals
- norm of an ideal
- ideal classes
- statement of Minkowski convex body theorem
- finiteness of class number
- computations of class number to go on example sheets

Reading I. Stewart and D. Tall, *Algebraic Number Theory*, (Chapman and Hall Mathematics Series) May 1987

Further Reading Marcus, *Algebraic Number Theory*

2.10 B10 Martingales and Financial Mathematics

Level: H-level **Method of Assessment:** 3-hour or $1\frac{3}{4}$-hour examination

Prerequisites: For Part I Martingales Through Measure Theory, the Part A courses on Integration and on Topology are helpful but not essential. Part II Mathematical Models of Financial Derivatives requires second-year Probability.

Weight: One unit, or can be taken as either a half-unit in Martingales or a half-unit in Mathematical Models of Financial Derivatives.

Quota: Not imposed this year.

2.10.1 Part I: Martingales Through Measure Theory — Dr Kristensen — 16 MT

[Option **B10a** if taken as a half-unit.]

Aims & Objectives It was observed by Brown that a pollen particle on the surface of water performs an erratic motion. However, the statistical properties of this motion are predictable and find close parallels in many other settings. The challenge is to create the mathematical framework needed to describe and study such stochastic processes. Certainly it involves probability, but tossing coins and throwing dice only hint at the kind of mathematics required to discuss the situation where the outcome of an experiment is a random path or random sequence. The transition taking probability from heuristic philosophy and into mathematics started with Kolmogorov; the construction of Brown's motion as a mathematical object is due to Wiener. New and basic ideas continued to emerge throughout the 20th century (Itô's paper with his celebrated calculus was published in 1942).

Today, probability theory is a substantial part of mathematics, still under active development, strongly interacting with applications (e.g. Finance) and with other areas of mathematics.

The aim of the course is to explain and develop, in a rigorous way, some of the basic tools: measure theory and (discrete parameter) martingales.

Synopsis σ-algebra, measurable space. σ-algebra generated by a family of sets. The Borel σ-algebra. Measurable functions and their elementary properties, including approximation by simple functions. σ-algebra generated by a family of functions.

Measure, measure space. Uniqueness of extension from a π-system (proof not examinable). [Caratheodory's construction of σ-algebra and measure.] Existence and uniqueness of Lebesgue measure.

Events, probability triple, and the first Borel Cantelli lemma with applications. Random variables and their distribution functions. Skorokod representation of random variable with prescribed distribution function. Independence for events, random variables and σ-algebras. π-systems criterion for independence. The tail σ-algebra, Kolomogorov's 0–1 Law. The Second Borel Cantelli lemma.

Lebesgue integration, expectation. Elementary properties of the integral, including the Monotone Convergence Theorem, Fatou's Lemma and the Dominated Convergence Theorem. [Product of σ-algebras, product measures. Tonelli's Theorem, Fubini's Theorem. The Radon Nikodym Theorem.] Convex functions and Jensen's inequality. [L^p spaces, Hölder's and Minkowski's inequalities.] A Strong Law of Large Numbers (easy version).

The Kolmogorov definition of conditional expectation. Elementary properties of conditional expectation. Completeness of L^2, conditional expectation as best mean-square approximation. Filtrations and martingales. Gambling systems exploded: Stopping times, discrete stochastic integrals, Optional Stopping Theorem. Convergence and upcrossing. [The Levy construction of Brownian motion.]

[ ]=covered informally.

Reading D. Williams. *Probability with Martingales*, CUP, 1995.

Further Reading R. Durret. *Probability: Theory and Examples.* (Second Edition) Duxbury Press, Wadsworth Publishing Company, 1996.

J. Neveu. *Discrete-parameter Martingales.* North-Holland, Amsterdam, 1975.

2.10.2 Part II: Mathematical Models of Financial Derivatives — Dr Hambly — 16 HT

[Option **B10b** if taken as a half-unit.]

Aims & Objectives The course aims to introduce students to mathematical modelling in financial markets. At the end of the course the student should be able to formulate a model for an asset price and then determine the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

Synopsis Introduction to markets, assets, interest rates and present value; arbitrage and the law of one price: European call and put options, payoff diagrams. Introduction to Brownian motion, continuous time martingales, informal treatment of Ito's formula and stochastic differential equations. Discussion of the connection with PDEs through the Feynman-Kac formula.

The Black-Scholes analysis via delta hedging and replication, leading to the Black-Scholes partial differential equation for a derivative price. General solution via Feynman-Kac and risk neutral pricing, explicit solution for call and put options.

Extensions to assets pating dividends, time-varying parameters. Forward and future contracts, options on them. American options, formulation as a free-boundary problem and a linear complementarity problem. Simple exotic options. Weakly path-dependent options including barriers, lookbacks and Asians.

Reading T Bjork, *Arbitrage Theory in Continuous Time*, OUP (1998)

P Wilmott, S D Howison and J Dewynne, *Mathematics of Financial Derivatives,* CUP (1995)

A Etheridge, *A course in Financial Calculus* CUP (2002)

Background J Hull, *Options Futures and Other Financial Derivative Products*, 4th edition, Prentice Hall (2001)

N Taleb, *Dynamic Hedging*, Wiley (1997)

P Wilmott, *Derivatives*, Wiley (1998)

2.11 B11 Communication Theory — Dr Stirzaker — 16 MT

NB: OB22 Integer Programming is a very suitable complement to this course.

Level: H-level **Method of Assessment:** $1\frac{3}{4}$-hour examination

Prerequisites: Part A Probability would be helpful, but not essential

Weight: Half-unit

Aims & Objectives The aim of the course is to investigate methods for the communication of information from a sender, along a channel of some kind, to a receiver. If errors are not a concern we are interested in codes that yield fast communication, whilst if the channel is noisy we are interested in achieving both speed and reliability. A key concept is that of information as reduction in uncertainty. The highlight of the course is Shannon's Noisy Coding Theorem.

Synopses Uncertainty (entropy); conditional uncertainty; information. Chain rules; relative entropy; Gibbs' inequality; asymptotic equipartition. Instantaneous and uniquely decipherable codes; the noiseless coding theorem for discrete memoryless sources; constructing compact codes.

The discrete memoryless channel; decoding rules; the capacity of a channel. The noisy coding theorem for discrete memoryless sources and binary symmetric channels.

Extensions to more general sources and channels.

Error-detection and error-correction. Equivalence of codes. Constraints upon the choice of good codes; the Hamming (sphere-packing) and the Gilbert–Varshamov bounds.

Reading D.J.A. Welsh, *Codes and Cryptography*, OUP, 1988, Chs 1–3, 5

G. Jones and J.M. Jones, *Information and Coding Theory,* Springer, 2000, Ch 1–5

T. Cover and J. Thomas, *Elements of Information Theory,* Wiley, 1991, Ch 1–5, 8

Further Reading R.B. Ash, *Information Theory*, Dover, 1990

D. MacKay, *Information Theory, Inference, and Learning Algorithms,* Cambridge, 2003. [Can be seen at: http://www.inference.phy.cam.ac.uk/mackay/itila. Do not infringe the copyright!]

Part C (M-level) courses available in the third year

Third year students may also offer up to the equivalent of one unit of the following Part C units and half-units. The courses are M-level and will be examined using the same examination questions as used for Part C. The courses Lie Groups, Differentiable Manifolds, and Introduction to Modular Forms will not be available in the fourth year; instead there will be courses on Topology and Groups, Algebraic Topology and Analytic Number Theory.

2.12 C3.1 Geometry: Lie Groups and Differentiable Manifolds (M-level)

Level: M-level

Method of Assessment: Lie Groups will be examined by mini project and Differentiable Manifolds by $1\frac{3}{4}$-hour examination.

Weight: One unit, or can be taken as either a half unit in Lie Groups or a half unit in Differentiable Manifolds.

Quota: Not imposed this year.

2.12.1 Part I: Lie Groups (M-level) — Prof Tillmann —16 MT

[Option C3.1a if taken as a half unit]

Prerequisites: 2nd year core algebra, complex analysis, groups in action, topology, multi-variable calculus.

