Scattering Sheep: Kink Collisions in the Presence of False Vacua

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Based on arXiv:1604.08413 in collaboration with

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Outline

1. Introduction
   - The model
   - A and B “kinks”
2. Scattering Outcomes
   - AB Scattering
   - BA Scattering
   - $B\overline{A}$ Scattering
   - $A\overline{B}$ Scattering
3. Summary and Discussion

The model

Consider a model of two real scalar fields $\phi(x,t)$ and $\psi(x,t)$ defined by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \psi \partial^\mu \psi + \partial_\mu \phi \partial^\mu \phi) - V(\psi, \phi),$$

where $V(\psi,\phi)$ is a scalar potential given by

$$V(\psi,\phi) = V_\psi(\psi) + V_\phi(\phi) + V_{\psi\phi}(\psi,\phi) - V_0\,,$$

with the individual potential terms

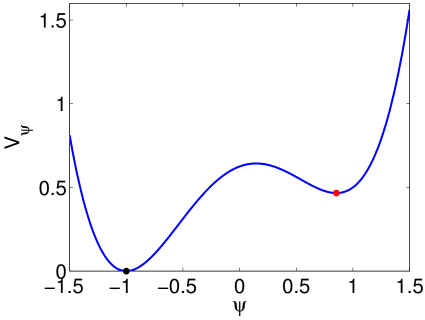
$$\begin{aligned} V_\phi(\phi) &= \alpha\Big(\sin^2(\pi\phi) + \epsilon_\phi \sin^2(\pi\phi/2)\Big)\,,\\ V_\psi(\psi) &= \beta(\psi+a)^2\Big((\psi-a)^2 + \epsilon_\psi^2\Big)\,, \end{aligned}$$

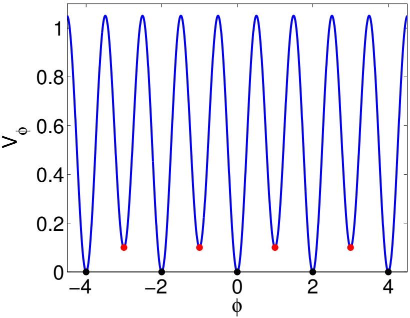
and the interaction potential

$$V_{\psi\phi}(\psi,\phi) = \lambda \frac{(\psi-a)^2\Big((\psi+a)^2+\epsilon_\psi^2\Big)}{\Big(V_\phi(\phi) - V_\phi(1/2)\Big)^2 + \gamma^2}\,.$$

Tunneling decay of false domain walls: the silence of the lambs, M. Haberichter, R. MacKenzie, M.B. Paranjape & Y. Ung (2015).

The potential





$$V_\psi(\psi) = \beta(\psi + a)^2\Big((\psi - a)^2 + \epsilon_\psi^2\Big)$$

Global minimum: $\psi = -a$.

Local minimum: $\psi = \frac{a+\sqrt{a^2-2\epsilon_\psi^2}}{2}$.

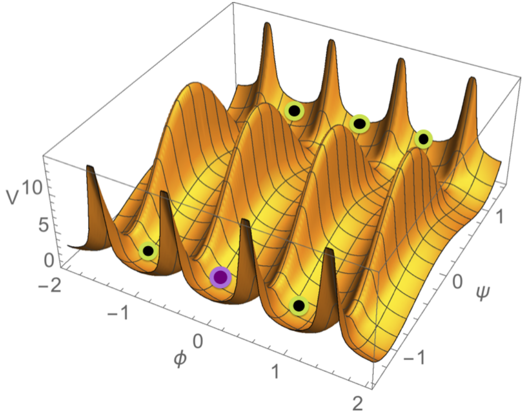
$$V_\phi(\phi) = \alpha\Big(\sin^2(\pi\phi) + \epsilon_\phi \sin^2(\pi\phi/2)\Big)$$

Global minima: $\phi = 2k$

Local minima: $\phi = 2k + 1,\ k \in \mathbb{Z}$.

True and false vacua

In a theory with scalar fields, any field configuration that is a local minimum of the potential energy density is a vacuum. More than one vacuum can exist, and not all vacua need have the same energy. The field configuration with the lowest energy is called the *true vacuum*; any higher-energy local minimum is a *false vacuum*.



