Examiners' Report: Final Honour School of Mathematics Part A Trinity Term 2014

October 28, 2014

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2014	2013	2012	2011	2010	2014	2013	2012	2011	2010
70–100	57	49	56	55	50	36.54	31.21	33.73	33.33	32.89
60–69	62	71	78	79	72	39.74	45.22	46.99	47.88	45.39
50–59	31	32	28	23	20	19.87	20.38	16.87	13.94	13.16
40–49	4	4	2	7	10	2.56	2.55	1.2	4.24	6.58
30–39	2	1	2	1	0	1.28	0.64	1.2	0.61	0
0–29	0	0	0	0	0	0	0	0	0	0
Total	156	157	166	165	152	100	100	100	100	100

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.
- **Marking of scripts.**
  All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for papers A1 and A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.
- **Numbers taking each paper.**
  All 156 candidates are required to offer the core papers A1, A2 and ASO, and five of the optional papers A3-A11. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A1	156	64.45	15.81	65.67	11.85
A2	156	65.09	15.79	65.83	10.44
A3	88	29.95	8.5	65.53	11.74
A4	98	30.93	10.78	66.01	15.74
A5	104	34.52	9.5	69.52	15.55
A6	78	30.72	6.28	65.38	8.74
A7	72	34.90	8.17	64.83	9.98
A8	134	32.83	9.18	65.70	14.55
A9	97	36.65	9.15	69.22	16.31
A10	42	40.81	6.27	68.95	13.79
A11	67	39.91	6.53	67.33	13.59
ASO	156	32.79	8.10	66.63	11.94

B. New examining methods and procedures

This was the first year of the new Part A structure. The core papers AC1 and AC2 have been replaced with core papers A1 and A2. The cross-sectional papers AO1 and AO2 have been replaced with option papers A3-A11. In addition there is a core cross-sectional paper, ASO, examining the short option courses.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 19th February 2014 and the second notice on the 5th May 2014.

These can be found at https://www.maths.ox.ac.uk/notices/undergrad/2013-14/part-a, and contain details of the examinations and assessments. The course Handbook contains the link to the full examination conventions and all candidates are issued with this at Induction in their first year. All notices and examination conventions are on-line at http://www.maths.ox.ac.uk/notices/undergrad.

Part II

A. General Comments on the Examination

The examiners would like to express their gratitude to

• Nia Roderick for overseeing Part A examinations during 2013/14, taking over from Vicky Archibald on a short notice.

• Also Waldemar Schlackow, assisting Helen Lowe, for continuing to develop the examinations database, responding to examiner requests and providing such a good framework for the examinations data.

• We would also like to thank Charlotte Turner-Smith, Sandy Patel, and Jessica Sheard for all their sterling work in keeping track of the scripts and marks and everything else they do during the busy examination period.

• We also thank those assessors who set their questions promptly, took care in checking and marking them, and met their deadlines. This is invaluable help for the work of the examiners.

• All the assessors and the internal examiners would like to thank the external examiner Dr. Mark Wildon for his careful reading of the draft papers, scrutiny of the examination scripts and insightful comments throughout the year, and also during the final meeting.

Timetable

The examinations began on Tuesday 17th June at 2.30pm and ended on Friday 27th June at 11.00am.

Medical certificates and other special circumstances

The Examiners received five medical certificates from the Proctors. Details of cases in which special consideration was required are given in Section F.

Setting and checking of papers and marks processing

As is usual practice, questions for the core papers A1 and A2, were set by the examiners and also marked by them. The papers A3-A11, as well as each individual question on ASO, were set and marked by the course lecturers. The setters produced model answers and marking schemes led by instructions from the teaching committee in order to minimize the need for recalibration.
The internal examiners met in December to consider the questions for Michaelmas Term courses (A1, A2 and A11). The course lecturers for the core papers were invited to comment on the notation used and in general on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to external examiner.
In a second meeting the internal examiners discussed the comments of the external examiner and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the Trinity term short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Before questions were

submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.
Examination scripts were collected by the markers from Ewert House or delivered to the Mathematical Institute for collection by the markers and returned there after marking. A team of graduate checkers under the supervision of Nia Roderick and Charlotte Turner-Smith sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition. An examiner was present at all times to authorise any required corrections.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. Here 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the parameters C_1 and C_2, that are the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from $(C_1, 72)$ to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between $(C_3, 37)$ and $(C_2, 57)$ and then again between $(0, 0)$ and $(C_3, 37)$. It is important that the positions of the corners in the piecewise linear map are not on the class borderlines in order to avoid distortion of the class boundaries. Thus, the conversion is fixed by the choice of the three parameters C_1, C_2 and C_3, the raw marks that are mapped to USM of 72, 57 and 37 respectively.
The examiners chose the values of the parameters as listed in Table 3 guided by the advice from the Teaching Committee and by examining individuals on each paper around the borderlines.

Table 3: Parameter Values

Paper	C1	C2	C3
A1	77.5	49	27.6
A2	80.6	47	24.5
A3	37.2	20.7	11.9
A4	39.8	21	10.8
A5	39	25	13.8
A6	36.8	23.3	13.4
A7	43.4	25.4	14.6
A8	39	27	12.7
A9	40	28.7	16.5
A10	44	34.6	19.9
A11	44	34.6	19.9
ASO	39	24	13.8

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
93	1	1	0.64
90	2	2	1.28
88	3	3	1.92
87	4	5	3.21
86	6	6	3.85
84	7	9	5.77
83	10	12	7.69
82	13	13	8.33
79	14	16	10.26
78	17	18	11.54
77	19	24	15.38
76	25	26	16.67
75	27	30	19.23
74	31	31	19.87
73	32	41	26.28
72	42	47	30.13
71	48	52	33.33
70	53	57	36.54
69	58	67	42.95
68	68	72	46.15

Table 5: Continuation of the Rank and Percentage table overall USMs

Av USM	Rank	Candidates with this USM or above	%
67	73	78	50.00
66	79	84	53.85
65	85	89	57.05
64	90	98	62.82
63	99	102	65.38
62	103	105	67.31
61	106	111	71.15
60	112	119	76.28
59	120	123	78.85
58	124	130	83.33
57	131	133	85.26
56	134	137	87.82
55	138	141	90.38
54	142	143	91.67
53	144	145	92.95
52	146	147	94.23
51	148	149	95.51
50	150	150	96.15
48	151	151	96.79
47	152	152	97.44
46	153	153	98.08
45	154	154	98.72
39	155	156	100.00

B. Equal opportunities issues and breakdown of the results by gender

Table 6, page 7 shows the performances of candidates broken down by gender.

Table 6: Breakdown of results by gender

Range	Total		Male		Female	
	Number	%	Number	%	Number	%
70–100	57	36.54	47	42.34	10	22.22
60–69	62	39.74	41	36.94	21	46.67
50–59	31	19.87	17	15.32	14	31.11
40–49	4	2.56	4	3.6	0	0
30–39	2	1.28	2	1.8	0	0
0–29	0	0	0	0	0	0
Total	156	100	111	100	45	100

C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

Paper A1: Algebra 1 and Differential Equations 1

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.18	16.25	5.18	143	2
Q2	15.86	15.93	4.16	96	2
Q3	13.34	14.75	6.43	72	11
Q4	14.10	14.43	5.13	65	4
Q5	19.01	19.19	4.98	143	2
Q6	13.58	14.01	6.80	104	4

Paper A2: Metric Spaces and Complex Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.64	16.12	5.48	120	7
Q2	15.70	16.03	5.53	115	5
Q3	16.87	16.88	4.26	131	1
Q4	13.98	15.08	6.49	75	11
Q5	16.93	17.14	5.36	114	3
Q6	15.50	15.67	4.82	69	1

Paper A3: Algebra 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.28	14.38	4.80	86	1
Q2	15.49	15.49	5.46	82	0
Q3	13.18	16.13	6.84	8	3

Paper A4: Integration

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.66	14.91	5.96	87	3
Q2	17.40	18.23	6.37	66	4
Q3	11.11	12.35	6.01	43	10

