If **Charles Dodgson** (Lewis Carroll) had not written *Alice's Adventures in Wonderland* and *Through the Looking-Glass*, he'd probably be remembered as a pioneer photographer. But his Oxford 'day job' was as Lecturer in Mathematics at Christ Church. What mathematics did he do?

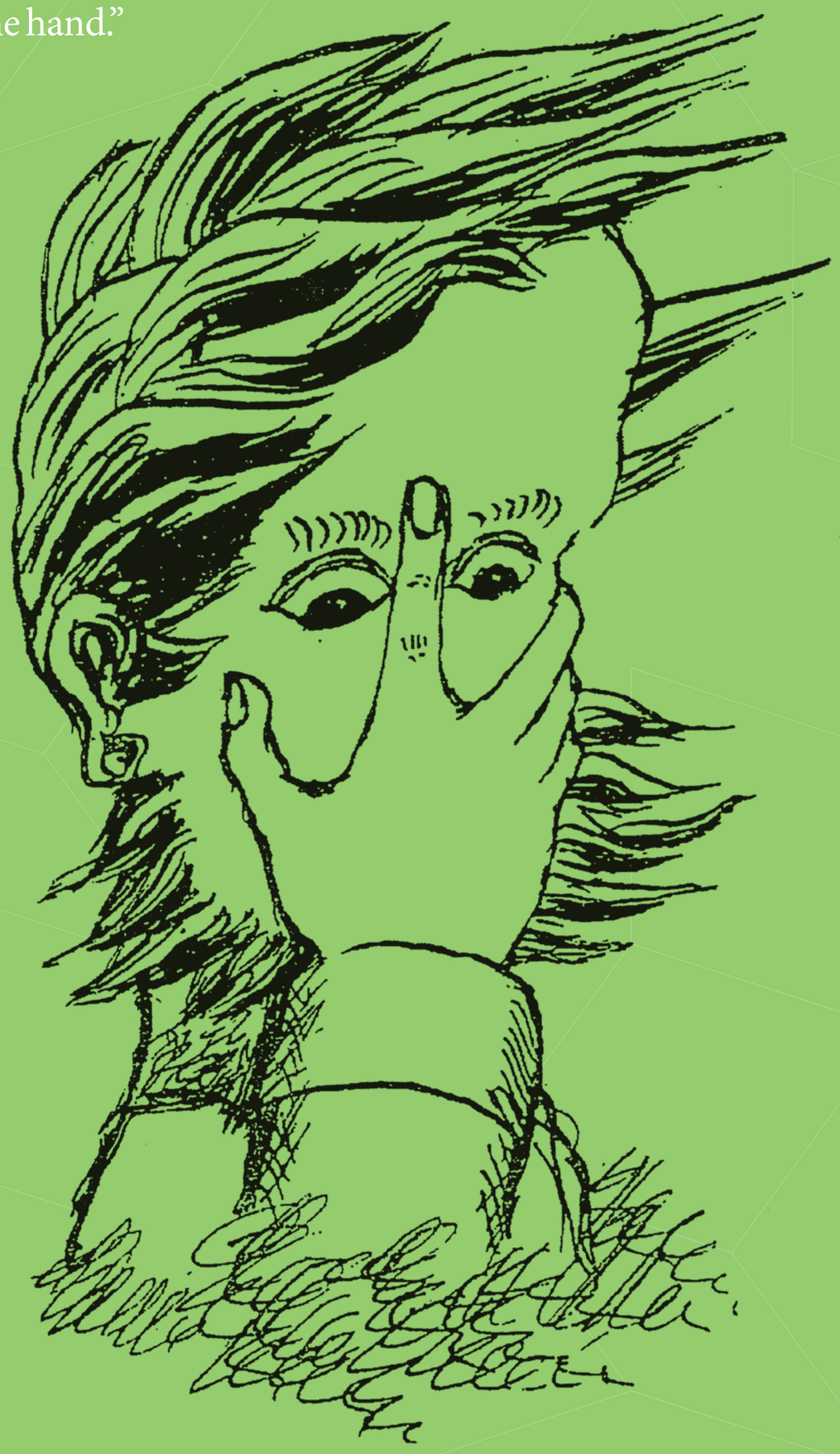
UNIVERSITY OF OXFORD
Mathematical Institute

C L Dodgson (Lewis Carroll)

Oxford mathematician

"What I look like when I'm lecturing.
The merest sketch, you will allow –
yet still I think there's something grand
In the expression of the brow
and in the action of the hand."