Parameter choice

From now on, we choose the parameter values

$$\alpha = 0.5\,, \quad \beta = 0.5\,, \quad \gamma = 0.01\,, \quad a = 1\,, \quad \epsilon_\psi = 1\,, \quad \epsilon_\phi = 0.01\,, \quad \lambda = 0.1\,.$$

Then the true vacua are

$$\{\psi = -0.7593,\ \phi = 2k\},$$

and false vacua

$$\{\psi = -0.7552,\ \phi = 2k+1\}\,,\ \{\psi = 0.6396,\ \phi = 2k\}\,,\ \{\psi = 0.6463,\ \phi = 2k+1\}.$$

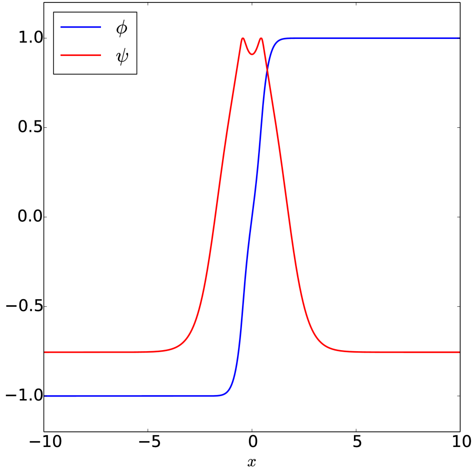
Original motivation

Sheep ϕ are in false vacuum outside domain wall and true vacuum inside.

Left alone they would separate to infinity to extend the true vacuum.

However **shepherd** ψ is in true vacuum outside domain wall, and false within!

[The shepherd field is unstable to quantum tunnelling to its true vacuum. Once this occurs, the sheep are without a shepherd and will spread out to infinity - *the silence of the lambs* arXiv:1506.05838.]



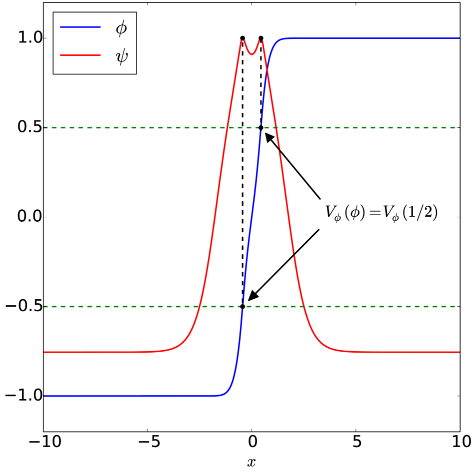
Construction of the interaction potential

Recall that the interaction potential is given by

$$V_{\psi\phi}(\psi,\phi) = \lambda \frac{(\psi - a)^2\left((\psi + a)^2 + \epsilon_\psi^2\right)}{\left(V_\phi(\phi) - V_\phi(1/2)\right)^2 + \gamma^2}.$$

We choose γ to be very small, so that this will become very large when $V_\phi(\phi) = V_\phi(1/2)$ and ψ is far away from $\pm a$.

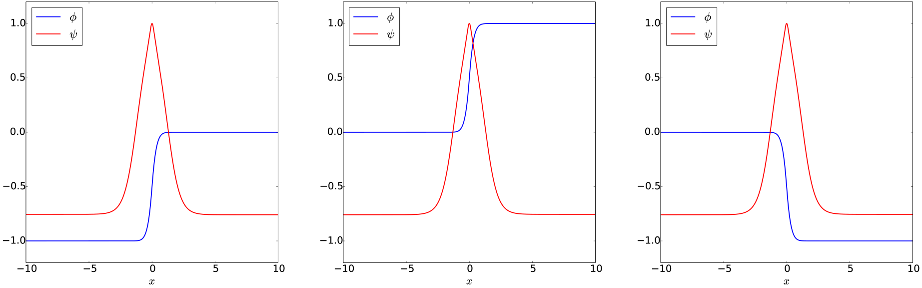
The ϕ solitons may not pass through the ψ solitons without making the energy contribution from the interaction term "blow up".



A and B kinks

Kinks interpolating between a false vacuum (F) and a true vacuum (T).

We call a $F \to T$ kink, a A **kink**, and a $T \to F$ kink a B **kink**. Kinks travelling $F \to T$ and $T \to F$ in the reverse direction are denoted by $\overline{A}$ and $\overline{B}$ respectively.

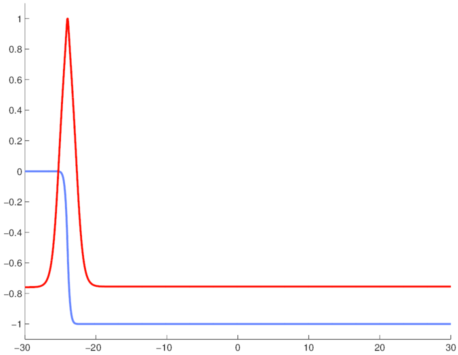


A kink $\quad$ B kink $\quad$ $\overline{B}$ kink

The effect of having two different vacua

- It is energetically favourable to have the largest region of true vacuum possible
- Our kinks have true vacuum at one boundary, and false at the other.
- When evolved from rest they will accelerate away to allow the region of true vacuum to expand.

