A Panorama of Homotopy Theory:
A conference in honour of Mike Hopkins

Monday 5 June

10:00 – 11:00 **Mike Hill**

11:30 – 12:30 **Constantin Teleman**
The Quantum GIT conjecture
Abstract: A famous theorem of Frances Kirwan describes the cohomology of the symplectic quotient X//G of a compact manifold as a quotient of the equivariant cohomology of X. The kernel of the surjection is described geometrically, but the ideal of algebraic relations is not so explicit. Motivated by ideas of 2- and 3- dimensional gauge theory (and long known in the case of toric varieties) I will describe the explicit formula for the quantum cohomology ring $QH^*(X//G)$ in the case when X is Fano. (The general case involves bulk deformations for which we do not yet know the formula.) The proof uses Floer theory techniques, even in the algebraic case. This is joint work with Dan Pomerleano.

14:30 – 15:30 **John Francis**
Integrating braided categories over 3-manifolds
Abstract: I'll describe a form of factorization homology, which gives an invariant of framed 3-manifolds given as input a rigid braided-monoidal category with duals. More generally, the construction gives an invariant of framed n-manifolds from an E_{n-1}-monoidal category with duals. This construction relies on a form of the tangle hypothesis, that a dualizable object in an E_{n-1}-monoidal category uniquely determines a functor from the category of tangles in n-space. This is joint work with David Ayala.

16:00 – 17:00 **Paul Goerss**
A panorama of K(n)-local homotopy theory
Abstract: In the golden age of chromatic homotopy, in the second half of the Thatcher era, Mike Hopkins saw that the K(n)-local stable homotopy category behaved far more like the derived category of quasi-coherent sheaves on a nice scheme than we had any right to expect. This thought was perhaps intended at once as a slogan, a conjecture generating device, and a suite of theorems, and Hopkins himself seemed to know which was which even then, but it took the rest of us a generation or two to catch up. Now seems like an excellent time to tell the story of this journey, to say where we are now, and to have a look at the future. This is work of far, far too many people to list here, but I will do my best to name names during the talk. We all worked under the benevolent eye of the master and often the aid and guidance of Hopkins was far more direct than that.

17:30 – 19:00 **Reception at Warden's Garden, New College**

Tuesday 6 June

09:00 – 10:00 **Lukas Brantner**
Deformations and lifts of Calabi-Yau varieties in characteristic p
Abstract: Homotopy theory allows us to study infinitesimal deformations of algebraic varieties via (partition) Lie algebras. We apply this general principle to two classical problems on Calabi-Yau varieties Z in characteristic p. First, we show that if Z has torsion-free crystalline cohomology and degenerating Hodge-de Rham spectral sequence, then its mixed characteristic deformations are unobstructed. This generalises

the BTT theorem to characteristic p. If Z is ordinary, we show that it moreover admits a canonical (and al-gebraisable) lift to characteristic zero, thereby extending Serre-Tate theory to Calabi-Yau varieties. This is joint work with Taelman, and generalises results of Achinger-Zdanowicz, Bogomolov-Tian-Todorov, Deligne-Nygaard, Ekedahl–Shepherd-Barron, Schröer, Serre-Tate, and Ward.

10:00 – 11:00 **Charles Rezk**
Reflections on accessible infinity categories

11:30 – 12:30 **Ishan Levy**
Applications of algebraic K-theory to a problem in topology
Abstract: I will explain work joint with Robert Burklund, Jeremy Hahn, and Tomer Schlank where we prove the Lichtenbaum-Quillen conjecture for ring spectra such as certain finite extensions of the K(1)-local sphere, and study Galois hyperdescent of chromatically localized algebraic K-theory. I will indicate applications to homotopy theory.