Aims & Objectives The theory of Lie groups is one of the most beautiful developments of pure mathematics in the twentieth century, with many applications to geometry, theoretical physics and mechanics, and links to both algebra and analysis. Lie groups are groups which are simultaneously manifolds, so that the notion of differentiability makes sense, and the group multiplication and inverse maps are differentiable. However this course introduces the theory in a more concrete way via groups of matrices, in order to minimise the prerequisites.

Synopsis The exponential map for matrices, Ad and ad, the Campbell-Baker-Hausdorff series.

Linear groups, their Lie algebras and the Lie correspondence. Homomorphisms and coverings of linear groups. Examples including $SU(2)$, $SO(3)$ and $SL(2;\mathbb{R})$.

The compact and complex classical Lie groups. Cartan subgroups, Weyl groups, weights, roots, reflections.

Informal discussion of Lie groups as manifolds with differentiable group structures; quotients of Lie groups by closed subgroups; the universal cover of $SL(2;\mathbb{R})$.

Bi-invariant integration on a compact group (statement of existence and basic properties only). Representations of compact Lie groups. Tensor products of representations. Complete reducibility, Schur's lemma. Characters, orthogonality relations.

Statements of Weyl's character formula, the theorem of the highest weight and the Borel-Weil theorem, with proofs for SU(2) only.

Reading W Rossmann, *Lie Groups: an introduction through linear groups,* Oxford (2002), Chapters 1-3 and 6

A Baker, *Matrix groups: an introduction to Lie Group Theory,* Springer Undergraduate Mathematics Series.

Additional Reading JF Adams, *Lectures on Lie Groups,* University of Chicago Press 1982.

R Carter, G Segal and I MacDonald, *Lectures on Lie groups and Lie algebras,* LMS Student Texts, Cambridge (1995).

JF Price, *Lie groups and compact groups,* LMS Lecture Notes 25, Cambridge 1977.

2.12.2 Part II: Differentiable Manifolds (M-level)— Prof Hitchin — HT

[Option C3.1b if taken as a half unit]

Prerequisites: 2nd year core algebra, topology, multivariable calculus.

Useful but not essential: groups in action, geometry of surfaces.

Aims & Objectives A manifold is a space M such that small pieces of M look like small pieces of Euclidean space. Thus a smooth surface is an example of a (2-dimensional) manifold.

Manifolds are the natural setting for much of classical applied mathematics such as mechanics, as well as general relativity. They are also central to areas of pure mathematics such as topology and certain parts of analysis.

In this course we introduce the tools needed to do analysis on manifolds. We prove a very general form of Stokes's Theorem which includes as special cases the classical theorems of Gauss, Green and Stokes. We also introduce the theory of de Rham cohomology, which is central to many arguments in topology.

Synopsis Smooth manifolds and smooth maps. Tangent vectors, the tangent bundle, induced maps. Vector fields and flows, the Lie bracket.

Exterior algebra, differential forms, exterior derivative, Cartan formula in terms of Lie bracket. Orientability. Partitions of unity, integration on oriented manifolds. Riemannian metrics.

Stokes's theorem.

De Rham cohomology and discussion of de Rham's theorem. Applications of de Rham theory including degree.

Reading M. Spivak. *Calculus on manifolds,* (W A Benjamin 1965).

M. Spivak. *A comprehensive introduction to differential geometry*, Vol. 1 (1970).

W. Boothby. *An introduction to differentiable manifolds and Riemannian geometry*, 2nd edition. (Academic Press 1986).

M. Berger and B. Gostiaux. *Differential geometry: manifolds, curves and surfaces.* Translated from the French by S. Levy. (Springer Graduate Texts in Mathematics, 115, Springer-Verlag (1988), chapters 0-3, 5-7).

F. Warner. *Foundations of differentiable manifolds and Lie groups.*(Springer Graduate Texts in Mathematics, 94)

J. Lee. *Introduction to smooth manifolds* (Springer GTM).

2.13 C9.1a Introduction to Modular Forms (M-level)—Dr Kilford — MT

Level: M-level **Method of Assessment:** $1\frac{3}{4}$-hour examination

Weight: Half-unit.

Quota: Not imposed this year.

Prerequisites: Second year Complex Analysis, and third year Geometry of Surfaces (for Riemann surfaces). Third year Algebraic Curves and Algebraic Number Theory would be useful, the course would sit well with them, but is in no way essential.

Aims & Objectives Modular forms are classical analytic objects which were the subject of much interest early in the last century. For some time their interest appeared to have diminished, until remarkable connections with a huge range of other areas in pure Mathematics were discovered. The most celebrated is, of course, the role they played in the proof of Fermat's last theorem, through the conjecture of Shimura-Taniyama-Weil that elliptic curves are modular, but they also occur in group theory amongst other things. Therefore there is much current interest in the theory of modular forms. The aim of this course is to cover the classical theory of modular forms.

Synopsis -Definitions of the modular group, congruence subgroups, modular forms (defined for congruence subgroups as well as $SL_2(Z_p)$, a difference from Serre's book)

-examples, Eisenstein series, lattice functions

-space of modular functions

-expansions at infinity, zeroes and poles using contour integrals

-Hecke operators

-Peterson inner product

-Eigenforms

-L-functions and some properties

-Theta functions.

-Modular curves over the complex numbers

-Moduli of elliptic curves

-Riemann surface structure

Reading Serre, *A Course in Arithmetic* (only covers modular forms for SL_2, with no discussion of congruence subgroups, of which there will be some mention)

Additional Reading T. Miyake, *Modular forms,* 1989, Springer-Verlag, Berlin and Heidelberg

A.W. Knapp, *Elliptic curves*, 1993, Princeton University Press

D. Bump,*Automorphic forms and Representations*, Cambridge Studies in Advanced Mathematics, CUP.

Cornell, Silverman and Stevans, *Modular Forms and Fermat's Last Theorem*, 1998, Springer-Verlag, New York.

Milne, *Modular Functions and Modular Forms,*
http://www.jmilne.org/math/CourseNotes/math678.html

Stein , *Algorithms for computing with modular forms,*
http://modular.fas.harvard.edu/257/notes/257.pdf

2.14 C5.1a Partial Differential Equations for Pure and Applied Mathematicians (M-level)— Dr Dyson — MT

Level: M-level **Method of Assessment:** $1\frac{3}{4}$-hour examination

Weight: Half-unit. **Quota:** Not imposed this year.

Prerequisites: Part A integration would be useful but is not essential.

Aims & Objectives The course will introduce some of the modern techniques in partial differential equations, such as Sobolev spaces, weak convergence, weak solutions and the direct method of calculus of variations, that are central to the theoretical and numerical treatments of linear and nonlinear partial differential equations arising in science, geometry and other fields.

The course is logically independent of B5b. It provides valuable background for the Part C courses on Calculus of Variations and Finite Element Methods.

Synopsis Part I Function Spaces

Why are function spaces important for partial differential equations?

User's guide to the Lebesgue integral. Definition of Banach spaces, separability and dual spaces. The spaces $L^p(\Omega)$, $1 \leq p < \infty$, where $\Omega \subset \mathbb{R}^n$ is open. Minkowski and Hölder inequalities. Statement that $L^p(\Omega)$ is a Banach space, and that the dual of L^p is $L^{p'}$, for $1 \leq p < \infty$. Statement of what it means that L^2 is a Hilbert space.

Mollifiers and the density of smooth functions in L^p for $1 \leq p < \infty$.

Definition of weak derivatives and their uniqueness. Definition of Sobolev space $W^{1,p}(\Omega)$, $1 \leq p < \infty$. $H^1(\Omega) = W^{1,2}(\Omega)$. Definition of $W_0^{1,p}(\Omega)$, $1 \leq p < \infty$.

Weak and weak* convergence in L^p spaces. Examples. A bounded sequence in L^2 has a weakly convergent subsequence.

Part II Elliptic Problems.

The direct method of calculus of variations. The Poincaré inequality. Proof of the existence and uniqueness of a weak solution to Poisson's equation $-\Delta u = f$, with zero Dirichlet boundary conditions and $f \in L^2(\Omega)$, with Ω bounded. Discussion of regularity of solutions.