$\overline{B}$ kink evolved from rest:



Pairing up kinks

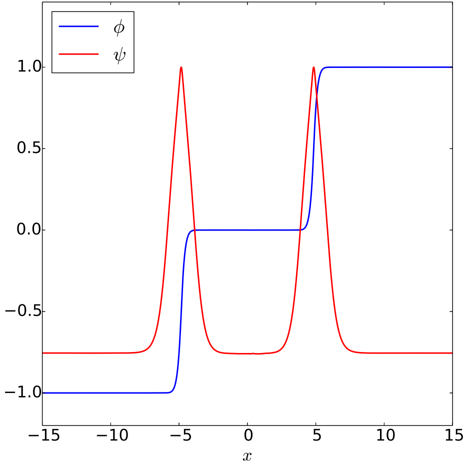
- The structure of true and false vacua excludes some combinations of kinks.
- Whether the kinks will attract or repel depends on whether there is false or true vacuum in between them.

	A	B	$\overline{B}$	$\overline{A}$
A	X	repulsive	repulsive	X
B	attractive	X	X	attractive
$\overline{B}$	attractive	X	X	attractive
$\overline{A}$	X	repulsive	repulsive	X

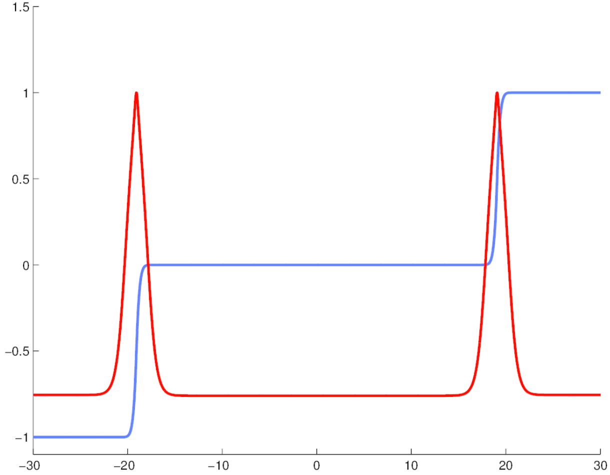
- We will consider configurations of the types AB, BA, $B\overline{A}$ and $A\overline{B}$.

AB Configuration

- Attaching an A kink to a B kink.
- We begin with the kinks well separated.
- Note that the **true vacuum** is in between the kinks, so they are **repulsive**.
- We boost the kinks towards each other for a range of initial velocities $0 \leq v \leq 0.9$ and observe the results...

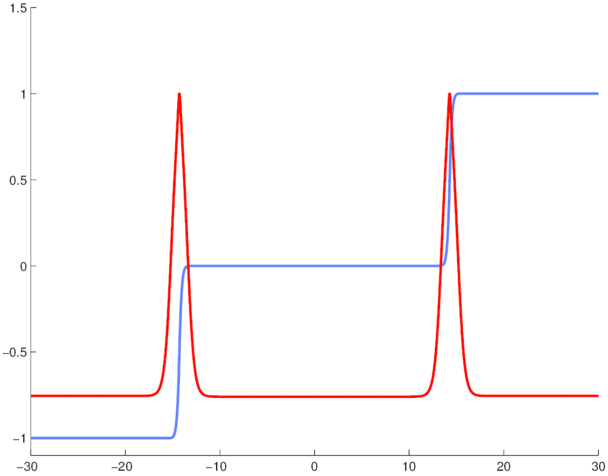


AB Outcome I - Repulsion ($v = 0.3$)



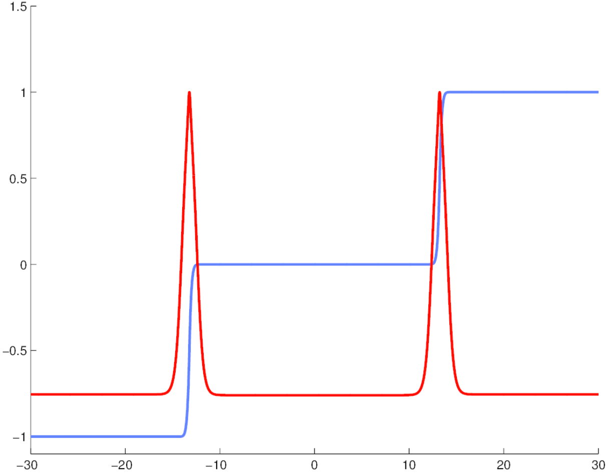
For initial speed $v \leq 0.476$, the kinks cannot overcome their mutual repulsion.

AB Outcome II - "Sticking" ($v = 0.7$)



For $0.476 < v < 0.707$ and $v > 0.801$, the kinks form a false domain wall.