Paper A5: Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.22	19.22	4.43	94	0
Q2	14.09	14.52	6.60	87	4
Q3	18.28	19.26	6.05	27	2

Paper A6: Differential Equations 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.93	16.16	2.95	70	1
Q2	14.08	15.11	6.79	35	4
Q3	13.55	14.43	5.47	51	7

Paper A7: Numerical Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.55	17.55	5.10	58	0
Q2	17.60	18.44	5.25	45	3
Q3	15.37	16.22	4.71	41	5

Paper A8: Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.44	16.79	5.23	117	5
Q2	16.94	16.94	4.52	99	0
Q3	12.95	14.58	6.53	52	10

Paper A9: Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.76	12.19	3.75	21	8
Q2	18.45	18.57	5.09	93	1
Q3	19.17	19.65	5.29	80	3

Paper A10: Waves and Fluids

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.00	15.00	3.16	6	0
Q2	20.81	20.81	3.55	42	0
Q3	20.83	20.83	3.61	36	0

Paper A11: Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.83	16.23	4.13	39	1
Q2	20.47	20.66	3.87	35	1
Q3	21.95	21.97	3.14	60	1

Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.64	17.89	4.53	97	2
Q2	14.11	14.52	4.35	25	2
Q3	17.13	17.13	3.75	23	0
Q4	13.09	14.25	6.18	20	3
Q5	18.56	18.56	5.36	57	0
Q6	13.74	14.25	5.63	81	5
Q7	14.00	14.00	6.65	9	0

D. Recommendations for Next Year's Examiners and Teaching Committee

The Examiners agreed to note the importance, in some papers, of setting questions with sufficient new material to produce a decent spread of marks and so allow differentiation between candidates.

The Maths and Statistics Chair noted that under the new structure the scaling of the joint school papers is more straightforward (which had been a main aim of the restructuring).

It was agreed that it would be useful in future for the database to include the average core papers (A1 and A2) marks of those taking an option, so as to be able to gauge the strength of that option's cohort when scaling.

There were occasions in the database that might be improved for clarity: for example, in the tables for individual candiates, USMs from A1 and A2 were set alongside option USMs without anything to signify their extra weight, so it was hard to gauge the profile of a candidate's performance just looking at the figures; also having raw marks out of 50 alongside USMs out of 100 meant examiners always had to remember to double the first figure to get a sense of how much the percentage had been scaled.

The cross-section paper was perceived to have worked well. However, it was reported that the teaching workload in Trinity term was tough and students had complained of less time to revise. As such it was suggested to recommend to Teaching Committee the importance of ASO questions not being overlong to take the pressure off. For example, the Special Relativity question was found much too hard this year.

E. Comments on sections and on individual questions

The following comments were submitted by the assessors.

Core Papers

A1: Algebra 1 and Differential Equations 1

The students found the differential equations questions rather easy, judging from the large number of scripts with more than twenty marks in two questions.

Q1: [Projections, Primary Decomposition Theorem] The question was built around the projections involved in the Primary Decomposition Theorem, and was chosen by most candidates. Question (a)(ii) was bookwork and could be done either by decomposition over the kernel and image, or by the observation that the minimal polynomial is then a product of distinct linear factors, but in the latter case the result and its consequence were too often not stated clearly. The shortest answer to Question (b)(ii) consisted in showing first directly that the projection has the corresponding block matrix form, and then concluding from (a)(i), although most students went through the correct but significantly longer computation of the square and inspection of the equalities between spaces. Question (a)(iii) was (consequently?) not solved by most candidates, although it only required to draw easy consequences from earlier subquestions.

Q2: [Symplectic matrices] The goal was to show some simple facts on symplectic matrices. Question (a) was meant to prepare for Questions (b)(ii), (c)(i) and (ii), which essentially relied on the observation that the determinant and characteristic polynomial remain the same under transposition and for similar matrices, although the steps were not obvious. The multiplicity part in Question (c)(iii) was too often answered without enough rigour; (c)(iv) required to draw conclusions from the expression of the determinant as the product of its (possibly multiple) eigenvalues, and from the size of the matrix, and was only solved by very few candidates.