C L Dodgson



Dodgson time-line

1832	Born in Daresbury, Cheshire
1843	Moves to Croft Rectory, Yorkshire
1844–49	At Richmond and Rugby Schools
1850	Matriculates at Oxford University
1851	Studies at Christ Church, Oxford
1852	Nominated a 'Student' at Christ Church
1854	Long Vacation at Whitby studying with **'Bat' Price**
	First Class in Mathematics in his Finals Examinations
1856	Mathematical Lecturer at Christ Church
	Adopts the pseudonym Lewis Carroll
	Develops an interest in photography
1860	*Notes on the First Two Books of Euclid*
1861	*The Formulae of Plane Trigonometry*
1862	Boat trip to Godstow with the **Liddell sisters**
1865	*The Dynamics of a Parti-cle*
	Alice's Adventures in Wonderland
1866	*'Condensation of Determinants'* read to the Royal Society
1867	*An Elementary Treatise on Determinants*
1868	*The Fifth Book of Euclid*
1871	*Through the Looking-Glass, and What Alice Found There*
1873	*A Discussion of the Various Methods of Procedure in Conducting Elections*
1876	*The Hunting of the Snark*
1879	*Euclid and his Modern Rivals*
1881	Resigns Mathematical Lecturership
1882	*Euclid, Books I, II*
1883	*Lawn Tennis Tournaments*
1884	*The Principles of Parliamentary Representation*
1885	*A Tangled Tale*
1886	*The Game of Logic*
1888	*Curiosa Mathematica, I. A New Theory of Parallels*
1889–93	*Sylvie and Bruno*
1893	*Curiosa Mathematica, II. Pillow-Problems*
1896	*Symbolic Logic. Part I. Elementary*
1898	14 January: dies in Guildford

Oxford Mathematics

C L Dodgson (Lewis Carroll) (1832–98)

Early years

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Dodgson inherited a love of mathematics from his father who had studied at Christ Church and had gained First Class degrees in mathematics and classics.

A mathematical childhood

The third of eleven children and the eldest boy, the young Charles Dodgson was initially taught at home by his parents. The following story is told of his mathematical precocity.

"One day when Charles was a very small boy, he came up to his father and showed him a book of logarithms, with the request, 'Please explain.' Mr Dodgson told him that he was much too young to understand anything about such a difficult subject. The child listened to what the father said, and appeared to think it irrelevant, for he still insisted, 'But, please, explain.'"

From 1843, when his family moved to Croft Rectory in Yorkshire, his father could afford to send young Charles to private school, first in Richmond (Yorkshire) and then to Rugby School where he won many prizes. As his mathematics master confided to Charles's father: 'I have not had a more promising boy at his age since I came to Rugby."