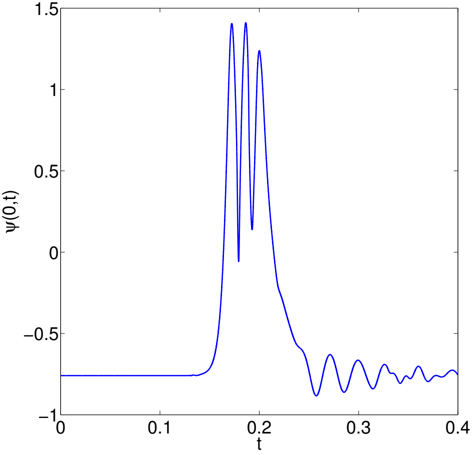
AB Outcome III - Reflection ($v = 0.8$)



For initial speed $v \in [0.707, 0.801]$, the kinks reflect off each other.

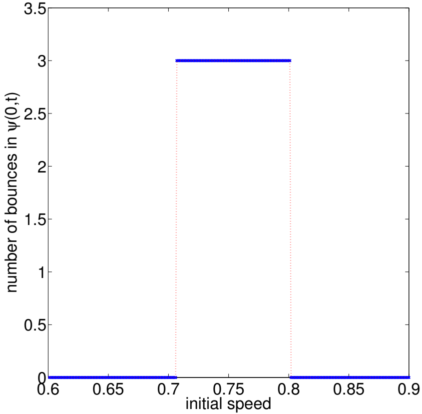
AB Reflection - “Bounces”

- We plot the shepherd field $\psi(0, t)$ at the origin.
- During the reflection, there are a number of oscillations, or “bounces”, in $\psi(0, t)$.
- This number is conserved in each window of reflection behaviour.



$v = 0.75$

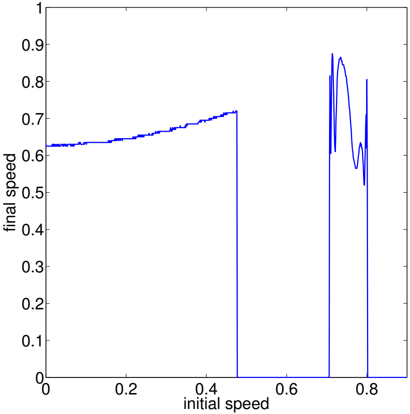
Bounce number conservation



- We can see the conservation of bounce number in a plot of number of bounces against initial speed.
- Where the kinks form a false domain wall, (so do not reflect), zero bounces are counted.
- Throughout the reflection window, three bounces are counted.

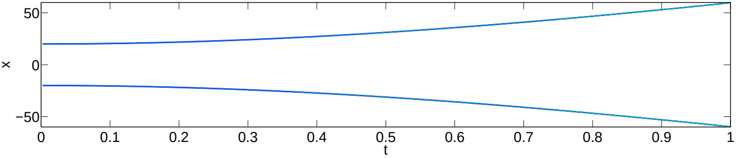
Overview I: Incoming and outgoing velocity

- For $v \leq 0.476$, the kinks cannot overcome their mutual repulsion, and accelerate away from each other to infinity.
- If the kinks form a false domain wall, the final speed is zero.
- In the reflection window $v \in [0.707, 0.801]$, final speed is again non-zero as the kinks separate to infinity.