Q3: [Positive definite matrices and principal minors] The question showed the classical result that a matrix is definite positive iff all the determinants of its leading principal minors are positive. It was rather popular, probably because it contained all along rather elementary questions. The identification of a space with its bidual was not always well-understood, which caused significant problems in (a)(ii)-(iii), whereas (a)(iv) was easy but often not written rigorously. Question (b)(iii) required to be able to make good use of the tools introduced in earlier subquestions, and was only solved by very few ones.

Q4: The first question was rather classical, and simply required sufficient justifications to get most of the marks. The final question eluded almost all students who attempted it. To obtain the last couple of points, students needed to explain when Picard's Theorem did not apply, whether there was infinitely many solutions (for positive time) or none depended on the initial condition.

Q5: The second question was globally extremely successful. In fact, an oversight in the hint given in (b,ii) lead the students to use a function which could allow to conclude everywhere except at the origin. Because this hint was misleading, this sub-question was marked generously. The main mistake, in the last question, was to use the the uniqueness result derived for linear equations for a non-linear one.

Q6: The third question was also quite well done. Only a handful of students remembered that one cannot divide my zero, whether one is computing Fourier transforms or otherwise. Apart from this very common mistake, the students who computed correctly obtained very satisfactory marks.

A2: Metric Spaces and Complex Analysis

Q1: [basic concepts: homeomorphism, closure, compact, etc.] The solutions were generally of good quality. Though, several candidates did not read part (b)(ii) carefully, i.e. missed that f need not be onto. As well, quite often people did not understand what they had to do in (b)(v) to show that the map is well-defined.

Q2: [product of metric spaces, completeness, contraction] Again, the solutions were mostly of good quality. The parts which appeared to be most difficult were part (b), and also to give an example in part (c) where F is not a contraction. Many candidates just reformulated the identity.

Q3: A very popular question and largely well done. Most attempts included correct statement and proof of the CREs, definition of L, verification of the CREs for L and that $L'(z) = z^{-1}$. A sizeable number then failed to find $zL(z) - z$ as an antiderivative of L (even though this is entirely equivalent integrating $\ln x$) or decided that there was a typo in the question and integrated L'. Part (c) was less well-done: many candidates overcomplicated (i) by trying (and usually failing) to find a Möbius transformation that worked when $z \mapsto (\mathrm{Im}\alpha)z + \mathrm{Re}\alpha$ works; many who had not done (i) still gained some credit in (ii) for appreciating the nub of the question was to find a conformal equivalence between R and H.

Q4: A reasonably popular question though with students tending to do well (26% got 20 or more marks) or poorly (30% getting single figures). Many forgot to mention in the statement of Morera's Theorem that the function needed to be continuous and/or forgot to note in (iii) that the uniform limit of continuous functions is continuous. (b) was quite poorly done with many candidates claiming (or seeking to prove), despite the hint, that the sum converged uniformly on $\mathbb{C}\backslash\mathbb{Z}$. In (c) quite a few efforts were made to use $-\pi/(w \tan \pi w)$ as the integrand rather than the required $\pi/((w^2 - z^2) \tan \pi w)$.

Q5: A popular question, largely well done (32% of attempts receiving 20 or more marks). Despite this there were still a worrying number of students who could not classify the two singularities at the origin in (a). Some students made heavy work of the calculation in (c) by combining the exponentials in the suggested integrand, which makes determining the residue substantially harder.

Q6: An unpopular question, though well done (56% of attempts receiving 20 or more marks). To do part (a)(ii) there was no need to explicitly work out the inverse of stereographic projection, though several successful efforts took this circuitous route; instead it was sufficient to see

$$f \circ \pi(a,b,c) = \frac{1-c}{a+bi} = \frac{(1-c)(a-bi)}{a^2+b^2} = \frac{a-bi}{1+c} = \pi(a,-b,-c)$$

as $a^2+b^2 = 1-c^2$. In (b)(iii), following previous reasoning, one could assume without any loss of generality that $z_1 = 0$ and $z_2 = \infty$, from which the answer follows by direct calculation.

Long Options

A3: Algebra 2

Q1: Question 1 and 2 were attempted by the majority of candidates. Much of question 1 was well answered, though the final part was taxing enough that only a small number of candidates saw what was involved.