14:30 – 15:30 **Kirsten Wickelgren**
Gromov-Witten invariants in $\mathbb{A}^1$*-homotopy theory*
Abstract: Gromov-Witten invariants count curves in a given homology class through an appropriate number of marked points. For example, the number of complex degree d rational plane curves passing through $3d - 1$ points is such an invariant. It is independent of the generically chosen points over the complex numbers. (There is 1 line through 2 points, 1 conic through 5, 12 rational degree 3 curves through 8...) Over the real numbers, there are invariants due to Jean-Yves Welschinger, Cheol-Hyun Cho and Jake Solomon giving an open Gromov-Witten invariant equal to a signed count of real curves. It is a feature of $\mathbb{A}^1$-homotopy theory that analogous real and complex results can indicate the presence of a common generalization, valid over a general field. We develop and compute an $\mathbb{A}^1$-degree, following Fabien Morel, of the evaluation map on Kontsevich moduli space to obtain a count of genus 0 curves on certain del Pezzo surfaces through the appropriate number of marked points. This count is valid for any field k of characteristic not 2 or 3. In particular, we define and compute some Gromov– Witten invariants over a finite field. This is joint work with Jesse Kass, Marc Levine, and Jake Solomon.

16:00 – 17:00 **Alexei Kitaev**
Topology of short-range entangled quantum states
Abstract: Interesting examples of short-range entangled (SRE) states (also known as invertible states) occur in physics. It has been conjectured that the spaces of SRE states on (S^n, x_0) for $n = 0, 1, 2 \ldots$ form an Ω-spectrum. However, rigorous definitions have been missing so far. In this talk, I will define the space of SRE states on a finite graph, show that it has certain desired properties, and discuss a limit procedure for constructing SRE states on (S^n, x_0).

Wednesday 7 June

09:00 – 10:00 **Aravind Asok**
On the motivic Freudenthal suspension theorem and some applications
Abstract: We will discuss recent joint work with Tom Bachmann and Mike Hopkins establishing a version of the Freudenthal suspension theorem with respect to $\mathbb{P}^1$-suspension in unstable motivic homotopy theory, together with some applications to the theory of algebraic vector bundles.

10:00 – 11:00 **Emily Riehl**
Homotopy types as homotopy types
Abstract: Quillen famously proved that the homotopy theory of spaces can be modelled by the category of simplicial sets, with Kan complexes encoding homotopy types. Voevodsky extended this result to show that homotopy type theory Martin- Löf's dependent type theory plus the univalence axiom can be modelled by the category of simplicial sets, again with the Kan complexes encoding homotopy types. In this talk, I'll explain what features must be added to Quillen's model structure to obtain a model of homotopy type theory and then explain why a Quillen equivalent model, on a suitably chosen category

of cubical sets, may be preferred. This last part involves joint work with Steve Awodey, Evan Cavallo, Thierry Coquand, and Christian Sattler.

11:30 – 12:30 **Tomer Schlank**
Cyclotomic Redshift
Abstract: In this talk which is a prequel to Jeremy Hahn talk, I shall describe results from a recent paper with S. Ben-Moshe, S. Carmeli and L. Yanovski that relates chromatic cyclotomic extensions of different heights via K-theory. I shall explain how combined with trace methods this gives a concrete description of hyper-compelition for algebraic K-Theory thus relating it to Ishan's talk. Further applications will be discussed in Robert's talk.

14:30 – 19:00 **Social activities**
- The classic Oxford experience: punting.
- Oxford colleges. A tour around some of the most beautiful and iconic Oxford colleges.
- Football: a game of football in Merton College sports field.
- A walk along the river Thames on Port Meadow to a beautiful riverside pub.

Thursday 8 June

09:00 – 10:00 **Soren Galatius**
Pontryagin classes of topological bundles
Abstract: Pontryagin classes of real vector bundles are usually defined as Chern classes of the complexified bundle. In rational cohomology they can be defined more generally for Euclidean bundles – fiber bundles with fiber $\mathbb{R}^n$ and structure group the entire homeomorphism group of $\mathbb{R}^n$. The question of whether familiar relations, such as $p_n = e^2$ for $2n$-dimensional vector bundles, hold for Euclidean bundles has recently been answered in the negative by Weiss. My talk will discuss joint work (arXiv:2208.11507) with Randal-Williams on related questions.

10:00 – 11:00 **Jeremy Hahn**
Quillen-Lichtenbaum, Ausoni-Rognes, and Doug Ravenel
Abstract: A sequel to Ishan Levy's talk.

