Abstracts

Semilinear elliptic problems via semigroups

Wolfgang Arendt
University of Ulm

In the talk we will discuss some properties of positive semigroups concerning the asymptotic behavior for t to infinity. For instance, by an old result joint with Charles Batty, the ABLV Theorem obtains a simpler form for positive semigroups on L_p spaces. We also mention versions of Kato's result on the convexity of the spectral bounds as a function of a perturbation by a potential. Then we present a recent investigation of the semilinear logistic equation by Daniel Daners and the speaker. Abstract results on positive semigroups allow one to give precise conditions when a unique positive solution exists. In this way, passing through properties of the parabolic equation helps to understand the elliptic problem. The abstract approach has the advantage to make proofs more transparent but also to minimize the regularity conditions on the domain and the coefficients of the equation.

References:
W.Arendt, C. Batty: Domination and ergodicity for positive semigroups. Proc. Amer. Math. Soc. 114 (1992) 743–747.
W. Arendt, C. Batty: Principal eigenvalues and perturbation. Operator Theory: Advances and Applications, Vol. 75 (1995) Birkhäuser, Basel.
W. Arendt, D. Daners: Semilinear elliptic equations on rough domains. arXiv 2022.

A functional calculus for bounded C_0-semigroups on Hilbert space

Loris Arnold
IMPAN Warsaw

In this talk we introduce a Figa-Talamanga algebra type on the right half-plane. Following ideas of Peller, we are able to construct a bounded functional calculus on this algebra for a bounded C_0-semigroup on Hilbert spaces. Following again ideas of Peller, we will explain how to extend this functional calculus to a bigger Figa-Talamanca algebra type which turns out to be related to a subclass of bounded Fourier multiplier on $H^1(\mathbb{R})$.

Hasimoto's frames and the Gibbs measure for the periodic NLSE

Gordon Blower
Lancaster University

The talk interprets the cubic nonlinear Schrödinger equation as a Hamiltonian system with infinite dimensional phase space. There is a Gibbs measure which is invariant under the flow associated with the canonical equations of motion. The logarithmic Sobolev and concentration of measure inequalities hold for the Gibbs measures, and here are extended to the k-point correlation function and distributions of related empirical measures. By Hasimoto's theorem, NLSE gives a Lax pair of coupled ODE for which the solutions give a system of moving frames. The talk studies the evolution of the measure induced on the moving frames by the Gibbs measure. The talk contains quantitative estimates with well-controlled constants on the rate of convergence of the empirical distribution in Wasserstein metric.

Stability of a damped wave equation on an infinite star-shaped network

Amina Boukhatem
University of Tunis

We study the stability of an infinite star-shaped network of a linear viscous damped wave equation. We prove that, under some conditions, the whole system is asymptotically stable. Moreover we give a decay rate of the energy of the solution. Our technique is based on a frequency domain method.

Positivity of infinite dimensional linear systems

Yassine El Gantouh
Universidad Autónoma de Madrid

In this talk, we study the structural properties of the spaces of admissible control/observation operators in the Banach lattice setting. In particular, sufficient conditions for zero-class admissibility are obtained. Moreover, in the context of positive perturbations of positive semigroups, we present two perturbation results, namely, of Miyadera-Voigt perturbation and of Desch-Schappacher perturbation.

Degenerate elliptic operators and Kato's inequality

Tom ter Elst
University of Auckland

We consider a divergence form operator

$$B_p u = - \sum_{k,l=1}^{d} \partial_k (c_{kl} \, \partial_l u) + \sum_{k=1}^{d} c_k \, \partial_k u + c_0 \, u$$

with the maximal domain in $L_p(\mathbb{R}^d)$, where $c_{kl} \in W^{2,\infty}(\mathbb{R}^d, \mathbb{R})$, $c_k \in W^{1,\infty}(\mathbb{R}^d, \mathbb{R})$ and $c_0 \in L_\infty(\mathbb{R}^d, \mathbb{R})$. We assume that the operator is degenerate elliptic in the sense that $\operatorname{Re} \sum_{k,l=1}^{d} c_{kl}(x) \, \xi_k \, \overline{\xi_l} \geq 0$ for all $x \in \mathbb{R}^d$ and $\xi \in \mathbb{C}^d$.