Compact operators and self adjoint operators. Fredholm Alternative and Hilbert Schmidt Theorem. Examples including $-\Delta$ with zero boundary data.

The Lax Milgram lemma and Gårding's inequality. Existence and uniqueness of weak solutions to general linear uniformly elliptic equations.

A nonlinear elliptic problem treated by the direct method using an embedding theorem (stated but not proved).

Reading M. Renardy and R.C. Rogers *An introduction to partial differential equations*, 2004, Springer-Verlag, New York.

Lawrence C. Evans, *Partial differential equations*, (Graduate Studies in Mathematics), 2004, American Mathematical Society

P.D. Lax *Functional analysis,* 2002, Wiley-Interscience, New York.

J. Rauch, *Partial differential equations*, 1992, Springer-Verlag, New York.

3 Other Mathematical units and half-units

3.1 O1 History of Mathematics — Dr Flood, Dr Neumann, Dr Stedall and Dr Wilson — 16 MT and 8 HT

Level: H-level

Method of Assessment: 2 hour written examination paper for the MT lectures and 3000 word mini project for the reading course.

Prerequisites: None.

Weight: One unit.

Quota: The maximum number of students that can be accepted for 2005-2006 will be 30.

Aims and objectives This course is designed to provide the historical background to some of the mathematics familiar to students from A-level and four terms of undergraduate study, and looks at a period from approximately the mid sixteenth century to the end of the nineteenth century.

Teaching

The course will be delivered through 16 lectures in Michaelmas Term, and a Reading Course consisting of 8 two-hour seminars (equivalent to a further 16 lectures) in Hilary Term. Guidance will be given throughout on reading, note-taking and essay-writing skills.

Lectures Week 1. Introduction An overview of the development of mathematics from the Greeks to the present day. Historical evidence and learning to read it.

Week 2. From calculus to analysis (1): calculus in its days of innocence

(a) Tangents; quadrature; indivisibles; the fundamental theorem of calculus.

(b) Euler's unification of logarithmic and circular functions.

Week 3. From calculus to analysis (2): the development of rigour

(a) Attempts to establish the calculus.

(b) Functions, limits and continuity.

Week 4. Mechanics and Probability

(a) Newton's Principia; differential equations.

(b) Probability.

Week 5. Equations and Group theory

(a) Polynomial equations; solvability.

(b) Groups and fields.

Week 6. Some nineteenth century developments (1): Linear algebra and Complex analysis

(a) Linear equations; matrices; vector spaces.

(b) Complex analysis.

Week 7. Some nineteenth-century developments (2): Real and complex numbers

(a) Development and extension of the concept of number; types of number; p, e, i.

(b) Development of the real and complex number systems.

Week 8. Theories of integration and Foundations

(a) Theories of integration

(b) Axiomatisation; set theory and logic.

The lecturers will set six exercise sheets (including suggested reading, extracts from primary sources on which the student will be expected to comment, and short essay titles). Written work will be discussed in six weekly intercollegiate classes in the usual way.

Reading Courses In Hilary Term each student will select a Reading Course from a menu of three or four options. Courses will be run according to demand, requiring ideally a minimum of five and a maximum of eight students on each. Students will be expected to write three essays during the first six weeks of the term. The course will be examined by a miniproject of 3000 words to be completed during Weeks 7 to 9. Topics offered for 2005-2006 could include the following:

- Combinatorics
- Origins of group theory
- Early number theory
- Probability and statistics
- Newton's Principia
- Seventeenth-century algebra
- Integration
- Theory of equations

Reading Victor Katz, *A History of Mathematics: An Introduction* (2nd edition), Addison Wesley Longman 1998.

John Fauvel and Jeremy Gray (eds), *The History of Mathematics: A Reader*, Macmillan 1987.

Dirk Struik, *A Concise History of Mathematics* (4th revised edition), Dover 1987.

Assessment The Michaelmas Term lectures will be examined in a two-hour written paper at the end of Hilary Term. Candidates will be expected to answer two half-hour questions (commenting on extracts) and one one-hour question (essay). The paper will account for 50% of the marks for the course.

The Reading Course will be examined by a 3000 word miniproject at the end of Hilary Term. The title will be set at the beginning of Week 7 and two copies of the project must be submitted to the Examination Schools by midday on Friday of Week 9. The miniproject will account for 50% of the marks for the course.

3.2 OBS1 Applied Statistics

[Paper BS1 in the Honour School of Mathematics and Statistics. Teaching responsibility of the Department of Statistics.]

Level: H-level

Method of Assessment: 2-hour examination plus assessed practical assignments

Prerequisites: Part A Probability and Statistics.

Weight: One unit.

Quota: Not imposed this year.

3.2.1 Applied Statistics I – Dr A D Lunn – 16 MT
Applied Statistics II – Lecturer TBC – 10 HT

Aims The course aims to develop the theory of statistical methods, and also to introduce students to the analysis of data using a statistical package. The main topics are: Simulation, Practical aspects of linear models, Logistic regression and generalized linear models, and Robust and computer-intensive methods.

Synopsis *Michaelmas Term (16 lectures)*

Simulation: pseudo-random numbers, inversion, rejection, composition, ratio-of-uniforms and alias methods, computational efficiency.

Practical aspects of linear models: review of multiple regression, analysis of variance, model selection, fit criteria, use of residuals, outliers, leverage, Box-Cox transformation, added-variable plots, model interpretation.

Logistic regression. Linear exponential families and generalized linear models, scale parameter, link functions, canonical link. Maximum likelihood fitting and iterated weighted least squares. Asymptotic theory: statement and applications to inference, analysis of deviance, model checking, residuals. Examples: binomial, Poisson and gamma models.

Hilary Term (10 lectures)

Robust statistics. Univariate methods, robust regression (M-estimation), resistant regression.

Computer-intensive statistics. Smoothing methods (kernels, splines, local polynomials). Bootstrapping. Monte Carlo tests.

Reading (Michaelmas Term) S. M. Ross, *Simulation*, 2nd edition, Academic Press (1996)

A. J. Dobson, *An Introduction to Generalized Linear Models*, Chapman and Hall (1990)

D. Lunn, *Notes* (2003)

Reading (Hilary Term) P. J. Rousseeuw and A. M. Leroy, *Robust Regression and Outlier Detection*, Wiley (1987), pp 1-194.

J. D. Gibbons, *Nonparametric Statistical Inference*, Marcel Dekker (1985), pp 1-193, 273-290.

R. H. Randles and D. A. Wolfe, *Introduction to the Theory of Nonparametric Statistics*, Wiley (1979), pp 1-322.

Further Reading F. L. Ramsey and D. W. Schafer, *The Statistical Sleuth: A Course in Methods of Data Analysis*, 2nd edition, Duxbury (2002)

W. N. Venables and B. D. Ripley, *Modern Applied Statistics with S*, Springer (2002)

Practicals In addition to the lectures there will be four supervised practicals, each containing one or more problems whose written solutions will be assessed as part of the unit examination. Similar practical applications will be used as illustrations in lectures.

3.3 OBS2 Statistical Inference – 16 HT

[Paper BS2 in the Honour School of Mathematics and Statistics. Teaching responsibility of the Department of Statistics.]

Level: H-level **Method of Assessment:** $1\frac{3}{4}$-hour examination

Prerequisites: Part A Probability and Statistics.

Weight: Half unit.

Quota: Not imposed this year.

Aims There has been a resurgence in Bayesian approaches in recent years and the methodology now plays a central role in many fields, from expert systems, machine learning, and pattern recognition, to applications in finance and the law. The course will provide an introduction to the theory and application of Bayesian methods in statistical problems. Some previous background in probability and statistics, at the second-year level, will be necessary, but the course will otherwise be selfcontained.