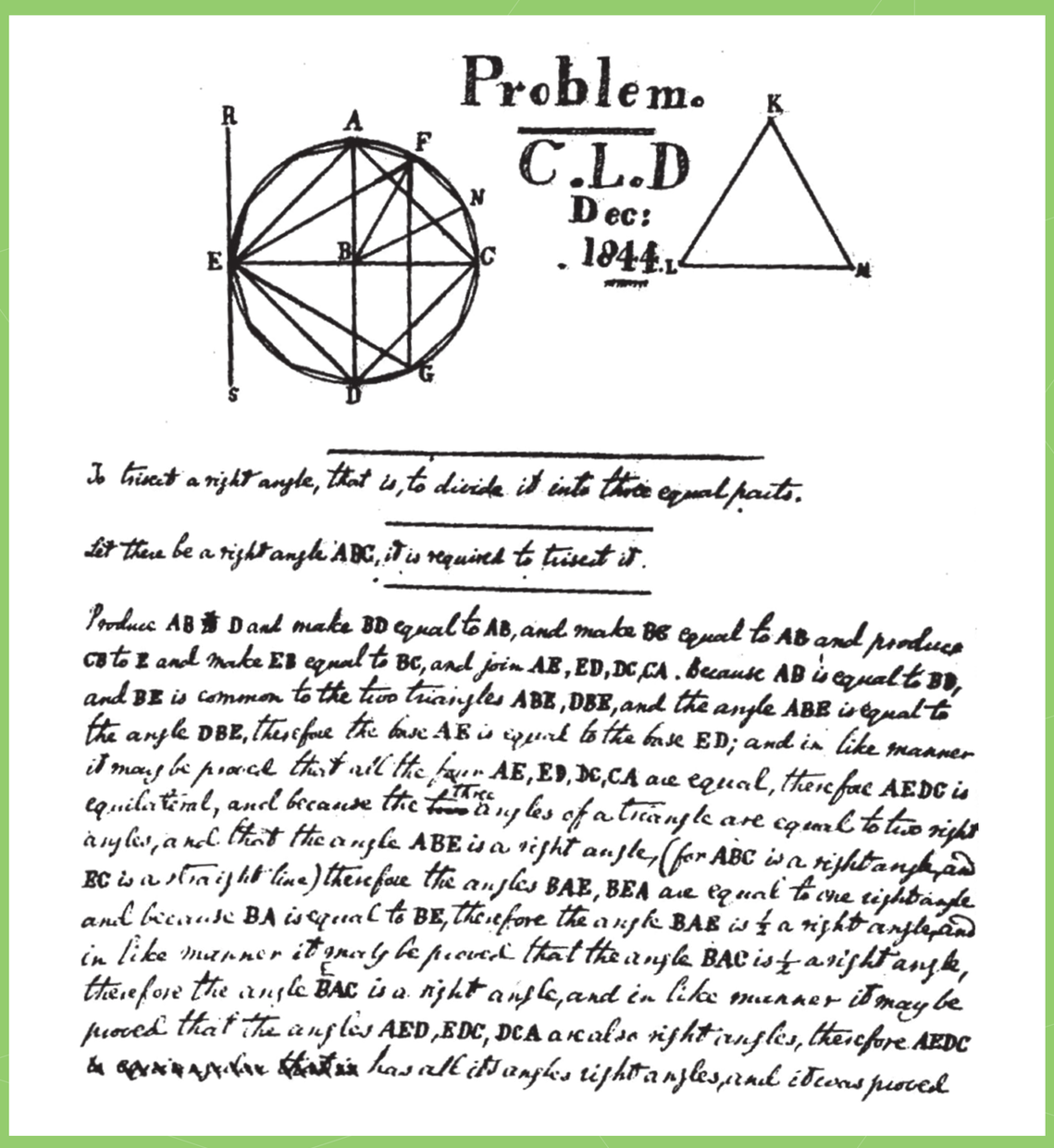
When Charles was 12, he produced a two-page note on trisecting a right angle, which seems remarkable for a child of his age (see below).

Dodgson's arithmetic textbook at school was the 1842 edition of Francis Walkingame's popular *Compendium of Arithmetic*. His copy survives and includes the following problems, unlikely to be seen in present-day school texts!

A SAMPLE OF WALKINGAME'S ARITHMETIC PROBLEMS:

What is the cube root of 673373097125?
Ans: 8765.

What sum did that gentleman receive in dowry with his wife, whose fortune was her wedding suit: her petticoat having 2 rows of furbelows, each furbelow 87 quills, and each quill 21 guineas?
Ans. £3836: 14s.



Problem.

C.L.D
Dec: 1844.

To trisect a right angle, that is, to divide it into three equal parts.

Let there be a right angle ABC, it is required to trisect it.