Q2: This was the best answered question, and there were a variety of successful approaches used in tackling the last part of the question, which was good to see.

Q3: This was the least popular, but was answered quite well by those who attempted it. Many people were able to solve at least one of the final two parts.

A4: Integration

Q1: Almost all candidates attempted this question, and most of them could get good marks. A few candidates mistaken the absolute convergence with uniform convergence, and argued wrongly the equality in (b). The function in (c) is a sort of typical (though a sum of two typical ones) the candidates should be able to test the integrability by using comparison and known integrability functions. It is still surprising most candidates lost their marks on showing the function is not integrable on the unbounded interval.

Q2: Less candidates choose this question, a few of those who attempted had achieved high marks. But most of those who attempted had difficulty to find correct control functions which should not depend on the parameter (by using the theorem that a continuous function on a compact subset is bounded from Analysis), and thus lost big chuck of their marks for (b).

Q3: Part (a) is bookwork, easy mark to earn, but still a few candidates didn't say that the function is measurable before writing the repeated integrals when stated the Tonelli's theorem. While surprisingly very few forgot that the absolute sign is needed, well done!
(b) Most candidates knew that one should use Tonelli's and polar coordinates to test the integrability. A few candidates just throw away the sin from the beginning which led to confusion late on, while most candidates were able to swap the repeated integral into a single integral (in polar coordinates) and spotted the different behaviours of the function near zero and near infinity to argue the integrability correctly.
(c) It proved the hard part of the question – most candidates ignored the condition that the function is non-negative to justify the function g is well defined, and tried to prove the impossible – integrability. But most candidates who attempted this question were able to use Tonelli's and simple comparison, by computing a repeated integral, to show the integrability.

A5: Topology

Overall, the quality of the students' scripts was very high indeed. It was pleasing to see that almost all the students had gained a very good understanding of the main concepts in the course. In particular, almost all students could accurately work with the Hausdorff condition, the quotient topology and the product topology, which has something that has not always been the case in previous years. Also, it was good to see that there were many very good answers to the question based on simplicial complexes and surfaces.
The exam seemed to work well, in that there were parts that could be attempted successfully by the weaker students, and yet there was enough challenging material to reliably differentiate between students at the class boundaries.

Q1: Generally, this question was answered well. The question focused on some foundational issues relating to the continuity of maps, and then applied this to the cofinite topology. It tested a reasonably broad set of concepts, including the Hausdorff condition and the quotient topology. The question was successful, in that it was accessible to students of all abilities, but few were able to gain full marks.

Q2: This question had two parts. The first was a substantial and difficult piece of bookwork, where the students were required to show a product $X \times Y$ of topological spaces is compact if and only if X and Y are compact. A substantial minority of students gave proofs which were seriously flawed. Even many of the better solutions contained some small errors. The second part of the question was mostly based around a proof that $[0,1] \times [0,1)$ and $[0,1) \times [0,1)$ are homeomorphic. Most students found this difficult. A hint was given, where students were encouraged to show that $[0,1] \times [0,1]$ and the closed unit disc D^2 are homeomorphic. However, a common error was to define a map from $[0,1] \times [0,1]$ to D^2 by assigning the first co-ordinate to be the radial distance from the origin, and the second co-ordinate to be the angle. However, this is not a bijection. Nevertheless, a substantial minority of students were able to give a reasonably convincing pictorial description of a homeomorphism from $[0,1] \times [0,1)$ and $[0,1) \times [0,1)$.

Q3: This question was slightly less popular than the other two, with 35 out of 138 students attempting it. This was not surprising, because it was based on the final part of the course. The first part of the question was standard bookwork, followed by a proof that simplicial complexes with infinitely many vertices are non-compact. There were many very good solutions here. The second part, on surfaces, was more straightforward, and was generally very well done.