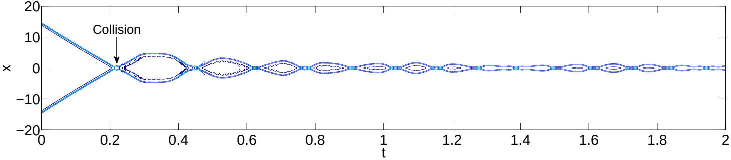


Overview II: Soliton trajectories

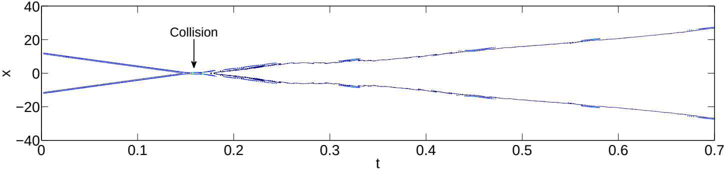
$v = 0$



$v = 0.7$

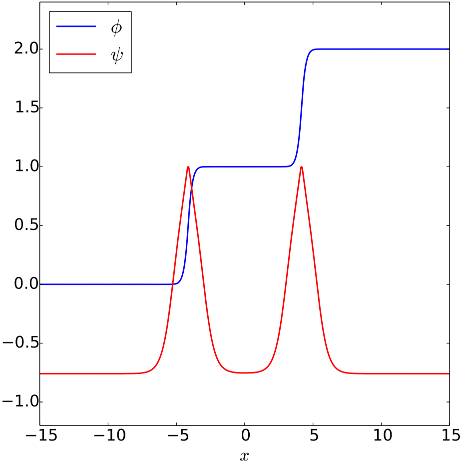


$v = 0.8$

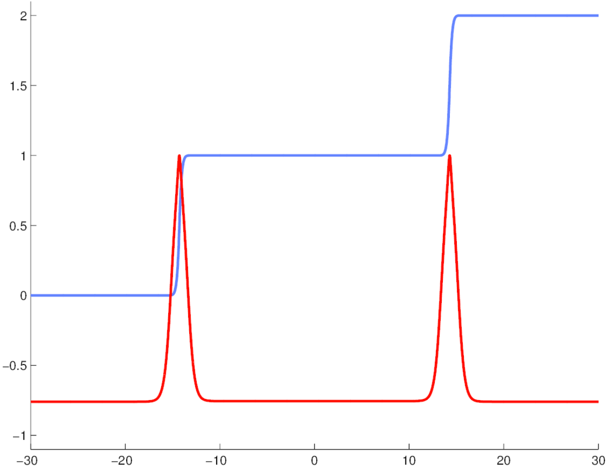


BA Configuration

- Attaching a B kink to an A kink.
- Note that the **false vacuum** is in between the kinks, so they are **attractive**.
- We boost the kinks towards each other for a range of initial velocities $0 \leq v \leq 0.9$ and observe the results...



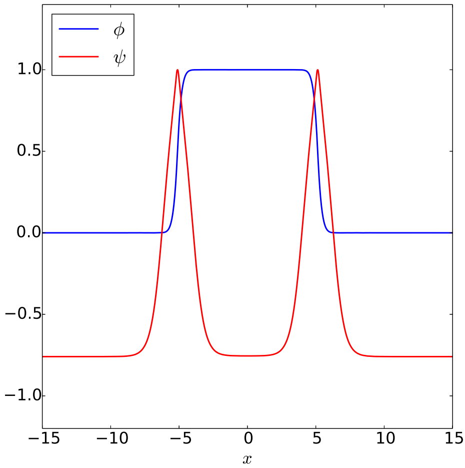
BA Outcome - "Sticking" ($v = 0.7$)



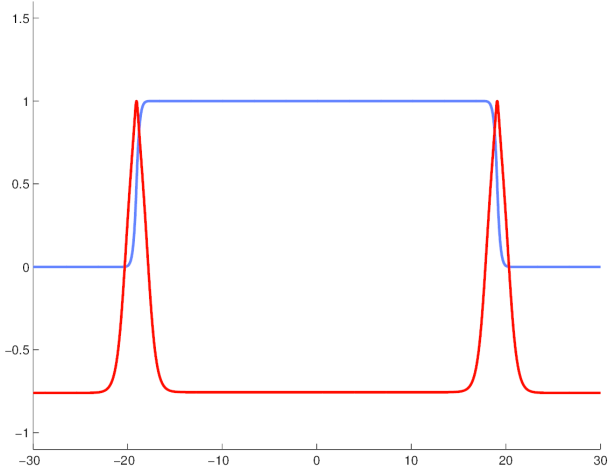
There is only one outcome: the kinks trap each other and form a true domain wall.

$B\overline{A}$ Configuration

- Attaching a B kink to an $\overline{A}$ kink.
- This is similar to kink-antikink scattering.
- Note that the **false vacuum** is in between the kinks, so they are **attractive**.
- We boost the kinks towards each other for a range of initial velocities $0 \leq v \leq 0.9$ and observe the results...

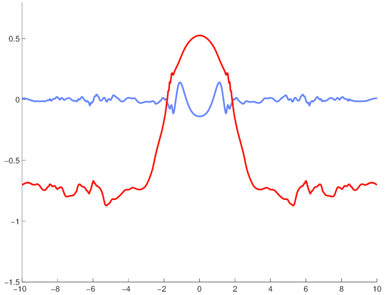


$B\overline{A}$ - Annihilation ($v = 0.3$)



The kinks annihilate by forming an **oscillon** which decays to the true vacuum.

Oscillons



An **oscillon** is a long-lived, oscillatory bound state which can persist for thousands of oscillations before decaying to the vacuum.

Kink-antikink collisions in which oscillons are formed occur in several models, e.g.