Synopsis Interpretations of probability, motivation for the Bayesian paradigm, illustrations of inference by simulation; single-parameter models, informative and noninformative prior distributions, posterior distributions and conjugacy, posterior summaries; multiparameter models, dealing with nuisance parameters, marginalisation problems with noninformative distributions; decision theoretic frameworks, loss function, Bayesian point estimation; Bayesian data analysis, constructing the prior, model checking, model comparisons, Bayes factors; practical implementation, Markov chain Monte Carlo, the Gibbs and Metropolis-Hastings algorithms.

Recommended texts A. Gelman, J.B. Carlin, H.S. Stern and D.B. Rubin, *Bayesian Data Analysis*, Chapman and Hall (2nd edition, 2004).

J.O. Berger, *Statistical Decision Theory and Bayesian Analysis*, 2nd edition, Springer-Verlag (1985), pp 1-166.

Further reading W.R. Gilks, S. Richardson and D. Spiegelhalter, *Markov Chain Monte Carlo in Practice*, Chapman and Hall.

C.P. Robert, *The Bayesian Choice*, Springer-Verlag (1994), pp 1-178.

A. O.Hagan, *Kendall's Advanced Theory of Statistics, Vol.2B, Bayesian Inference*, Edward Arnold, (1994), 1-168.

J.M. Bernardo and A.F.M. Smith, *Bayesian Theory*, Wiley (1994), pp 165-284.

3.4 OBS3 Stochastic Modelling

[Paper BS3 in the Honour School of Mathematics and Statistics. Teaching responsibility of the Department of Statistics.]

Level: H-level **Method of Assessment:** 3-hour or $1\frac{3}{4}$-hour examination

Prerequisites: For the first 16 lectures, Part A Probability. For the second 16 lectures, Part A Statistics also.

Weight: One unit, or the first 16 lectures can be taken as a half-unit in Applied Probability. (The second 16 lectures cannot be taken as a half-unit.)

Quota: Not imposed this year.

Aims: This unit has been designed so that a student obtaining at least an upper second class mark on the whole unit can expect to gain exemption from the Institute of Actuaries' paper CT4, which is a compulsory paper in their cycle of professional actuarial examinations. *Details are yet to be confirmed.* The first half of the unit, clearly, and also the second half of the unit, apply much more widely than just to insurance models.

3.4.1 Applied Probability – Dr M Winkel – 16 MT

Aims This course is intended to show the power and range of probability by considering real examples in which probabilistic modelling is inescapable and useful. Theory will be developed as required to deal with the examples.

Synopsis Poisson processes and birth processes. Continuous-time Markov chains. Transition rates, jump chains and holding times. Forward and backward equations. Class structure, hitting times and absorption probabilities. Recurrence and transience. Invariant distributions and limiting behaviour. Time reversal.

Applications of Markov chains in areas such as queues and queueing networks - M/M/s queue, Erlang's formula, queues in tandem and networks of queues, M/G/1 and G/M/1 queues; insurance ruin models; epidemic models; applications in applied sciences.

Renewal theory. Limit theorems: strong law of large numbers, strong law and central limit theorem of renewal theory, elementary renewal theorem, renewal theorem, key renewal theorem. Excess life, inspection paradox. Applications.

Reading

- J.R. Norris: *Markov chains.* Cambridge University Press (1997)
- G.R. Grimmett and D.R. Stirzaker: *Probability and Random Processes.* 3rd edition, Oxford University Press (2001)
- G.R. Grimmett and D.R. Stirzaker: *One Thousand Exercises in Probability.* Oxford University Press (2001)
- S.M. Ross: *Introduction to Probability Models.* 4th edition, Academic Press (1989)

- D.R. Stirzaker: *Elementary Probability.* 2nd edition, Cambridge University Press (2003)

3.4.2 Statistical Lifetime-Models – lecturer TBC – 16 HT

Aims The second half of the unit follows on from the first half on Applied Probability. Models introduced there are examined more specifically in a life insurance context where transitions typically model the passage from 'alive' to 'dead', possibly with intermediate stages like 'loss of a limb' or 'critically ill'. The aim is to develop statistical methods to estimate transition rates and more specifically to construct life tables that form the basis in the calculation of life insurance premiums. Survival analysis will allow consideration of the effect of covariates.

Synopsis Survival models: general lifetime distributions, force of mortality (hazard rate), survival function, specific mortality laws, the single decrement model, curtate lifetimes, life tables.

Estimation procedures for lifetime distributions: empirical lifetime distributions, censoring, Kaplan-Meier estimate, Nelson-Aalen estimate. Parametric models, accelerated life models including Weibull, log-normal, log-logistic. Plot-based methods for model selection. Proportional hazards, partial likelihood.

Two-state and multiple-state Markov models, with simplifying assumptions. Estimation of Markovian transition rates: Maximum likelihood estimators, time-varying transition rates, census approximation.

Graduation, including fitting Gompertz-Makeham model, comparison with standard life table: tests including chi-square test and grouping of signs test, serial correlations test; smoothness.

Reading

- *Subject 104[CT4] Survival models[Modelling] Core Reading 2004[2005].* Faculty & Institute of Actuaries (2003[2004])
- D.R. Cox and D. Oakes: *Analysis of Survival Data.* Chapman & Hall (1984)

Further Reading

- J.P. Klein and M.L. Moeschberger: *Survival Analysis.* Springer (1997)
- C.T. Le: *Applied Survival Analysis.* Wiley (1997)
- H.U. Gerber: *Life Insurance Mathematics.* 3rd edition, Springer (1997)
- N.L. Bowers et al.: *Actuarial mathematics.* 2nd edition, Society of Actuaries (1997)

3.5 OBS4 Actuarial Science

[Paper BS4 in the Honour School of Mathematics and Statistics. Teaching responsibility of the Department of Statistics.]

Level: H-level **Method of Assessment:** 3-hour examination

Prerequisites: Part A Probability is useful, but not essential. If you have not done Part A Probability, make sure that you are familiar with Mods work on Probability.

Weight: One unit.

Quota: Not imposed this year.

3.5.1 Actuarial Science I – Dr Martin – 16 MT

Aims This unit is supported by the Institute of Actuaries. It has been designed to give the undergraduate mathematician an introduction to the financial and insurance worlds in which the practising actuary works. Students will cover the basic concepts of risk management models for investment and mortality, and for discounted cash flows. In the examination, a student obtaining at least an upper second class mark on this unit can expect to gain exemption from the Institute of Actuaries' paper CT1, which is a compulsory paper in their cycle of professional actuarial examinations.

Synopsis Fundamental nature of actuarial work. Use of generalised cash flow model to describe financial transactions. Time value of money using the concepts of compound interest and discounting. Interest rate models. Present values and accumulated values of a stream of equal or unequal payments using specified rates of interest. Interest rates in terms of different time periods. Equation of value, rate of return of a cash flow, existence criteria.

Loan repayment schemes. Investment project appraisal, funds and weighted rates of return. Inflation modelling, inflation indices, real rates of return, inflation-adjustments. Valuation of fixed-interest securities, taxation and index-linked bonds.

Uncertain payments, corporate bonds, fair prices and risk. Single decrement model, present values and accumulated values of a stream of payments taking into account the probability of the payments being made according to a single decrement model. Annuity functions and assurance functions for a single decrement model. Risk and premium calculation.

Reading All of the following are available from the Publications Unit, Institute of Actuaries, 4 Worcester Street, Oxford OX1 2AW

- *Subject 102[CT1] Financial Mathematics Core Reading 2004[2005]* . Faculty & Institute of Actuaries (2003[2004]).
- J.J. McCutcheon and W.F. Scott: *An Introduction to the Mathematics of Finance.* Heinemann (1986)
- P. Zima and R.P. Brown: *Mathematics of Finance.* McGraw-Hill Ryerson (1993)

- H.U. Gerber: *Life Insurance Mathematics.* 3rd edition, Springer (1997)
- N.L. Bowers et al: *Actuarial mathematics.* 2nd edition, Society of Actuaries (1997)

3.5.2 Actuarial Science II – Dr Clark – 16 HT

Synopsis Liabilities under a simple assurance contract or annuity contract. Premium reserves, Thiele's differential equation. Expenses and office premiums.