A6: Differential Equations 2

Q1: (a) Most students attempted this question. Everyone found the critical points and analysed these two by linearisation. The only difficulties were due to algebraic slips and the lack of careful consideration of the behaviours for the situation with repeated eigenvalues. Sketches of the phase plane were reasonable with those exploiting nullclines having the clearest general patterns.
(b) Many students were not very precise with the Bendixson Dulac theorem and although most found a simple tests function to consider many did not note what mights occur where

the strict inequality was not valid. The extended analysis near the origin was started well by most but few got far with solving the local ODE or sketching its behaviour.

Q2: (a) The first part of this question attempted well by the majority of candidates. In some cases small algebraic mistakes in calculating terms in the ODE caused complications later on. Otherwise, demonstrating that the given functions were solutions and linear independent, was completed well by all candidates.
Calculating the Green's function proved to be much more difficult with almost no students being able to complete this calculation in full. Many students sketched out the general plan but were unable to execute the details. Those who chose to calculate the solution by variation of parameters were able to express the solution in integral form though in this case the resulting integrals could not be evaluated.
(b) This part of the question was generally well-completed by candidates. Almost all candidates were able to recall and use Green's theorem. Many candidates incorrectly recalled the coefficient for the fundamental solution in two dimensions, and some candidates also incurred sign errors in the method of images calculation.

Q3: (a) This question proved difficult for almost all candidates. Most candidates were able to ascertain the first term of the solution, but ran into trouble finding the next term due to the repeated and were not able to resolve the issue (a few found the correct scaling for the next term). The majority of candidates were able to find both terms in the large root.
(b) This part of the question was completed well by candidates. Parts (i) and (ii) were well-completed by almost all candidates, with the only errors arising due to small algebraic errors. For part (iii) many students failed to use the solution they had in (ii), and hence erroneously considered a boundary layer near $x = 0$. Many candidates did not determine the correct power for the size of the boundary layer by using dominant balance , which caused subsequent issues in calculating the inner solution. Those who found the correct inner scaling were generally able to calculate the inner solution correctly.

A7: Numerical Analysis

Q1: Question 1 was generally well done. The most common problem was applying Householder to the entire column in part (c) as the right-multiplication will spoil the sparsity previously introduced.

Q2: Question 2 was also generally well done. In part (c), hardly anyone noted the checkerboard pattern, instead opting for rather verbose written explanations. In part (e), only a few students noted that as n increases the weight is increasingly concentrated at the ends which leads to a "qualitative explanation".

Q3: Question 3 had slightly lower scores mostly due to parts (d), (e) and (f). Very few people considered both A and A^T in (e). In part (f), many people produced (or tried to produce) a "slicker" answer than the intended one, which is always nice to see. But alarming how many people cannot multiply by a diagonal matrix.

A8: Probability

Q1: Question 1 was answered by the great majority of the candidates. All sections caused problems.
In (1)(a)(iii), most candidates knew the strategy for proving that convergence in probability implies convergence in distribution, but the details of the argument often showed a very poor level of basic analysis. Those who could accurately manipulate interactions of inequalities and limits were very much in the minority.
In (1)(b)(ii) a limit like $\lim_{n\to\infty}\left(1-\frac{1}{xn+1}\right)^n$ caused many problems, maybe in part because the limit $e^{-1/x}$ is not a familiar distribution function. Possibly students are more familiar with limits such as $\lim_{n\to\infty}\left(1+\frac{\alpha}{n}\right)^n$ at the end of the first year than at the end of the second year.
In part (c) as one might expect, the details of conditional independence were often an issue. Many candidates wrote something like "S_{n+1} depends only on S_n and is independent of $S_0, \ldots, S_{n-1}$". The most convincing answers went directly through the definition of the Markov property.

Q2: The standard of answers to question 2 was generally good. Many candidates are confident in manipulating moment generating functions, although frequently marks were dropped on details such as being clear about the range on which the mgf is defined, and being clear about the conditions required for the uniqueness theorem to hold. In d(ii), the independence of U and V is a key point. Some candidates had no idea what to do with the limit in part (c); among those who saw that it is $P(X \leq n)$ for an appropriate Gamma random variable X which can also be represented as a sum of i.i.d. exponentials, the most common mistake was then to try to apply the weak law of large numbers somehow rather than the central limit theorem.