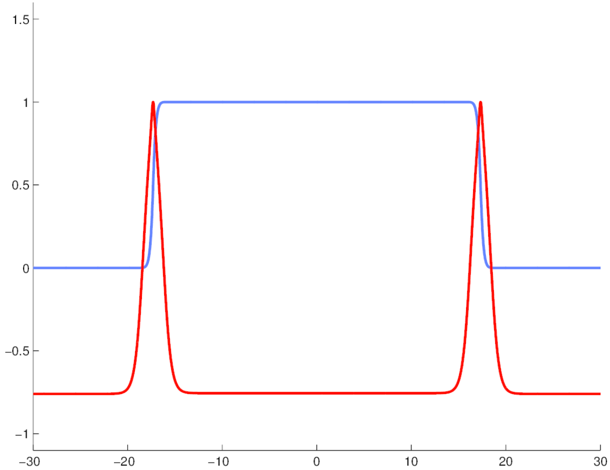
P. Anninos, S. Oliveira & R. A. Matzner, Phys. Rev. D44 (1991)

A. Halavanu, T. Romanczukiewicz & Y. Shnir, Phys. Rev. D86 (2012)

J. Braden, J. R. Bond & L. Mersini-Houghton, JCAP 1503 (2015)

V. A. Gani, V. Lensky & M. A. Lizunova JHEP 08 (2015)

$B\overline{A}$ - Reflection then Annihilation ($v = 0.5$)

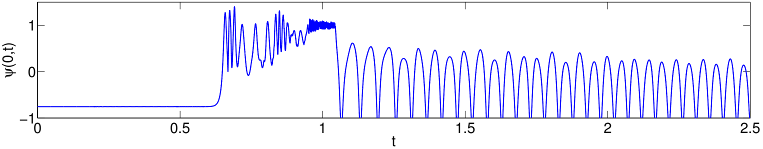


The kinks first reflect off one another, then collide a second time and annihilate.

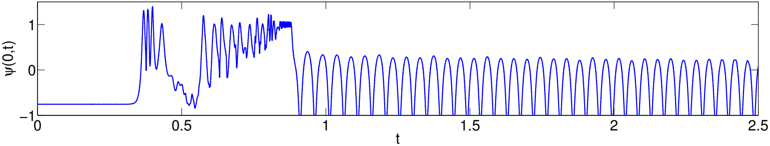
$B\bar{A}$ Bounce plots

By plotting the shepherd field $\psi(0,t)$, we see key features of the kink interactions.

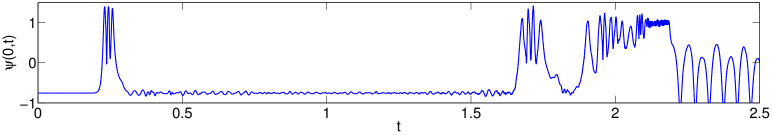
$v = 0$



$v = 0.355$

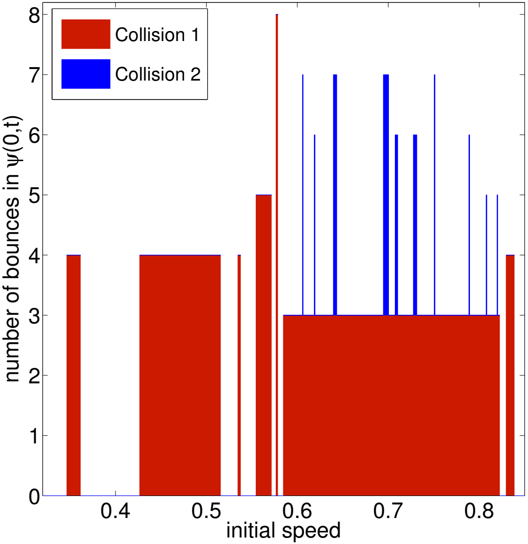


$v = 0.606$



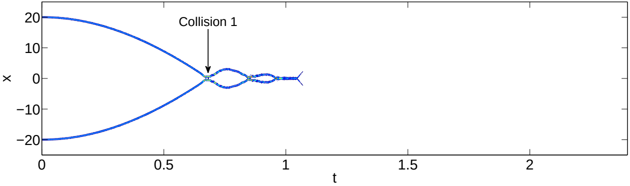
$B\bar{A}$ Overview I: Bounce number

- We count the number of bounces during the first collision in red, and during the second collision in blue.
- If the kinks annihilate, then no bounces are counted.
- Thus far, we have not found any windows with three or more collisions.