Produce AB to D and make BD equal to AB, and make BC equal to AB and produce CB to E and make EB equal to BC, and join AE, ED, DC, CA. Because AB is equal to BD, and BE is common to the two triangles ABE, DBE, and the angle ABE is equal to the angle DBE, therefore the base AE is equal to the base ED; and in like manner it may be proved that all the four AE, ED, DC, CA are equal, therefore AEDC is equilateral, and because the ~~two~~ three angles of a triangle are equal to two right angles, and that the angle ABE is a right angle, (for ABC is a right angle, and EC is a right line) therefore the angles BAE, BEA are equal to one right angle and because BA is equal to BE, therefore the angle BAE is ½ a right angle, and in like manner it may be proved that the angle BAC is ½ a right angle, therefore the angle BAC is a right angle, and in like manner it may be proved that the angles AED, EDC, DCA are also right angles, therefore AEDC is equiangular and it has all its angles right angles, and it was proved

Student days in Oxford

On 23 May 1850 Charles Dodgson travelled to Oxford for his matriculation examinations in Latin, Greek and mathematics, and was officially enrolled as a member of Oxford University. He entered Christ Church in January 1851 and settled into the routine of student life.

The University year was divided into four terms: Michaelmas (October–December), Hilary or Lent (January–March), Easter (April–June), and Trinity (June–July).

Undergraduates were Passmen working for a three-year Pass degree, and Classmen working for four-year Honours degrees in Classics and then Mathematics, Natural Science, or Law and Modern History. Dodgson elected for Honours in Classics and Mathematics.

His university course required him to pass three examinations, *Responsions* (or 'Little-go'), *Moderations* and *Finals*. Examinations were both written and oral.

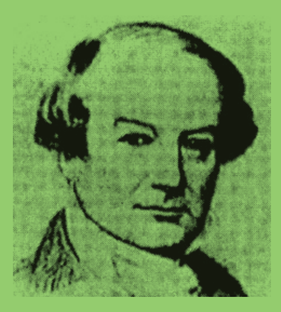
Responsions included papers in Latin and Greek, grammar, basic arithmetic, algebra, and geometry from Euclid's *Elements*, Books I, II.

He then prepared a schedule for Moderations:

"I believe 25 hours' hard work a day may get through all I have to do, but I am not certain."

This paid off, and in his 1852 Moderations examinations he was awarded a First Class in Mathematics.

Teaching was provided by professors delivering University lectures, college lecturers teaching small student groups, and tutors giving private tuition. The Savilian Professor of Geometry was Baden Powell and the Mathematical Lecturer at Christ Church was Robert Faussett.



Baden Powell

Robert Faussett

In Summer 1854, Dodgson went on a Mathematics reading party to Yorkshire with Bartholomew Price, newly appointed Sedleian Professor of Natural Philosophy. Known as 'Bat', he was immortalised in the Hatter's parody:

"Twinkle, twinkle, little bat!
How I wonder what you're at!"

Bartholomew 'Bat' Price

Dodgson's Finals examinations in late 1854 ranged through pure and applied mathematics, from algebra, geometry and calculus to mechanics and astronomy. He came top of the entire list, and received his First Class B.A. degree in December.

An Oxford Viva examination

C L Dodgson (Lewis Carroll) (1832–98)

Dodgson at Christ Church

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From 1856 to 1881 Charles Dodgson was Mathematical Lecturer at Christ Church, teaching mathematics to students for Responsions, Moderations and Finals.

Dodgson, the young lecturer

After graduation Dodgson took on private pupils at Christ Church, preparing them for the Oxford examinations – particularly Responsions.

In 1855 the Dean of Christ Church died and was replaced by Henry Liddell (of Liddell & Scott's Greek dictionary, which is still used today). Dean Liddell had several children, one of whom (Alice) was to be forever associated with the name of Lewis Carroll.

The Mathematical Lecturer Robert Faussett had left Oxford to fight in the Crimean War, and Dean Liddell appointed Dodgson in his place, from January 1856. Dodgson was to hold this position for twenty-five years.

In 1856 Charles Dodgson adopted the pen-name Lewis Carroll – from Carolus (Latin for Charles) and Lutwidge (his middle name and his mother's maiden name).