Q3: There were many good answers to question 3, although there were also a fair number of candidates who were completely at sea in parts (c)(ii) and (iii). In (c)(ii), one could equally (maybe even more straightforwardly) use an approach via a hitting-probability recursion; a few candidates tried this, but none with complete success (maybe since the more confident candidates were happy to follow the direction of the hint). To use the recursion, one has to treat appropriately the boundary conditions which apply at the point 0 itself. In any case, to continue to part (iii), one probably does need to use an approach along the lines of the hint.

A9: Statistics

Q1: This was the least popular question by far. Overall, attempts at (a) were quite good, whereas attempts at (b) and (c) were quite weak. In (a)(ii)/(iii), some candidates ended (incorrectly) with a confidence interval whose end-points depended on θ. In (a)(iii), some candidates incorrectly used an exponential distribution whose mean was $1/\theta$ rather than θ. In (b), some candidates did not realise that they needed to substitute explicit expressions for $\hat{\mu}, \hat{\sigma}^2, \hat{\sigma}_0^2$ into the likelihood ratio λ in order to obtain the given expression for λ in terms of $t = (\bar{y} - \mu_0/(s/\sqrt{n})$. The question indicated that it was fine to state the required expressions for $\hat{\mu}, \hat{\sigma}^2, \hat{\sigma}_0^2$, but few candidates correctly stated (or derived if they didn't know) these. In (c)(iii), only a few candidates could correctly identify that the paired t-test (used by "Student 2") would be preferred for the paired data setup in (c).

Q2: This question was popular. Most candidates attempting it seemed well-prepared and did well overall, and on (a) especially. In (b)(i) some candidates reached the right expression for $w(\theta)$ but made errors along the way which either cancelled out or were ignored in order to get to the correct end-point.

Q3: Most answers to this question were good. After determining the posterior mean, some candidates simply asserted that it was between the prior mean and the sample mean, rather than providing the required justification. Most candidates did the (potentially tricky) calculation in (c) fairly easily.

A10: Waves and Fluids

Q1: Question 1 attracted only five attempts, none close to complete, though there was a similar question on the specimen papers to calculate the kinetic energy of the radial potential flow around a pulsating sphere. Part (b) involving separable solutions of Laplace's equation caused the most difficulty. A potentially tricky point is that the streamfunction is not constant on the boundary $r = a$, because the cylinder is moving, but no candidates appeared confused. Part (c) was done best, though two candidates lost minus signs to obtain the given expression despite starting with streamfunctions of the opposite sign to that in lectures. One candidate had difficulty converting the given expression for T back into an integral around a parametrised curve.

Q2: All candidates attempted question 2. Candidates were expected to leave their answer to part (d) parametrised by zeta, as suggested by the hint, and then put zeta = exp(i theta) for part (e). Quite a number of candidates found the complex velocity as a function of z instead. The most common error was to apply the Joukowski mapping to a circle of radius a, the parameter in the mapping, rather than the unit circle specified in the question. Some candidates were imprecise in claiming that the Milne-Thompson circle theorem gives a function with no new singularities, rather than no new singularities in $|z|$ >radius.

Q3: Question 3 was popular and mostly done well. The most common errors related to incorrect choices of the functions F(t) in Bernoulli's theorem for the two layers. Some candidates asserted that the pressure at the interface was constant, atmospheric pressure, or tried to incorporate the spatially varying pressure into an F(t), rather than equating the pressures above and below the interface. A few candidates neglected to include $U^2/2$ in their F(t), so their "linearisations" omitted the largest term. Almost all candidates wrote down correct functional forms for phi above and below the interface. Some candidates made errors in calculating the determinant of the 3x3 matrix to find the dispersion relation, at which point the tanh(kh) factors should disappear. A common sign error led to a dispersion relation with a single real frequency instead of the correct complex conjugate pair of frequencies.

A11: Quantum Theory

Q1: This was intended to be a straightforward question on some basic theory around time-dependent Schrodinger equation. Parts (a) and (b) were well-done but a surprisingly high proportion couldn't finish (c), only a couple got j(a/2,t) in (d) and nobody got the reason.

Q2: The harmonic oscillator done in detail from the Schrodinger equation. Parts (a), (b) and (d) were well done but the argument in (c) was generally confused.