The no-arbitrage assumption, arbitrage-free pricing. Price and value of forward contracts, effect of fixed income or fixed dividend yield from the asset. Futures, options and other financial products.

Investment and risk characteristics of investments. Term structure of interest rates, spot rates and forward rates, yield curves. Stability of investment portfolios, analysis of small changes in interest rates, Redington immunisation.

Simple stochastic interest rate models, mean-variance models, log-normal models. Mean, variance and distribution of accumulated values of simple sequences of payments.

Reading All of the following are available from the Publications Unit, Institute of Actuaries, 4 Worcester Street, Oxford OX1 2AW

- *Subject 102[CT1] Financial Mathematics Core Reading 2004[2005]* . Faculty & Institute of Actuaries (2003[2004]).
- J.J. McCutcheon and W.F. Scott: *An Introduction to the Mathematics of Finance.* Heinemann (1986)
- H.U. Gerber: *Life Insurance Mathematics.* 3rd edition, Springer (1997)
- N.L. Bowers et al: *Actuarial mathematics.* 2nd edition, Society of Actuaries (1997)

3.6 OCS1 Functional Programming and Data Structures and Algorithms

[Paper CS1 in Honour Moderations in Computer Science. Teaching responsibility of the Computing Laboratory.]

Level: H-level **Method of Assessment:** 3-hour examination plus assessed practical assignment

Prerequisites: None

Weight: One unit

3.6.1 Functional Programming — Prof Bird — 16 MT

Overview This is a first course in programming. It makes use of a programming language called Haskell, in which programs can be viewed as mathematical functions. This makes the language very powerful, so that we can easily construct programs that would be difficult or very large in other languages.

An important theme of the course is how to apply mathematical reasoning to programs, so as to prove that a program performs its task correctly, or to derive it by algebraic manipulation from a simpler but less efficient program for the same problem.

The course provides hands-on experience of programming through two lab exercises: the first one aims to make you acquainted with the mechanics of writing Haskell programs, and the second one tackles a more challenging programming task.

Learning Outcomes At the end of the course the student will be able to:

Write programs in a functional style;

Reason formally about functional programs;

Use polymorphism and higher-order functions;

Reason about the time and space complexity of programs.

Syllabus Principles of functional programming: expressions, evaluations, functions, and types. Type definitions and built-in types: numbers, characters, strings and lists. Basic operations on lists, including map, fold and filter, together with their algebraic properties. Recursive definitions and structural induction. Simple program calculation. Infinite lists and their uses. Further data structures: binary trees, general trees. Use of trees for representing sets and symbolic data. Normal order reduction and lazy evaluation. Simple cost models for functional programs; time and space complexity.

Synopsis Programming with a functional notation: sessions and scripts, expressions and values. Evaluation of expressions. Case study: Approximating square roots. Reduction strategies: innermost vs outermost. [1 lecture]

Types and strong-typing. Basic types: Booleans and truth values. Simple programs involving pattern matching. Polymorphism and type classes. Functional application and currying. Functional composition. More types: characters, strings, tuples. Type synonyms. [2 lectures]

Lists and their operations; list comprehensions. The functions map, foldl, foldr, concat and filter. Many small examples illustrating the use of these functions in a compositional style of programming. [3 lectures]

Time complexity. Asymptotic notation. Advice on writing efficient programs; use of accumulating parameters. [2 lectures]

Recursion and induction. The algebraic properties of list functions and their proof by equational reasoning. Simple manipulation of programs to achieve efficiency. [2 lectures]

Infinite lists and their applications: Pascal's triangle, digits of a number, sieve of Eratosthesnes. Infinite lists as limits. Proving properties of infinite lists: induction, take lemma. Cyclic structures. [2 lectures]

Non-linear data structures. Binary trees and the relationship between size and depth. Binary search trees for representing sets. Insertion and searching in a binary search tree. Representing and evaluating arithmetic expressions. [3 lectures]

More advice on writing efficient programs: halve instead of decrease; tupling and accumulation. Space complexity and the use of strict. [1 lecture]

More substantial examples if time allows.

Reading *Course text*

Richard Bird, *Introduction to Functional Programming using Haskell*, second edition, Prentice-Hall International (1998).

Alternative Reading

Richard Bird and Philip Wadler, *Introduction to Functional Programming*, Prentice-Hall International (1988).

Simon Thompson, *Haskell: The Craft of Functional Programming*, Addison-Wesley (1996).

Paul Hudak, *The Haskell School of Expression*, Cambridge University Press (2000).

3.6.2 Data Structures and Algorithms — Prof McColl — 16 HT

Overview This core course covers good principles of algorithm design, elementary analysis of algorithms, and fundamental data structures. The emphasis is on choosing appropriate data structures and designing correct and efficient algorithms to operate on these data structures.

Learning Outcomes This is a first course in data structures and algorithm design.

Students will:

learn good principles of algorithm design;

learn how to analyse algorithms and estimate their worst-case and average-case behaviour (in easy cases);

become familiar with fundamental data structures and with the manner in which these data structures can best be implemented;

become accustomed to the description of algorithms in both functional and procedural styles;

learn how to apply their theoretical knowledge in practice (via the practical component of the course).

Syllabus Basic strategies of algorithm design: top-down design, divide and conquer, average and worst-case criteria, asymptotic costs. Simple recurrence relations for asymptotic costs. Choice of appropriate data structures: arrays, lists, stacks, queues, trees, heaps, priority queues, graphs, hash tables. Applications to sorting and searching, matrix algorithms, shortest-path and spanning tree problems. Introduction to discrete optimisation algorithms: dynamic programming, greedy algorithms. Graph algorithms: depth first and breadth first search.

Synopsis Program costs: time and space. Worst case and average case analysis. Asymptotics and "big O" notation. Polynomial and exponential growth. Asymptotic estimates of costs for simple algorithms. Use of induction and generating functions. [2 lectures]

Data structures and their representations: arrays, lists, stacks, queues, trees, heaps, priority queues, graphs. [3 lectures]

Algorithm design strategies: top down design, divide and conquer. Application to sorting and searching and to matrix algorithms. Solution of relevant recurrence relations. [4 lectures]

Graph algorithms: examples of depth-first and breadth-first search algorithms. Topological sorting, connected components. [3 lectures]

Introduction to discrete optimisation algorithms: dynamic programming, greedy algorithms, shortest path problems. [2 lectures]

Linear sorting and comparator networks (if time).

Reading *Primary Text*

Thomas Cormen, Charles Leiserson, Ronald Rivest and Cliff Stein, *Introduction to Algorithms*, second edition, MIT Press and McGraw-Hill (2001).

Alternatives/Background

Richard Bird, *Introduction to Functional Programming using Haskell*, second edition, Prentice-Hall International (1998).

Gilles Brassard and Paul Bratley, *Fundamentals of Algorithmics*, Prentice-Hall (1996).

Martin Reiser and Niklaus Wirth, *Programming in Oberon: steps beyond Pascal and Modula*, Addison-Wesley (1992).

Thomas Cormen, Charles Leiserson, Ronald Rivest, *Introduction to Algorithms*, MIT Press (1990), (An old version of the primary text, to be found in many libraries).

3.7 OB21 Numerical Solution of Differential Equations

[From Part B2 in the Honour School of Computer Science. Teaching responsibility of the Computing Laboratory.]

3.7.1 Numerical Solution of Differential Equations I — Prof Süli — 16 MT

Overview To introduce and give an understanding of numerical methods for the solution of ordinary and partial differential equations, their derivation, analysis and applicability.

The MT lectures are devoted to numerical methods for initial value problems, while the HT lectures concentrate on the numerical solution of boundary value problems.

Syllabus Initial value problems for ordinary differential equations: Euler, multistep and Runge-Kutta; stiffness; error control and adaptive algorithms.

Initial value problems for partial differential equations: parabolic equations, hyperbolic equations; explicit and implicit methods; accuracy, stability and convergence, Fourier analysis, CFL condition.

Synopsis The MT part of the course is devoted to the development and analysis of numerical methods for initial value problems. We begin by considering classical techniques for the numerical solution of ordinary differential equations. The problem of stiffness is then discussed in tandem with the associated questions of step-size control and adaptivity.