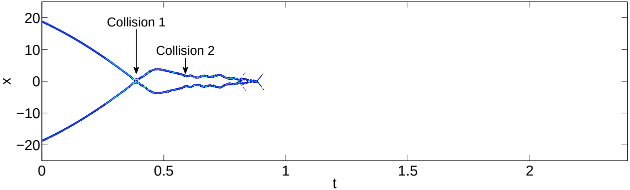


$B\bar{A}$ Overview II: Soliton trajectories

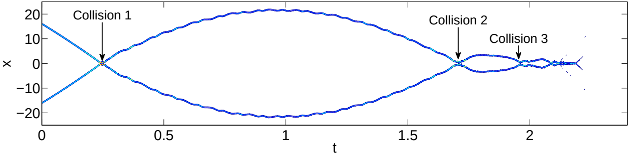
$v = 0$



$v = 0.355$

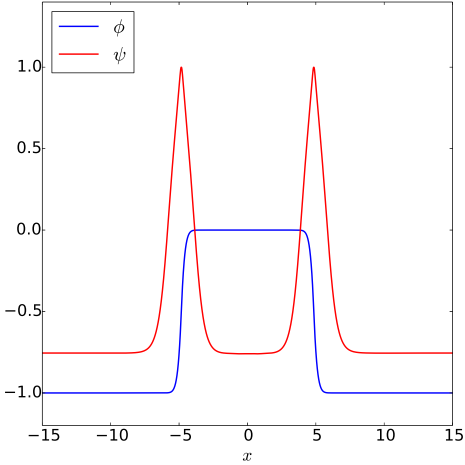


$v = 0.606$

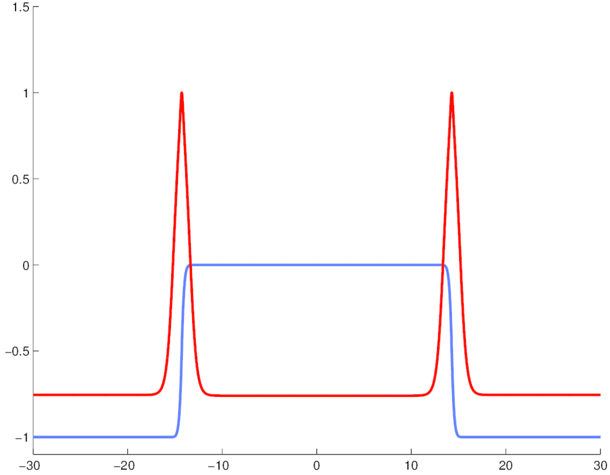


$A\overline{B}$ Configuration

- Attaching an A kink to a $\overline{B}$ kink.
- Note that the **true vacuum** is in between the kinks, so they are **repulsive**.
- We boost the kinks towards each other for initial velocities $0 \leq v \leq 0.9$.
- For $v \leq 0.476$, the kinks cannot overcome their mutual repulsion. Otherwise, there are two possible outcomes...

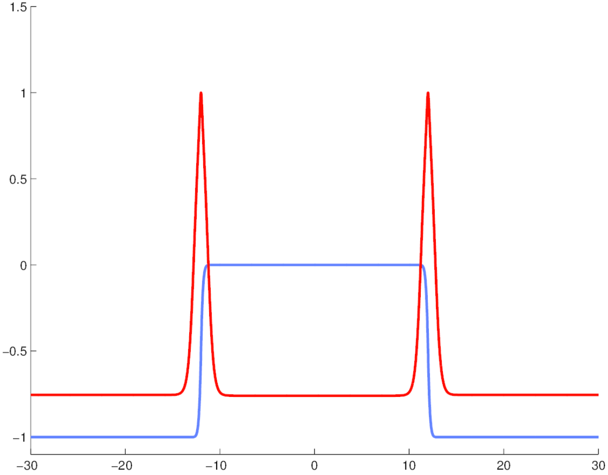


$A\overline{B}$ Outcome II - Annihilation ($v = 0.7$)



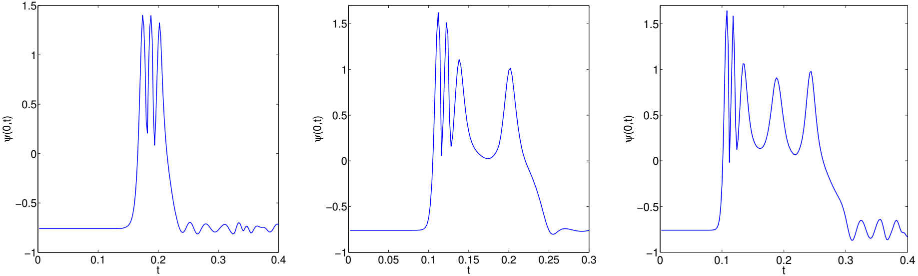
The kinks annihilate by forming an **oscillon** which decays to the false vacuum.

$A\bar{B}$ Outcome III - Reflection ($v = 0.75$)



The kinks reflect off each other, and separate to infinity.

