

## PROBLEM SHEET 1

**1.1** Find the radius and centre of the circle described by the equation

$$x^2 + y^2 - 2x - 4y + 1 = 0$$

by writing it in the form  $(x - a)^2 + (y - b)^2 = c^2$  for suitable  $a, b$  and  $c$ .

**1.2** Find the equation of the line perpendicular to  $y = 3x$  passing through the point  $(3, 9)$ .

**1.3** Given

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \quad \text{and} \quad \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B,$$

show that

$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)] \quad \text{and} \quad \sin^2 A = \frac{1}{2}[1 - \cos 2A].$$

**1.4** Show that

$$4 \cos(\alpha t) + 3 \sin(\alpha t) = 5 \cos(\alpha t + \phi)$$

where  $\phi = \arctan(-3/4)$ .

**1.5** Show that, for  $-1 \leq x \leq 1$ ,

$$\cos(\sin^{-1} x) = \pm \sqrt{1 - x^2}.$$

**1.6** Given

$$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B \quad \text{and} \quad \cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B,$$

show that

$$\cosh A \cosh B = \frac{1}{2}[\cosh(A + B) + \cosh(A - B)] \quad \text{and} \quad \sinh^2 A = \frac{1}{2}[\cosh 2A - 1].$$

**1.7** Given that

$$\sinh x = \frac{1}{2}[e^x - e^{-x}],$$

show that

$$\sinh^{-1} x = \ln \left[ x + \sqrt{1 + x^2} \right].$$

**1.8** Express

$$\frac{x}{(x - 1)(x - 2)}$$

in partial fractions.

**1.9** If  $a_n = \frac{1}{n}$ , find  $\sum_{i=1}^5 a_n$  as a fraction.

**1.10** If  $S = \sum_{i=0}^N x^i$ , show that  $xS = \sum_{i=1}^{N+1} x^i$ . Hence show that  $S - xS = 1 - x^{N+1}$  and therefore that

$$S = \frac{1 - x^{N+1}}{1 - x}.$$