Initial value problems for ordinary differential equations: Euler, multistep and Runge-Kutta; stiffness; error control and adaptive algorithms. [Introduction (1 lecture) + 5 lectures]

The remaining lectures focus on the numerical solution of initial value problems for partial differential equations, including parabolic and hyperbolic problems.

Initial value problems for partial differential equations: parabolic equations, hyperbolic equations; explicit and implicit methods; accuracy, stability and convergence, Fourier analysis, CFL condition. [10 lectures]

Reading List The course will be based on the following textbooks:

K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations*, Cambridge University Press (1994). ISBN 0-521-42922-6 (Paperback edition) [Chapters 2, 3 (Secs. 3.1, 3.2), Chapter 4 (Secs. 4.1-4.6), Chapter 5]

E. Süli and D. Mayers, *An Introduction to Numerical Analysis*, Cambridge University Press (2003). ISBN 0-521-00794-1 (Paperback edition) [Chapter 12]

A. Iserles, *A First Course in the Numerical Analysis of Differential Equations*, Cambridge University Press (1996). ISBN 0-521-55655-4 (Paperback edition) [Chapters 1-5, 13, 14]

3.7.2 Numerical Solution of Differential Equations II — Dr Wathen — 16 HT

Overview To introduce and give an understanding of numerical methods for the solution of ordinary and partial differential equations, their derivation, analysis and applicability.

The MT lectures are devoted to numerical methods for initial value problems, while the HT lectures concentrate on the numerical solution of boundary value problems.

Syllabus Boundary value problems for ordinary differential equations: shooting and finite difference methods.

Boundary value problems for partial differential equations: finite difference discretisation; Poisson equation. Associated methods of sparse numerical algebra: brief consideration of sparse Gaussian elimination, classical iterations, multigrid iterations.

Synopsis The HT part of the course is concerned with numerical methods for boundary value problems. We begin by developing numerical techniques for the approximation of boundary value problems for second-order ordinary differential equations.

Boundary value problems for ordinary differential equations: shooting and finite difference methods. [Introduction (1 lecture) + 2 lectures]

Then we consider finite difference schemes for elliptic boundary value problems. This is followed by an introduction into the theory of direct and iterative algorithms for the solution of large systems of linear algebraic equations which arise from the discretisation of elliptic boundary value problems.

Boundary value problems for partial differential equations: finite difference discretisation; Poisson equation. Associated methods of sparse numerical algebra: brief introduction to sparse Gaussian elimination, classical iterations, multigrid iterations. [13 lectures]

Reading List This course will approximately follow the textbook A. Iserles, *A First Course in the Numerical Analysis of Differential Equations*, Cambridge University Press (1996). The material covered will correspond to Chapters 1,2,3,4,5,7,9,10,11,13,14.

Some of this material can also be found in K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations*, Cambridge University Press (1994). Or the more recent 2nd Edition (2005)

3.8 OB22 Integer Programming — Dr Hauser — 16 MT

[From Part B2 in the Honour School of Computer Science. Teaching responsibility of the Computing Laboratory.]

NB: B11 Communication Theory is a very suitable complement to this course.

Level: H-level **Method of Assessment:** This paper is an option in Part B2 in the Honour School of Computer Science, which will usually be examined in a $1\frac{1}{2}$-hour examination.

Prerequisites: None

Weight: Half-unit

Aims In many areas of practical importance linear optimisation problems occur with integrality constraints imposed on some of the variables. In optimal crew scheduling for example, a pilot cannot be fractionally assigned to two different flights at the same time. Likewise, in combinatorial optimisation an element of a given set either belongs to a chosen subset or it does not. Integer programming is the mathematical theory of such problems and of algorithms for their solution. The aim of this course is to provide an introduction to some of the general ideas on which attacks to integer programming problems are based: generating bounds through relaxations by problems that are easier to solve, and branch-and-bound.

Synopsis Lecture 1: Course outline: What is integer programming (IP)? Some classical examples.

Lecture 2: Further examples, hard and easy problems.

Lecture 3: Alternative formulations of IPs, linear programming (LP) and the simplex method.

Lecture 4: LP duality, sensitivity analysis.

Lecture 5: Optimality conditions for IP, relaxation and duality.

Lecture 6: Unimodularity, network flows.

Lecture 7: Optimal trees, submodularity, matroids and greedy algorithms.

Lecture 8: Augmenting paths and bipartite matching.

Lecture 9: The assignment problem.

Lecture 10: Dynamic programming.

Lecture 11: Knapsack problems.

Lecture 12; Branch-and-bound.

Lecture 13: More on branch-and-bound.

Lecture 14: Lagrangian relaxation and the symmetric travelling salesman problem.

Lecture 15: Solving the Lagrangian dual.

Lecture 16: Semidefinite programming relaxation and the max-cut problem.

Reading L.A. Wolsey, *Integer Programming*, John Wiley & Sons, 1998, parts of chapters 1–5 and 7.

Further materials posted on the course web page.

4 Non-Mathematical units and half-units

4.1 N1 Undergraduate Ambassadors' Scheme — mainly HT

Level: H-level **Method of Assessment:** Journal of activities, Oral presentation, Course report and project, Teacher report

Prerequisites: None

Weight: Half-unit

Quota: There will be a quota of approximately 16 students for this course.

Co-ordinator: Dr Earl

The Undergraduate Ambassadors' Scheme (UAS) was begun by Simon Singh in 2002 to give university undergraduates a chance to experience assisting and, to some extent, teaching in schools and to be credited for this. The Oxford UAS option, N1, is a half-unit, mainly run in Hilary Term and will be available to BA and MMath Mathematics students in their third year. A quota will be in place, of approximately 16 students, and so applicants for the UAS option will be asked to name a second alternative half-unit.

A student on the course will be assigned to a mathematics teacher in a local secondary school (in the Oxford, Abingdon, Kidlington area) for half a day per week during Hilary

Term. Students will be expected to keep a journal of their activities, which will begin by assisting in the class, but may widen to include teaching the whole class for a part of a period, or working separately with a smaller group of the class. Students will be required at one point to give a presentation to one of their school classes relating to a topic from university mathematics, and will also run a project for a class or group with advice from the teacher. This project might include demonstrations relating to a certain topic, production of website material, arranging a academic-oriented visit to the university etc. Final credit will be based on the journal (20%), the presentation (30%), an end of course report (2000-3000 words) including details of the project (35%), together with a report from the teacher (15%).

Interviews will take place on Thursday or Friday of 0th week in Michaelmas term to select students for this course. The interview (of roughly 15 minutes) will include a presentation by the student on an aspect of mathematics of their choosing. Students will be chosen on the basis of their keenness and ability to communicate mathematics, and two references will be sought from college tutors on these qualities. Applicants will be quickly notified of the decision.

During Michaelmas term there will be a Training Day, in conjunction with the Department of Educational Studies, as preparation for working with pupils and teachers, and to provide more detail on the organisation of teaching in schools. Those on the course will also need to fill in a CRB form, or to have done so already. By the end of term students will have been assigned to a teacher and have made a first, introductory, visit to their school. The course will begin properly in Hilary term with students helping in schools for half a day each week. Funds are available to cover travel expenses. Support classes will be provided throughout Hilary for feedback and to discuss issues such as the planning of the project. The deadline for the journal and report will be Friday of 0th week of Trinity term.

Any further questions on the UAS option should be passed on to the option's co-ordinator, Richard Earl (earl@maths.ox.ac.uk).

4.2 N101 History of Philosophy from Descartes to Kant — lecturer and term t.b.a.

[Paper 101 in all Honour Schools including Philosophy. Teaching responsibility of the Philosophy Faculty.]