It was also around this time that Dodgson took up the hobby of photography using the wet collodion process, taking many hundreds of pictures of his Oxford contemporaries and famous figures of the day. His artistic pictures included many fine portraits of children, such as his well-known picture of Alice Liddell as a beggar girl.

Alice Liddell

A new Mathematical Institute?

An example of Dodgson's playful attitude to University affairs was in a letter of 1868 entitled *The Offer of the Clarendon Trustees*, where he proposed an appropriately designed Mathematical Institute. Oxford University had to wait a further sixty years before this happened.

Dear Senior Censor,

In a desultory conversation on a point connected with the dinner at our high table, you incidentally remarked to me that lobster-sauce, "though a necessary adjunct to turbot, was not entirely wholesome."

It is entirely unwholesome. I never ask for it without reluctance: I never take a second spoonful without a feeling of apprehension on the subject of possible nightmare.

This naturally brings me on to the subject of Mathematics, and of the accommodation provided by the University for carrying on the calculations necessary in that important branch of Science …

It may be sufficient for the present to enumerate the following requisites: others might be added as funds permitted.

A. A very large room for calculating Greatest Common Measure. To this a small one might be attached for Least Common Multiple: this, however, might he dispensed with.

B. A piece of open ground for keeping Roots and practising their extraction: it would be advisable to keep Square Roots by themselves, as their corners are apt to damage others.

C. A room for reducing Fractions to their Lowest Terms. This should be provided with a cellar for keeping the Lowest Terms when found, which might also be available to the general body of Undergraduates, for the purpose of "keeping Terms."

D. A large room, which might be darkened, and fitted up with a magic lantern, for the purpose of exhibiting Circulating Decimals in the act of circulation. This might also contain cupboards, fitted with glass doors, for keeping the various Scales of Notation.

E. A narrow strip of ground, railed off and carefully levelled, for investigating the properties of Asymptotes, and testing practically whether Parallel Lines meet or not: for this purpose it should reach, to use the expressive language of Euclid, "ever so far."

This last process, of "continually producing the Lines," may require centuries or more: but such a period, though long in the life of an individual, is as nothing in the life of the University…

May I trust that you will give your immediate attention to this most important subject?

Believe me, Sincerely yours,
MATHEMATICUS.

C L Dodgson (Lewis Carroll) (1832–98)

Geometry and Algebra

UNIVERSITY OF OXFORD
Mathematical Institute

Much of Dodgson's teaching activities involved geometry – especially from Euclid's *Elements* – while his research investigations included algebra.

Geometry

From his earliest years Dodgson had been enthused by geometry. The following is his whimsical view of the Pythagorean theorem on right-angled triangles:

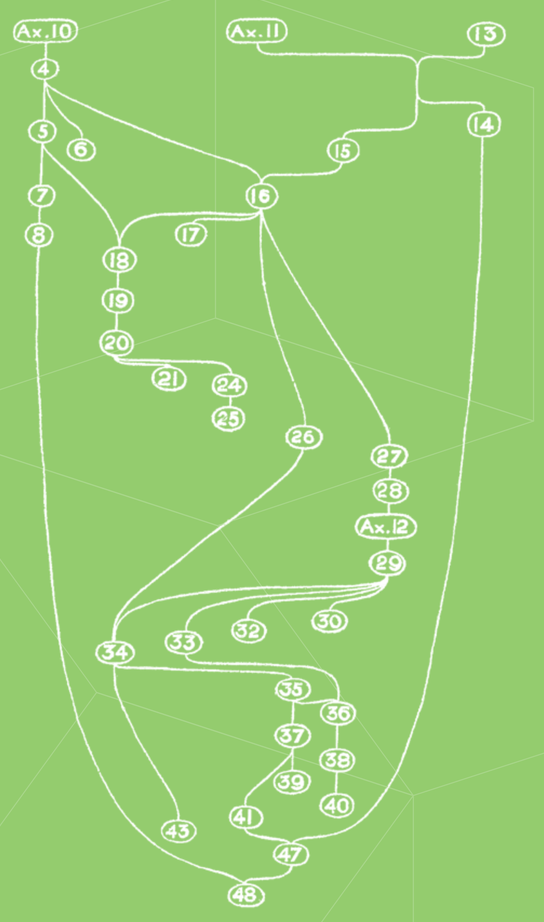
"It is as dazzlingly beautiful now as it was in the day when Pythagoras first discovered it, and celebrated its advent, it is said, by sacrificing a hecatomb [one hundred] of oxen – a method of doing honour to Science that has always seemed to me slightly exaggerated and uncalled-for. One can imagine oneself, even in these degenerate days, marking the epoch of some brilliant scientific discovery by inviting a convivial friend or two, to join one in a beefsteak and a bottle of wine. But a hecatomb of oxen! It would produce a quite inconvenient supply of beef."