$A\bar{B}$ Reflection - Bounce plots

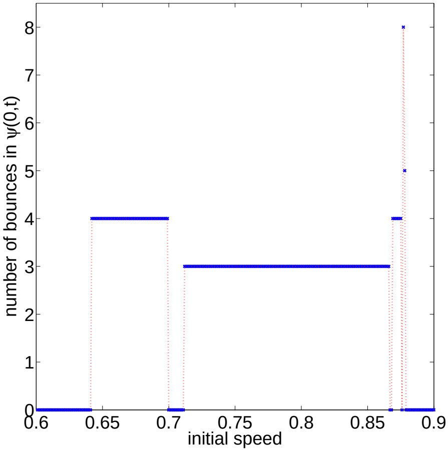


$v = 0.75$ $v = 0.87$ $v = 0.878$

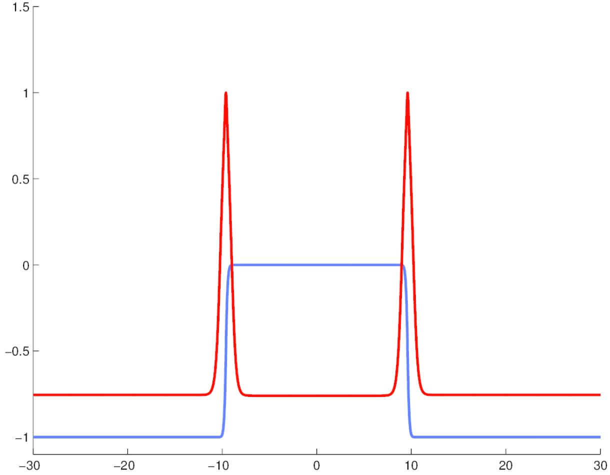
When plotting the shepherd $\psi(0, t)$ as a function of time, we observe different numbers of bounces in different windows of the reflection behaviour.

Bounce number conservation

- The bounce number is conserved in each window of the reflection behaviour.
- The smallest number of bounces seen for this configuration is three.
- The largest number of bounces seen is eight. This has an unusual structure of a group of six bounces shortly followed by two individual bounces.



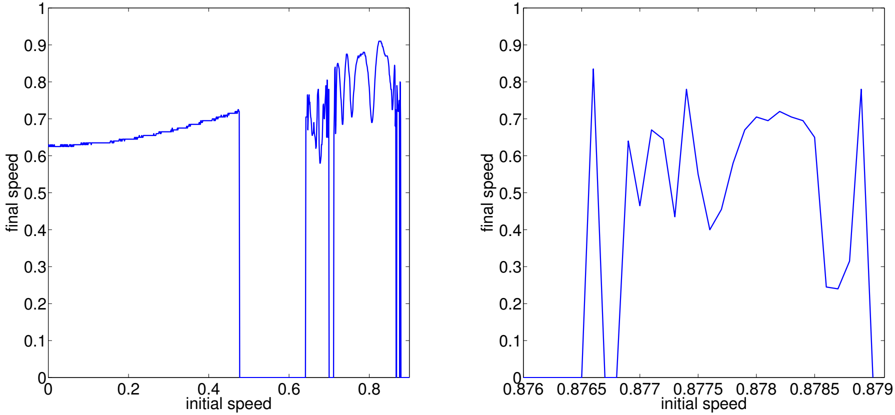
$A\overline{B}$ - 8 bounce ($v = 0.877$)



There are 8 “bounces” in the shepherd field before the kinks separate.

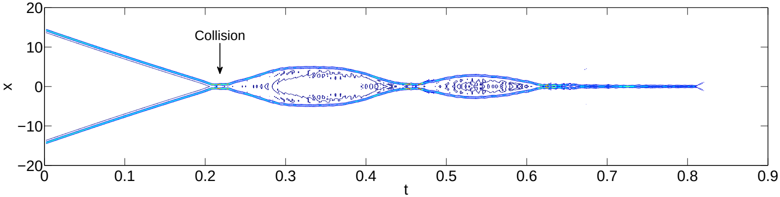
$A\overline{B}$ Overview I - Incoming and outgoing velocity

There is a fractal structure. When we go to higher resolution, more windows of the reflection behaviour are found.

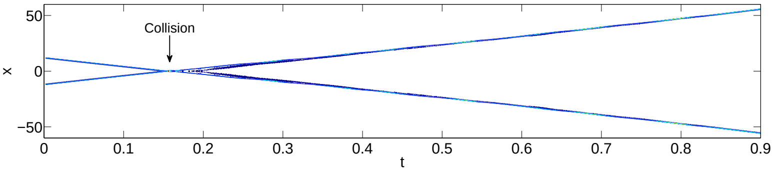


$A\bar{B}$ Overview II - Soliton trajectories

$v = 0.7$



$v = 0.8$



Summary

There are a rich variety of scattering outcomes in this model. Depending on the impact velocity and initial configuration, kinks can:

- repel,
- form true or false domain walls ("sticking"),
- reflect off each other,
- annihilate by forming an oscillon which decays to the true or false vacuum.

We summarise the observed behaviours in the following table:

	Repel	Annihilate	Stick	Reflection
AB	✓	✗	✓	✓
BA	✗	✗	✓	✗
$B\bar{A}$	✗	✓	✗	✓
$A\bar{B}$	✓	✓	✗	✓