We show that $-B_p$ is the generator of a C_0-semigroup for all $p \in [1, \infty)$ and that the space $C_c^\infty(\mathbb{R}^d)$ of test functions is a core for B_p for all $p \in (1, \infty)$. We also discuss perturbation of B_p with a positive potential.

This talk is based on joint work with Wolfgang Arendt and Tan Do.

Staffans-Weiss perturbations for linear stochastic Cauchy problems in Hilbert space

Mohamed Fkirine
University of Agadir

In this talk, we study the following linear perturbed stochastic evolution equations

$$\begin{cases} dX(t) = (A + BKC)X(t)dt + BdW(t), & t \geq 0, \\ X(0) = x, \end{cases} \tag{1}$$

in a real separable Hilbert space H. Here, A is the generator of a C_0-semigroup $\mathbb{T} := (T(t))_{t \geq 0}$ on H, $C \in \mathcal{L}(D(A), Y)$ is a bounded operator (not necessarily closed or closable) from $D(A)$ to the Hilbert space Y, $B \in \mathcal{L}\,(U, H_{-1})$ is a bounded operator from the separable Hilbert space U to H_{-1}, where H_{-1} is the extrapolation space associated to A and H, $K \in \mathcal{L}(Y, U)$ is a bounded operator and $W(t)$ is a cylindrical Wiener process over U. Under assumptions on the triplet (A, B, C), we prove the existence and uniqueness for solutions of (1). Moreover, a variation of constants formula is given in terms of $\mathbb{T}$. The equivalence of distribution of the solutions of (0.1) and solutions of the unperturbed problem ($K \equiv 0$) is considered. Sufficient condition for the existence of the invariant measures is given.

Degenerate elliptic operators in L^p spaces

Simona Fornaro
University of Pavia

The talk is concerned with second-order linear elliptic operators whose diffusion coefficients degenerate at the boundary in first order. It is shown that these operators generate analytic semigroups in L^p spaces.

The behavior strongly depends on the size and direction of the drift term. Some cases were already known in the literature. We have completed the picture by providing a precise description of the domain of the generator, which is more involved than in the other cases and exhibits reduced regularity compared to them.

Wave equations with low regularity coefficients

Dorothee Frey
Karlsruhe Institute of Technology

In this talk we discuss fixed-time L^p estimates and Strichartz estimates for wave equations with low regularity coefficients. It was shown by Smith and Tataru that wave equations with $C^{1,1}$ coefficients satisfy the same Strichartz estimates as the unperturbed wave equation on $\mathbb{R}^n$, and that for less regular coefficients a loss of derivatives in the data occurs. We improve these results for Lipschitz coefficients with additional structural assumptions. We show that no loss of derivatives occurs at the level of fixed-time L^p estimates, and that existing Strichartz estimates can be improved. The permitted class in particular excludes singular focussing effects. We also discuss perturbation results, and a recently introduced class of function spaces adapted to Fourier integral operators.

Holomorphic HÃ¶rmander-type functional calculus on sectors and strips

Markus Haase
University of Kiel

In 2018, Kriegler and Weis established functional calculus results for 0-sectorial and 0-strip type operators involving the classical Hörmander spaces over the half-line and the line. We generalize and refine some of their results to arbitrary sectorial and strip-type operators. To this end, holomorphic Hörmander-type functions on sectors and strips are introduced with a scale of smoothness finer than the classical polynomial one. Moreover, we establish alternative descriptions of these spaces involving Schwartz and "holomorphic Schwartz" functions. When combined with the famous theorem by Carbonaro and Dragicevic, our result yields an improvement (with respect to the smoothness condition) of the their Hörmander-type multiplier theorem for general symmetric contraction semigroups. This is joint work with Florian Pannasch and based on his PhD thesis (2019).

The Keller-Segel system in domains with non-smooth boundaries in critical spaces

Matthias Hieber
Technische Universität Darmstadt

In this talk we consider the classical parabolic-parabolic Keller-Segel system in domains with non-smooth boundaries, such as convex domains. We prove the existence of a unique, global solution to this system in critical spaces for sufficiently small data und discuss also the existence of time-periodic solutions.