Q3: Basic theory of the mathematical formulation of quantum theory, I was surprised how well this had been learned and reproduced.

Short Options

ASO: Number Theory

Q1: The bookwork parts of the question were on the whole done very well, and likewise the CRT calculation in (b)(i). There were a couple of different approaches to (a)(iv), but many candidates did not manage to work through either correctly. Part (b)(ii) was tricky, although in the end just reduced to the observation $1 + 2 = 3$: quite a few did spot how to do this.

ASO: Algebra 3

Q2: Part (a): caused no problems. Part (b): was solved completely by very few students only, with some more students making good attempts, but a large proportion of students had difficulties to get started. Part (c): about half of the students had good ideas around Sylow's theorem. No-one produced a complete solution.

ASO: Projective Geometry

Q3: There were 23 attempts. Most answers showed a reasonably good understanding of the material.
The part (a) bookwork on the general position theorem was done well, though most candidates missed the necessity to show that the scalars in the expansion of the final vector were nonzero.
Part (b) was done fairly well, though some candidates gave insufficient detail.
Most candidates were able to attack part (c) using the hint on using general position. However quite a few failed to distinguish properly between singular and nonsingular conics (ie their choices of B_0 and B_1 were singular).
Several candidates were able to do the first part of part (d) by direct calculation using the results of (c), but only a few realised the geometric interpretation in terms of line-pairs.

ASO: Multivariate Differentiations

Q4: There were 26 attempts. Clearly many candidates were on top of the material but there were also quite a few who showed little understanding.
The bookwork definitions in parts (a) and (c) were generally done well. Part (b), calculating the derivative of the determinant map, was done fairly well though most candidates skipped some of the details. Part (d) on showing $SL(n)$ was a submanifold of $M_{n\times n}$ was a bit mixed. Several candidates seemed a bit confused about the notion of tangent space, often giving answers which were obviously wrong on dimensional grounds. Part (e) was generally found difficult. Many candidates expressed the set as the vanishing locus of a function and calculated the function's derivative successfully, but only a few were able to show surjectivity and prove the manifold property.

ASO: Calculus of Variations

Q5: This was a popular question, with over 60 entries. Almost all candidates could derive the Euler-Lagrange equation, though the weaker ones fudged or muddled the derivation of the natural boundary conditions. Most candidates also made a good attempt at the application to the elastic beam, and there were a fair number who obtained complete solutions, with only small slips on the algebra. The last part of the question served well to reward the candidates who had a proper understanding of the meaning of the natural boundary conditions.

ASO: Graph Theory

Q6: Part (a). Most students gave a good presentation of the standard proof of Dirac's Theorem, but quite a few tried their own proofs (eg by induction on n) and failed. Most found subpart (iii) hard, though several spotted the appropriate complete bipartite graph.
Part (b). Most students gave the standard proof of the required inequality for Ramsey numbers, though often with minor slips on details. (The proof in lectures proved more, and so needed tailoring.) Quite a few skipped (forgot?) this proof. In subpart (iii), many gave proofs or partial proofs involving many cases, ignoring the hint (once we see that w must have at least 3 incident edges with the same colour the proof is short).

ASO: Special Relativity

Q7: There were only 9 attempts. All these candidates showed a good understanding of the use of 4-vector notation, and almost all could handle the calculation of velocity and acceleration vectors with confidence. But the insertion of numerical data to obtain rough estimates was found very difficult, and this held back even the best candidates from completing the question.

F. Comments on performance of identifiable individuals

Removed from the public version of the report.

G. Names of members of the Board of Examiners

- **Examiners:**
  Prof. K. Erdmann (Chair)
  Prof. Y. Capdeboscq
  Dr R. Earl
  Prof. P. Tarres
  Prof. G Reinert
  Dr M. Wildon (External Examiner)

- **Assessors:**
  Prof. A. Dancer
  Prof. P. Dellar
  Dr I. Griffiths
  Prof. A. Henke
  Dr A. Hodges

Prof. M. Lackenby
Prof. A. Lauder
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Prof. Z. Qian
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