11:30 – 12:30 **Michael Hopkins**
New classification result for algebraic vector bundles

14:30 – 15:30 **Agnes Beaudry**
A Classifying Space for Injective Matrix Product States
Abstract: The classification of bosonic $1 + 1d$ phases without symmetry is known to be trivial. This goes back to work of Hastings based on the description of $1 + 1d$ systems by matrix product states. If one instead considers families of such systems parametrized by a nice topological space X, the classification becomes more interesting and is believed to be $H^3(X, \mathbb{Z})$. In other words, the classifying space for these phases should be a $K(\mathbb{Z}, 3)$. This conjecture can be traced back to a talk by Kitaev in 2013 and is also confirmed by Freed-Hopkins's work on the classification of certain families of invertible TQFTs by the spectrum $\Sigma^2 I\mathbb{Z}MSO$. Others may also have their own reasons to adhere to this philosophy. These predictions seem to suggest that there should be a construction of $K(\mathbb{Z}, 3)$ directly from matrix product states. In this talk, I will discuss such a proposal. This is based on various joint work with Michael Hermele, Mike Hopkins, Juan Moreno, Markus Pflaum, Marvin Qi, Daniel Spiegel, David Stephen and Xueda Wen.

16:00 – 17:00 **Graeme Segal**
The nature of locality in quantum field theory

Friday 9 June

09:00 – 10:00 **Andrew Blumberg**

Relative cyclotomic structures and equivariant cobordism
Abstract: We give a new criterion on a pre-cyclotomic commutative ring spectrum R that suffices to construct a pre-cyclotomic structure on topological Hochschild homology relative to R. We are motivated by applications to Floer homotopy theory; our main examples of interest are periodic circle-equivariant complex cobordism (MUP) and a new circle-equivariant version of MU. This talk describes joint work with Mandell and Yuan.

10:00 – 11:00 **Tom Bachmann**
Biconnectivity for motivic spaces
Abstract: It is a well-known fact that motivic cohomology theories have two indices, owing to the two "motivic spheres" S^1 (from topology) and Gm (from geometry). From this one also obtains "two directions of connectivity", with the S^1-connectivity usually just called connectivity and the Gm-connectivity called effectivity. So far, Gm-connectivity has been studied in the stable context, through Voevodsky's slice filtration. In this talk I will explain joint work with Asok-Hopkins studying biconnectivity in the *unstable* context, that is, for motivic spaces. One key result is that sufficiently biconnected spaces can be decomposed into infinite P^1-loop spaces. This is an important ingredient to our motivic Freudenthal suspension theorem.

11:30 – 12:30 **Robert Burklund**
Beyond Z_p extensions
Abstract: In this talk, which concludes the series beginning with Ishan's talk, I will present some fragments of a conjectural picture of what happens beyond the case of Z_p extensions and discuss the bearing this has on our understanding of the sphere spectrum and its homotopy groups.

14:30 – 15:30 **Inna Zakharevich**
Scissors Congruence and Coinvariants
Abstract: For a geometry X (such as Euclidean, spherical, or hyperbolic) with isometry group G the scissors congruence group $P(X, G)$ is defined to be the free abelian group generated by polytopes in X, modulo the relation that for polytopes P and Q that intersect only on the boundary, $[P \cup Q] = [P] + [Q]$, and for $g \in G$, $[P] = [g \cdot P]$. This group classifies polytopes up to "scissors congruence," i.e. cutting up into pieces, rearranging the pieces, and gluing them back together. With some basic group homology one can see that $P(X, G) \sim= H_0(G, P(X, 1))$. Using combinatorial K- theory $P(X, G)$ can be expressed as the K_0 of a spectrum $K(X, G)$. In this talk we will generalize this formula to show that, in fact, $K(X, G) ' K(X, _{1)hG,}$ and in fact more generally that this is true for any assembler with a G-action. This is joint work with Anna Marie Bohmann, Teena Gerhardt, Cary Malkiewich, and Mona Merling.

16:00 – 17:00 **Daniel Freed**
Remarks about invertible field theories
Abstract: I will recall the relation of these theories to homotopy theory and then I will discuss some applications: locality of bordism invariants, abelianization of Chern- Simons invariants, and anomalies in quantum field theory.

19:30 for 20:15 **Reception and Conference Dinner: Balliol College**

Saturday 10 June

11:00 **Bus to INI, Cambridge via Bletchley Park from the Mathematical Institute, Oxford.**