This is joint work with Klaus Kress and Christian Stinner.

Uniform ergodicity and periodicity of topological dynamical systems

Julian Hölz
Bergische Universität Wuppertal

Each continuous mapping $\varphi : X \to X$ on a compact Hausdorff space X induces a linear operator

$$T_\varphi : C(X) \to C(X), \quad f \mapsto f \circ \varphi,$$

on the Banach space $C(X)$ of continuous functions $f : X \to \mathbb{C}$, called the *Koopman operator*. This operator can be examined in terms of its ergodic behaviour. In this talk we study properties of φ under the assumption that T_φ is uniformly mean ergodic and show that this is equivalent to eventual periodicity.

Analyticity of positive semigroups by domination and maximal inequalities

Jochen Glück
Bergische Universität Wuppertal

For positive C_0-semigroups on, for instance, L^p-spaces, we discuss two recent insights about how domination conditions are related to analyticity of the semigroup:

(i) If a positive semigroup S is dominated by an analytic semigroup T, then S is analytic as well.

(ii) If each orbit of a positive semigroup S is, for small times, order bounded (in other words: if the semigroup satisfies a maximal inequality for small times), then S is automatically analytic.

The second result yields, as a corollary, a connection between analyticity and Gaussian estimates.

Convergence rates for form-induced semigroup approximation

Katharina Klioba
Hamburg University of Technology

A common approach to spatial discretisation of evolution equations consists of solving their weak formulation on a sequence of approximating spaces. We present a novel quantified version of the Trotter-Kato theorem in this setting, yielding rates of strong convergence under a joint condition on properties of the corresponding form and the approximating spaces. A generalisation to the complete case allowing for the treatment of j-elliptic forms will be given. This is joint work with Christian Seifert (Hamburg University of Technology).

Functional calculus for submarkovian semigroups on weighted L2 spaces

Christoph Kriegler
Université Clermont-Auvergne

joint work with Komla Domelevo and Stefanie Petermichl from Würzburg

The invariant subspaces of distributional Fourier multipliers with appli-

cation to abstract integro-differential equations

Sebastian Król
Adam Mickiewicz University, Poznán

We present a convenient framework for studying the well-posedness of a variety of abstract integro-differential equations in general Banach function spaces, which allows us to extend and unify several known results from the literature. More precisely, using harmonic analysis methods, we identify a large class of Banach spaces invariant with respect to distributional Fourier multipliers. Then, we show how this result applies to the study of the well-posedness and maximal regularity of an abstract differential equation, which model various types of elliptic and parabolic problems arising in different areas of applied mathematics.

Functional Calculus via the Extension Technique

David Lee
Sorbonne Université Paris

In this talk, I will present a solution to the problem:

"*Which type of linear operators can be realized by the Dirichlet-to-Neumann operator associated with the operator* $-\Delta - a(z)\frac{\partial^2}{\partial z^2}$ *on an extension problem?*",

which was raised in the pioneering work [Comm. Par.Diff. Equ. 32 (2007)] by Caffarelli and Silvestre. But I even intend to go a step further by replacing the negative Laplace operator $-\Delta$ on $\mathbb{R}^d$ by an m-accretive operator A on a general Banach space X and the Dirichlet-to-Neumann operator by the Dirichlet-to-Wentzell operator. From this, we have a new characterization of the famous *Phillips' subordination theorem* within this class $\mathcal{CBF}$.

This talk is based on the research results provided in the recent Arxiv submission: 2101.11305.

A groupoid construction from Cartan pairs

Ying-Fen Lin
Queen's College Belfast

Let (A, B) be a Cartan pair of C^*-algebras (or of $\mathbb{C}$-algebras), that is, B contains local units for A, B is regular and is a maximal abelian subalgebra of A, and there is a faithful conditional expectation from A onto B. We know that there is a natural inverse

semigroup $N(B)$ of the normalisers of B in A. I will show that a groupoid and a twist can be constructed on the inverse semigroup $N(B)$ so that the algebras B and A can be recovered by the groupoid C^*-algebra (resp. Steinberg algebra) and the twisted groupoid C^*-algebra (resp. twisted Steinberg algebra) defined on the above constructed groupoid and twist.

