Oxford Mathematics Alphabet

UNIVERSITY OF OXFORD
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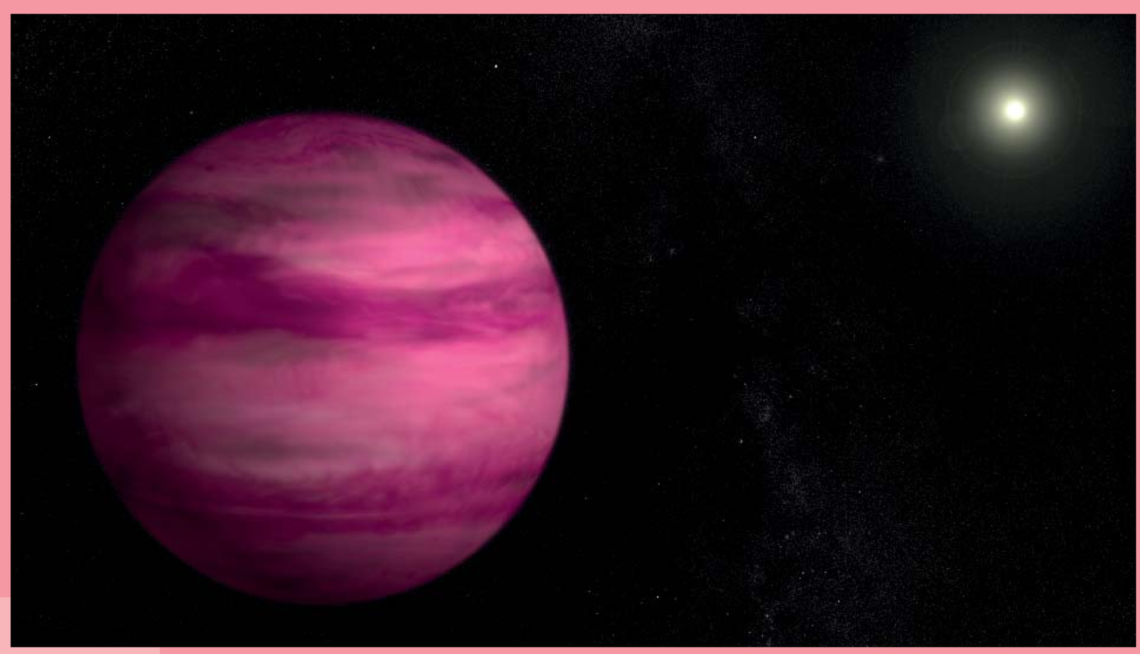
I *is for* inverse problems

All mathematical models require information to make their predictions; to get something out, you have to put something in.

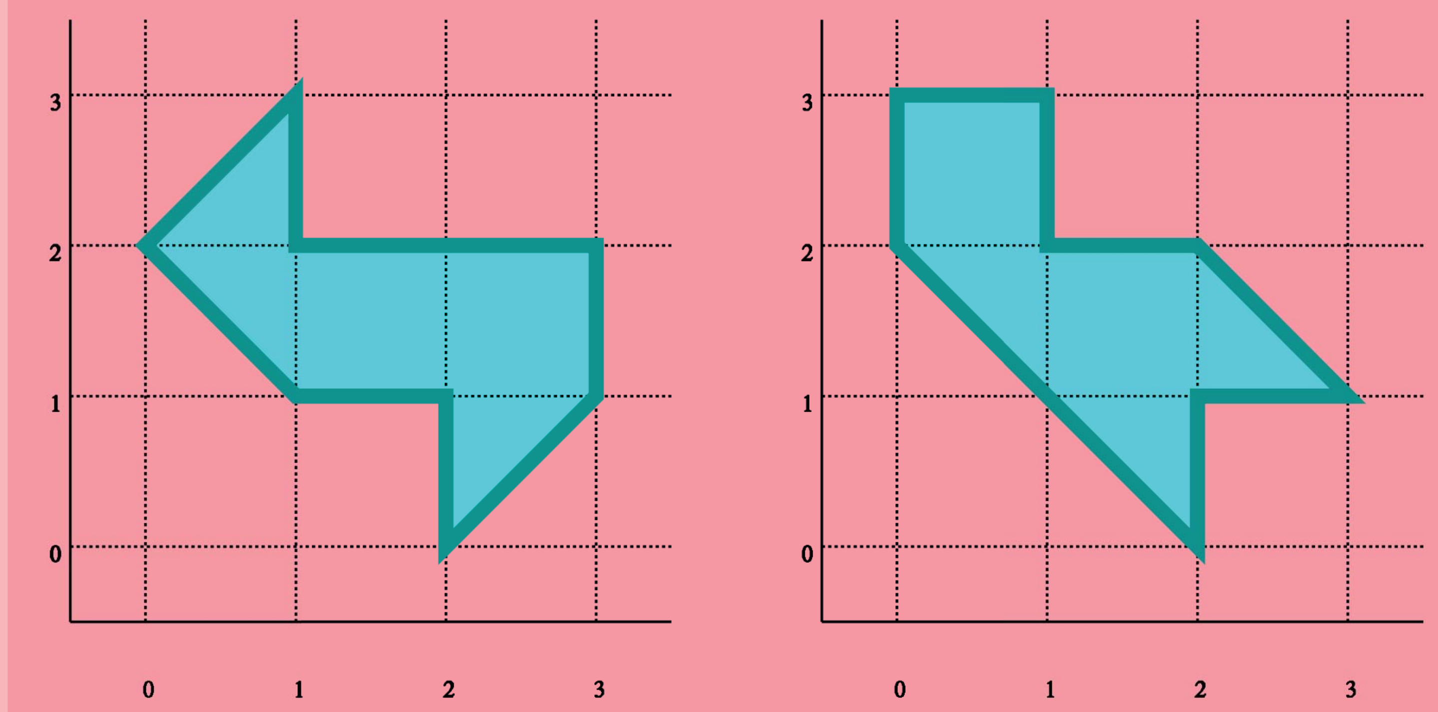
In many cases, however, we are faced with the opposite problem: given information about the outcome of a physical process, how did it come about? Such a problem is called an inverse problem for it inverts the relationship between cause and effect encoded in the underlying equations. Think of them as mathematical detective puzzles - we see the crime, and seek the culprit.

Bats and submariners solve inverse problems to navigate in the dark. Their solution underpins much of modern technology, from subsurface petroleum exploration to medical imaging and the daily weather forecast. They offer a promising way of learning about things we can't easily do experiments on, such as the bottom of the ocean or the interior of the sun.

However, inverse problems are also very difficult mathematically. Forward problems arising in practice are usually *well-posed*, meaning that they have a unique solution that is stable with respect to the data. Unfortunately, inverse problems are almost always ill-posed: there are typically many possible inputs that match the observed outputs.



In 1966, Marc Kac famously asked: can one hear the shape of a drum? This is an example of the nonuniqueness of inverse problems: Carolyn Gordon et al. showed that two genuinely different shapes can give rise to the same eigenvalues ('sound').



There are two responses to ill-posedness, the *deterministic* and *Bayesian* approaches. The deterministic response is that the problem should be *regularised* - perturbed to a nearby problem that is well-posed. The Bayesian approach regards the solution to an inverse problem as a *probability distribution* over the input space, not a single input. The efficient Bayesian solution of inverse problems is at the forefront of current research in numerical analysis and statistics, with many important discoveries yet to be made.

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