Level H-level **Method of Assessment:** 3-hour examination

Weight: One unit

Candidates will be expected to show critical appreciation of the main philosophical ideas of the period. The subject will be studied in connection with the following texts: Descartes, *Meditations, Objections and Replies*; Spinoza, *Ethics*; Leibniz, *Monadology, Discourse on Metaphysics*; Locke, *Essay Concerning Human Understanding*; Berkeley, *Principles of Human Knowledge, Three Dialogues Between Hylas and Philonous*; Hume, *Treatise of Human Nature*; Kant, *Critique of Pure Reason.* Candidates will not be required to show knowledge of all the texts, but will be required to show adequate knowledge of at least two authors. Some questions will be set to allow detailed discussion of the interpretation and historical significance of the texts. [Examination Regulations 2004, p. 481]

4.3 N102 Knowledge and Reality — lecturer and term t.b.a.

[Paper 102 in all Honour Schools including Philosophy. Teaching responsibility of the Philosophy Faculty.]

Level H-level **Method of Assessment:** 3-hour examination

Weight: One unit

Candidates will be expected to show knowledge in some of the following areas: knowledge and justification; perception; memory; induction; other minds; *a priori* knowledge; necessity and possibility; reference; truth; facts and propositions; definition; existence; identity, including personal identity; substances, change, events; properties; causation; space; time; essence; natural kinds; realism and idealism; primary and secondary qualities. There will also be a section on Philosophy of Science. Candidates' answers must not be confined to questions from the section on Philosophy of Science. [Examination Regulations 2004, p. 481]

4.4 N122 Philosophy of Mathematics — lecturer and term t.b.a.

[Paper 122 in all Honour Schools including Philosophy. Teaching responsibility of the Philosophy Faculty.]

Level H-level **Method of Assessment:** 3-hour examination

Weight: One unit

Prerequisites No candidate will be permitted this option who is not offering Paper B1

Questions may be set which relate to the following issues: Incommensurables in the development of Greek geometry. Comparisons between geometry and other branches of mathematics. The significance of non-Euclidean geometry. The problem of mathematical rigour in the development of the calculus. The place of intuition in mathematics (Kant, Poincaré). The idea that mathematics needs foundations. The role of logic and set theory (Dedekind, Cantor, Frege, Russell). The claim that mathematics must be constructive (Brouwer). The finitary study of formal systems as a means of justifying infinitary mathematics (Hilbert). Limits to the formalization of mathematics (Gödel). Anti-foundational views of mathematics. Mathematical objects and structures. The nature of infinity. The applicability of mathematics. [Examination Regulations 2004, p. 485]

5 Extended Essays

See the "Projects Guidance Notes" on the website at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/ for more information on these options and an application form.

5.1 BE "Mathematical" Extended Essay

Level H-level **Method of Assessment:** written essay

Weight: One unit

5.2 OE "Other Mathematical" Extended Essay

Level H-level **Method of Assessment:** written essay

Weight: One unit

6 Language Classes

Students in the FHS Mathematics may apply to take language classes. In 2005–6 French Language classes will be run in MT and HT. The courses are run by the University Language Centre.

Students wishing to take language classes should attend the presentation given in Trinity Term at 3.00pm, Friday 20th May (Week 4), at the Language Centre, 12 Woodstock Road. Application forms will be available at these presentations.

Preference will be given to those students with good GCSE-level (or equivalent) French, though additional places can be filled by those with AS-level/A-level (or equivalent) French.

Performance will not contribute to the class of degree awarded in Mathematics. However, upon successful completion of the course, students will be issued with a certificate of achievement which will be sent to their college, and the details may appear on their transcript.

Places at these classes are limited, so students are advised that they must indicate their interest at this stage. If you are interested but were unable to attend this presentation for some reason please contact the Academic Administrator in the Mathematical Institute (academic.administrator@maths.ox.ac.uk; (2)73530) as soon as possible.

Aims and rationale

The general aim of the language courses is to develop the student's ability to communicate (in both speech and writing) in French to the point where he or she can function in an academic or working environment in a French-speaking country.

The courses have been designed with general linguistic competence in the four skills of reading, writing, speaking and listening in mind. There will be opportunities for participants to develop their own particular interests through presentations and assignments.

Form and Content

Each course will consist of a thirty-two contact hour course, together with associated work. Classes will be held in the Language Centre at two hours per week in Michaelmas and Hilary Terms.

The content of the courses is based on course books together with a substantial amount of supplementary material prepared by the language tutors. Participants should expect to spend an additional two hours per week on preparation and follow-up work.

Each course aims to cover similar ground in terms of grammar, spoken and written language and topics. Areas covered will include:

Grammar:

- all major tenses will be presented and/or revised, including the subjunctive

- passive voice
- pronouns
- formation of adjectives, adverbs, comparatives
- use of prepositions
- time expressions

Speaking

- guided spoken expression for academic, work and leisure contact (e.g. giving presentations, informal interviews, applying for jobs)
- expressing opinions, tastes and preferences
- expressing cause, consequence and purpose

Writing

- guided letter writing for academic and work contact
- summaries and short essays

Listening

- listening practice of recorded materials (e.g. news broadcasts, telephone messages, interviews)
- developing listening comprehension strategies

Topics (with related readings and vocabulary, from among the following)

- life, work and culture
- the media in each country
- social and political systems
- film, theatre and music
- research and innovation
- sports and related topics
- student-selected topics

Teaching staff

The courses are taught by Language Centre tutors or Modern Languages Faculty instructors.

Teaching and learning approaches

Each course uses a communicative methodology which demands active participation from students. In addition, there will be some formal grammar instruction. Students will be expected to prepare work for classes and homework will be set. Students will also be given training in strategies for independent language study, including computer-based language learning, which will enable them to maintain and develop their language skills after the course.

Entry

Two classes will be formed according to level of French at entry (in 2004-2005 these were at about post-GCSE and post-A level standard). The minimum entry standard is a good GCSE Grade or equivalent. Registration for each option will take place at the start of Michaelmas Term. A preliminary qualifying test is then taken which candidates must pass in order to be allowed to join the course.

Learning Outcomes

The learning outcomes will be based on internationally agreed criteria for specific levels (known as the ALTE levels), which are as follows:

Basic Level (corresponds to ALTE Level 2 “Can-do” statements)

- Can express opinions on professional, cultural or abstract matters in a limited way, offer advice and understand instructions.
- Can give a short presentation on a limited range of topics.
- Can understand routine information and articles, including factual articles in newspapers, routine letters from hotels and letters expressing personal opinions.
- Can write letters or make notes on familiar or predictable matters.

Threshold Level (corresponds to ALTE Level 3 “Can-do” statements)

- Can follow or give a talk on a familiar topic or keep up a conversation on a fairly wide range of topics, such as personal and professional experiences, events currently in the news.
- Can give a clear presentation on a familiar topic, and answer predictable or factual questions.
- Can read texts for relevant information, and understand detailed instructions or advice.
- Can make notes that will be of reasonable use for essay or revision purposes.
- Can make notes while someone is talking or write a letter including non-standard requests.

Higher Level (corresponds to ALTE Level 4 ”Can-do” statements)

- Can contribute effectively to meetings and seminars within own area of work or keep up a casual conversation with a good degree of fluency, coping with abstract expressions.

- Can follow abstract argumentation, for example, balancing alternatives and drawing a conclusion.
- Can read quickly enough to cope with an academic course, to read the media for information or to understand non-standard correspondence.
- Can prepare/draft professional correspondence, take reasonably accurate notes in meetings or write an essay which shows an ability to communicate, giving few difficulties for the reader.

Assessment

There will a preliminary qualifying test in Michaelmas Term. There are three parts to this test: Reading Comprehension, Listening Comprehension, and Grammar. Candidates who have not studied or had contact with French for some time are advised to revise thoroughly, making use of the Language Centre's French resources.

Students' achievement will be assessed through a variety of means, including continuous assessment of oral performance, a written final examination, and a project or assignment chosen by individual students in consultation with their language tutor.

Reading comprehension, writing, listening comprehension and speaking are all examined. The oral component may consist of an interview, a presentation or a candidate's performance in a formal debate or discussion.

Feedback Oral and written feedback from students are encouraged. A Feedback Form will be distributed at the end of each course.