Levy–Khintchine decomposition for convolution semigroups of states on $SUq(N)$

Martin Lindsay
Lancaster University

The sum of the generating functionals of two convolution semigroups of states on a compact quantum group $\mathbb{G}$ is the generating functional of a convolution semigroup of states whose corresponding Lévy process is expressible as a limit of evolution Trotter-products of those of the summands. In this talk I shall discuss a question concerning the reverse of this procedure. Given a convolution semigroup of states on $\mathbb{G}$, can its generating functional γ be expressed as the sum of a 'gaussian' generating functional and a 'wholly non-gaussian' one - in short, does γ have a 'Lévy–Khintchine decomposition'? It turns out that the answer depends on $\mathbb{G}$. The question has a satisfactory (and positive) answer in the case of the compact quantum groups $SUq(N)$.

The talk is based on joint work with Uwe Franz, Anna Kula and Michael Skeide.

Noncommutative extension of diffusions

Shreya Mehta
Imperial College London

We want to find the noncommutative analogue of the Hörmander theorem involving the Hörmander type diffusion operators. We use the Quantum Dirichlet forms to generate the corresponding Markov semigroups for these diffusions.

A first-order approach to solvability for singular Schrödinger equations

Andrew Morris
University of Birmingham

We will first give a brief overview of the first-order approach to boundary value problems, which factorises second-order divergence-form equations into Cauchy-Riemann systems. The advantage is that the holomorphic functional calculus for such systems can provide semigroup solution operators in tremendous generality, extending classical harmonic measure and layer potential representations. We will then show how recent developments now allow for the incorporation of singular perturbations in the associated quadratic estimates. This allows us to solve Dirichlet and Neumann problems for Schrödinger equations with potentials in reverse Hölder spaces. This is joint work with Andrew Turner.

Optimal rates of decay in the Katznelson-Tzafriri theorem for operators on Hilbert spaces

Abraham Ng
University of Wollongong

The classical Esterle–Katznelson–Tzafriri result says that for a power-bounded operator T on a Banach space X, $\|T^n(I-T)\| \to 0$ as $n \to \infty$ if and only if the intersection of the spectrum of T and the unit circle is contained in $\{1\}$. By studying the relationship between asymptotics and resolvent behaviour, we show that $\|T^n(I-T)\|$ decays at an optimal rate when growth of the resolvent $(\lambda I - T)^{-1}$ can be estimated by functions of so-called reciprocally positive increase. This work is joint with David Seifert and can be naturally considered as a sequel to Seifert's "Rates of decay in the classical Katznelson–Tzafriri theorem" and a discretisation of Rozendaal, Seifert and Stahn's "Optimal rates of decay for operator semigroups on Hilbert spaces".

Integral inequalities for generalised Mehler semigroups

Diego Pallara
University of Salento

We consider generalised Mehler semigroups and, assuming the existence of an invariant measure, we prove integral inequalities of Poincaré and Log-Sobolev type. We also study the integrability of exponential functions with respect to the invariant measure.

Composition semigroups, shift semigroups, and the Cesàro operator

Jonathan Partington
University of Leeds

A closed subspace is invariant under the Cesàro operator on the classical Hardy space H^2 if and only if its orthogonal complement is invariant under a certain semigroup of composition operators induced by affine maps. By linking the invariant subspaces to the lattice of the closed invariant subspaces of the standard right-shift semigroup acting on a particular weighted L^2-space on the line, we exhibit a large class of non-trivial closed invariant subspaces and provide a complete characterization of the finite codimensional ones. Finally, we use a functional calculus which allows us to extend a recent result by Mashreghi, Ptak and Ross regarding the square root of the Cesàro operator, and discuss its invariant subspaces.

This is joint work with Eva Gallardo-Gutiérrez (Complutense, Madrid).

Polynomial stability of coupled PDEs

Lassi Paunonen
Tampere University

Polynomial convergence rates appear frequently in the study of various types of partial differential equations (PDEs). In particular, when two or more PDE models are coupled together via boundary conditions, the energies of the solutions of the coupled system often decay at subexponential rates. In this talk we present general results which can be used to study the stability properties and polynomial energy decay of such coupled systems. The results are presented for strongly continuous semigroups and they are based on the characterisation of polynomial stability via resolvent estimates due to Batty and Duyckaerts (and several other researchers). The presented results are particularly easy to apply in the study of coupled PDEs on one-dimensional or geometrically simple spatial domains, and we illustrate how they can be used to unify and extend earlier results on such PDE systems.