7 Registration for Part B courses 2005–06

CLASSES: Students will have to register in advance for the classes they wish to take. Students will have to register by Friday of Week 8 of Trinity Term 2005 using the form overleaf. Students who register for a course or courses for which there is a quota will have to indicate a "reserve choice" which they will take if they do not receive a place on the course with the quota. They may also have to give the reasons why they wish to take a course which has a quota, and provide the name of a tutor who can provide a supporting statement for them should the quota be exceeded. In the event that the quota for a course is exceeded, the Mathematics Teaching Committee will decide who may have a place on the course on the basis of the supporting statements from the student and tutor. Students who are not allocated a place on the course with the quota will be registered for their "reserve" course. Students will be notified of this by email. In the case of the "Undergraduate Ambassadors' Scheme" students will have to attend a short interview in Week 0, Michaelmas Term. In the meantime they will also have to complete a separate application form, and indicate a "reserve" course.

LECTURES: Some combinations of subjects are not advised and lectures may clash. Details are given below. STUDENTS WHO WISH TO TAKE ANY OF THESE COMBINATIONS OF LECTURES SHOULD NOTIFY THE ACADEMIC ADMINISTRATOR, MATHEMATICAL INSTITUTE (academic.administrator@maths.ox.ac.uk) BY FRIDAY OF WEEK 6 IN TRINITY TERM OF THE PRECEDING YEAR. We will attempt to ensure that any lecture combinations of which we have been notified do not clash. However, we are not be able to guarantee this, and will not be able to confirm it until all Hilary Term lectures have been timetabled in the Christmas vacation. Because of the large number of options available in Part B some clashes are inevitable, and we must aim to accommodate the maximum number of student preferences.

B1 Logic and Set Theory B2 Algebra B3 Geometry B9 Number Theory	may clash with	B6 Theoretical Mechanics B8 Topics in Applied Mathematics

B1 Logic and Set Theory B2 Algebra B3 Geometry B9 Number Theory	may clash with	OBS1 Applied Statistics OBS2b Statistical Inference OBS3 Stochastic Modelling OBS4 Actuarial Science

B3 Geometry B6 Theoretical Mechanics B7 Mathematical Physics B8 Topics in Applied Mathematics B9 Number Theory C3.1 Geometry: Lie Groups and Differentiable Manifolds	may clash with	OCS1 Functional Programming and Data Structures and Algs and all options in FHS Computer Science and Mathematics & Computer Science except OB21 Numerical Solutions to Partial Differential Equations and OB22 Integer Programming

B5 Applied Analysis B6 Theoretical Mechanics B7 Mathematical Physics B8 Topics in Applied Mathematics B10 Martingales & Financial Maths B11 Communication Theory	may clash with	FHS Mathematics & Philosophy core Philosophy options

8 REGISTRATION FORM: PART B CLASSES 2005–06

This form is for students in FHS Mathematics and FHS Mathematics & Philosophy. Students in FHS Mathematics & Statistics and FHS Mathematics & Computer Science will be contacted separately about options.

FHS MATHEMATICS: Candidates shall offer a total of four units in Part B. A total of at least three units offered should be from the schedule of "Mathematics Department units and half-units".

FHS MATHEMATICS & PHILOSOPHY: Candidates shall offer a total of at least two units in Mathematics in Part B. These should be from the schedule of "Mathematics Department units and half-units". One of these must be "Foundations" (B1 Logic and Set Theory). [This is usually studied in the second-year. Only tick it here if you need to attend classes in 2005–06.]

SURNAME ....FIRST NAME ....

EMAIL ADDRESS ....

COLLEGE ....

THIS FORM MUST BE RETURNED TO THE ACADEMIC ASSISTANT, MATHEMATICAL INSTITUTE, BY FRIDAY OF WEEK 8, TRINITY TERM (17 JUNE 2005).

I wish to take classes in the following options: [Please Tick]

MATHEMATICS DEPARTMENT UNITS AND HALF-UNITS

☐ B1a Logic — MT (half-unit)

☐ B1b Set Theory — HT (half-unit)

☐ B2 Algebra — MT & HT (whole unit)

☐ B3a Geometry of Surfaces — MT (half-unit)

☐ B3b Algebraic Curves — HT (half-unit)

☐ B4 Analysis — MT & HT (whole unit) or ☐ B4a Analysis I — MT (half-unit)

☐ B5a Techniques of Applied Mathematics — MT (half-unit)

☐ B5b Applied Partial Differential Equations — HT (half-unit)

☐ B6a Viscous Flow — MT (half-unit)

☐ B6b Waves and Compressible Flow — HT (half-unit)

☐ B7 Electromagnetism, Quantum Mechanics and Special Relativity — MT & HT (whole unit) or ☐ B7a Quantum Mechanics and Electromagnetism — MT (half-unit)

☐ B8a Mathematical Ecology and Biology — MT (half-unit)

☐ B8b Nonlinear Systems — HT (half-unit)

☐ B9 Number Theory — MT & HT (whole unit) or ☐ B9a Polynomial Rings and Galois Theory — MT (half-unit)

☐ B10a Martingales Through Measure Theory — MT (half-unit)

☐ B10b Mathematical Models of Financial Derivatives — HT (half-unit)

☐ B11 Communication Theory — MT (half-unit)

NB: This course has a quota. Please indicate here which course you wish to take if it is not possible for us to offer you classes in this subject:

. . . .

Please give your reasons for wishing to take this course:

. . . .

. . . .

. . . .

. . . .

Please give the name of a tutor who could provide a statement of support for you taking this course:

. . . .

☐ C3.1a Lie Groups — MT (half-unit: M-level)

☐ C3.1b Differentiable Manifolds — HT (half-unit: M-level)

☐ C9.1a Introduction to Modular Forms — MT (half-unit: M-level)

☐ C5.1a Partial Differential Equations for Pure and Applied Mathematicians — MT (half-unit: M-level)

NB: In Part B, candidates may offer at most one unit from those designated as M-level.

☐ BE Extended Essay in Mathematics (whole unit)

NB: Classes are not applicable. Students wishing to offer an Extended Essay must apply to the Projects Committee by Friday of Week 3 of Michaelmas Term using the form provided at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

OTHER MATHEMATICAL UNITS AND HALF-UNITS

☐ O1 History of Mathematics — MT & HT (whole unit)

NB: This course has a quota. Please indicate here which course you wish to take if it is not possible for us to offer you classes in this subject:

. . . .

Please give your reasons for wishing to take this course:

. . . .

. . . .

. . . .

. . . .

Please give the name of a tutor who could provide a statement of support for you taking this course:

. . . .

☐ OBS1 Applied Statistics — MT & HT (whole unit)

☐ OBS2 Statistical Inference — HT (half-unit)

☐ OBS3 Stochastic Modelling — MT & HT (whole unit) or ☐ OBS3a Applied Probability — MT (half-unit)

☐ OBS4 Actuarial Science — MT & HT (whole unit)

☐ OCS1 Functional Programming and Data Structures and Algorithms — MT & HT (whole unit)

☐ OB21 Numerical Solution of Differential Equations — MT & HT (whole unit)

☐ OB22 Integer Programming — MT (half-unit)

☐ OE Extended Essay in a topic closely related to Mathematics (whole unit)

NB: Classes are not applicable. Students wishing to offer an Extended Essay must apply to the Projects Committee by Friday of Week 3 of Michaelmas Term using the form provided at
http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

OTHER NON-MATHEMATICAL UNITS AND HALF-UNITS

☐ N1 Undergraduate Ambassadors' Scheme — (MT, HT) (half-unit)

NB: This course has a quota. Please indicate here which course you wish to take if it is not possible for us to offer you teaching in this subject:

. . . .

You will need to complete the separate application form for this course, provide the names of two referees and attend a brief interview in Week 0, Michaelmas Term. Details will be sent to Mathematics Tutors.

☐ N101 History of Philosophy from Descartes to Kant (whole unit)

☐ N102 Knowledge and Reality (whole unit)

☐ N122 Philosophy of Mathematics (whole unit)

NB: Options N101, N102, N122: Class teaching is **not** *arranged by the Mathematical Institute, and arrangements for your teaching should be made by your college tutors.*