The talk contains joint work with Charles Batty, David Seifert, and Filippo Dell'Oro.

L^∞-admissibility and a new class of Laplace-Carleson embedding theorems

Sandra Pott
Lund University

Motivated by questions on the admissibility of control operators in the theory of infinite-dimensional linear systems, so-called Laplace-Carleson embeddings have been studied in recent years.

Here, one considers boundedness of the Laplace transform $\mathcal{L}$ as an operator $L^p(0,\infty) \to L^q((C)_+, \mu)$, where μ is a positive regular Borel measure on $(C)_+$. In case $p = q = 2$, this is just the classical Carlsson Embedding Theorem for the right half plane. In the talk, we will in particular provide a full characterisation for the case $p = \infty$ and also the case of the Orlicz space of exponentially integrable functions.

This is joint work with Birgit Jacob (Wuppertal), Jonathan Partington (Leeds), Eskil Rydhe (Lund), and Felix Schwenninger (Twente).

Decay of quasilinear Maxwell equations with conductivity

Roland Schnaubelt
Karlsruhe Institute of Technology

The Maxwell system governs electromagnetic theory. In this system the permittivity ε and the permeability μ describe how the electric and magnetic fields interact with the material, and a nonzero conductivity density σ imposes a damping of the electric fields. These quantities can also depend on the fields; one obtains a quasilinear hyperbolic system if this occurs for ε or μ.

We show exponential decay to 0 of solutions for small data in the quasilinear case if σ is supported near the boundary, also for matrix-valued ε and μ. Here we use the approach and some of the results from a joint work with I. Lasiecka and M. Pokojovy (2019) treating the case of strictly positive σ. A crucial ingredient in our reasoning is an observability-type estimate which is inspired by an observability result due to S. Nicaise and C. Pignotti (2004) for linear autonomous problems with scalar coefficients (the isotropic case). The new results in the talk are joint work with Richard Nutt (Karlsruhe).

A Katznelson-Tzafriri theorem for analytic Besov functions

David Seifert
Newcastle University

Let $-A$ be the generator of a bounded C_0-semigroup $(T(t))_{t\geq 0}$, and suppose that A admits a bounded functional calculus with respect to an algebra $\mathcal{A}$ of holomorphic functions on the the open right half-plane $\mathbb{C}_+$. Theorems of Katznelson-Tzafriri type provide sufficient conditions under which

$$\lim_{t\to\infty} \|T(t)f(A)\| = 0 \qquad (*)$$

for suitable functions $f \in \mathcal{A}$. Such theorems play an important role in the asymptotic theory of C_0-semigroups, and in particular have been used to give an alternative proof of the famous countable spectrum theorem. The main result to be presented in this talk is a new Katznelson-Tzafriri theorem for operators A admitting a bounded functional calculus with respect to a certain algebra $\mathcal{B}$ of analytic Besov functions. The theorem states that $(*)$ holds for all $f \in \mathcal{B}$ such that f vanishes on the boundary spectrum $\sigma(A) \cap i\mathbb{R}$ of A and $|f(z)| \to 0$ as $|z| \to \infty$ with $z \in \mathbb{C}_+$. The talk is based on joint work with Charles Batty.

On weak convergence of shift operators to zero on rearrangement-invariant spaces

Eugene Shargorodsky
King's College London and Technische Universität Dresden

Let $\{h_n\}_{n\in\mathbb{N}}$ be a sequence in $\mathbb{R}^d$ tending to infinity and let $\{T_{h_n}\}$ be the corresponding sequence of shift operators given by $(T_{h_n}f)(x) = f(x - h_n)$ for $x \in \mathbb{R}^d$.

We prove that $\{T_{h_n}\}$ converges weakly to the zero operator as $n \to \infty$ on a separable rearrangement-invariant Banach function space $X(\mathbb{R}^d)$ if and only if its fundamental function φ_X satisfies $\varphi_X(t)/t \to 0$ as $t \to \infty$.

For a non-separable rearrangement-invariant Banach function space $X(\mathbb{R}^d)$, we show that $\{T_{h_n}\}$ does not converge weakly to the zero operator as $n \to \infty$ if

- $h_n = nh,\ h \in \mathbb{R}^d \setminus \{0\}$,

or

- $X(\mathbb{R}^d)$ is a Marcinkiewicz endpoint space $M_\varphi(\mathbb{R}^d)$ or an Orlicz space $L^\Phi(\mathbb{R}^d)$.

This is a joint work with Oleksiy Karlovych.

Stability and decay rates for delay semigroups

Sachi Srivastava
University of Delhi

We study decay rates for delay semigroups, in particular the notion of "polynomial stability", associated with abstract delay differential equations. We find general conditions under which a delay semigroup is polynomially stable and apply these to special delay operators. Some perturbation results are also obtained and provide another approach to the study.

A stochastic maximal function and regularity estimates for parabolic stochastic evolution equations

Lutz Weis
Karlsruhe Institute of Technology

We introduce a stochastic maximal function to facilitate regularity and maximal regularity estimates for stochastic evolution equations given in terms of the generator of an analytic semigroup on UMD Banach spaces.

Generators of GNS-symmetric quantum Markov semigroups
Melchior Wirth
Institute of Science and Technology Austria

Quantum Markov semigroups are noncommutative analogs of classical Markov semigroups, acting on operator algebras instead of function spaces. Since there introduction, a central question has been to characterize their generators. For uniformly continuous quantum Markov semigroups on a von Neumann algebra, such a characterization was given by Christensen and Evans in the 70s. Recent years have seen renewed interest in quantum Markov semigroups, with a special focus on semigroups that satisfy certain symmetry conditions, such as the GNS symmetry or detailed balance condition. I will discuss a new version of the Christensen-Evans theorem for GNS-symmetric quantum Markov semigroups, which gives a complete characterization of the generators for uniformly bounded semigroups. For semigroups that are only weak* continuous, I will show that there generators can be represented in terms of derivations using the theory of noncommutative Dirichlet forms.

Scaling laws in fluid mechanics: from computational optimization to analysis
Andrew Wynn
Imperial College London

This talk will consider the problem of how to estimate time-averaged statistics of fluid flows, and to understand how these scale as a function of external forcing parameters (e.g, how average drag varies a function of flow velocity; or how heat flux scales as a function of applied heating). We study this question using a recent minimax characterisation of time-averages of the 2D Navier-Stokes equations, due to Rosa & Temam. In particular, if $u_t = F(u)$ represents the flow on an appropriate Hilbert space H, with orbits $S(t)u_0$ and absorbing set $B \subset H$, then the time-average of a bounded continuous functional $\phi : H \to \mathbb{R}$ can obtained by in terms of cylindrical test functions:

$$\max_{u_0 \in B} \limsup_{T\to\infty} \frac{1}{T} \int_0^T \phi(S(t)u_0)dt = \inf_{\Psi \in \mathcal{T}^{cyl}} \max_{u \in B} \left\{ \phi(u) + \langle F(u), \Psi'(u) \rangle \right\}.$$

In this talk we will show how the convex optimization technique of *semidefinite programming* can be used to systematically implement this characterisation. We apply this approach to *internally heated convection*, in which fluid motion in a layer is driven by a uniformly distributed heat source. For this flow, a fundamental question is to determine how the time-averaged heat flux at the upper boundary $\mathcal{F}_T$ varies with the strength of the applied heating (quantified by the Rayleigh number Ra).

Numerical optimization results will be presented which indicate that Ra-dependent

bounds may be obtained using test functions Ψ with a particular double boundary-layer structure. It will be shown that this can be formalised to give upper bounds of the form

$$\mathcal{F}_T \leq 1 - c_1 Ra^{1/5} e^{-c_2 Ra^{3/5}}, \qquad Ra \to \infty,$$

which can be improved to $\mathcal{F}_T \leq 1 - cRa^{-2}$ in the case of infinite Prandtl number. These are the first known Ra-dependent bounds for internally heated convective flows.