But the Greek author that Dodgson most admired was Euclid, whose *Elements* in thirteen parts has been described as the most printed book of all time after the Bible, and became a central text for teaching geometry and training the mind. During the 19th century over two hundred editions were produced in England alone.

A knowledge of the first six parts of the *Elements* formed part of the various University examinations. To help his students Dodgson wrote a number of useful mathematical pamphlets explaining the ideas involved.

But Dodgson's best-known writings on geometry appear in *Euclid and his Modern Rivals*. Dedicated 'to the memory of Euclid', this book compares, unfavourably in every case, Euclid's treatment of geometry (parallel lines, angles, etc.) with those of about a dozen more recent texts. In order to increase its popularity, it was written in the style of a play in which Dodgson exhibits both his intimate knowledge of Euclid and his whimsical sense of humour.

The *Elements* is axiomatic and hierarchical, with a large number of propositions all following on from previous ones, and ultimately from an initial set of axioms and postulates. Dodgson produced the following diagram to show how Book I, Proposition 47 (the Pythagorean theorem) depends on previous results.



Euclidean hierarchy

But not all of Dodgson's geometrical endeavours were so serious. His witty pamphlet *The Dynamics of a Parti-cle* satirised the parliamentary election for the Oxford University seat in July 1865.

Much of the pamphlet resembles a treatise on formal geometry, with some initial definitions parodying the Euclidean ones.

Whereas Euclid had written:

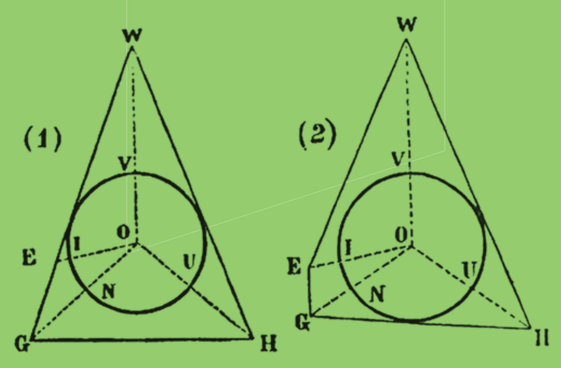
- A plane angle is the inclination of two straight lines to one another, which meet together, but which are not in the same direction.

Dodgson wrote:

- Plain anger is the inclination of two voters to one another, who meet together, but whose views are not in the same direction.

And so he continued for several pages, leading up to the following geometrical construction, designed *to remove a given Tangent from a given Circle, and to bring another given Line into contact with it*.

In order to do so, Dodgson assigned letters to the points of the diagram:



Here, UNIV represents the University, O is Oxford, V is the Vice-Chancellor, and WEG, GH and WH are the three candidates, William Ewart Gladstone (too liberal for Dodgson), the conservative Gathorne Gathorne-Hardy, and William Heathcote; the object was to unseat Gladstone and replace him by Gathorne-Hardy.

Before working through a pseudo-geometrical argument, Dodgson observes: *When this is effected, it will be found most convenient to project* WEG *to infinity.*

Algebra

A well-known story (which Dodgson firmly denied) relates how Queen Victoria was so utterly charmed by *Alice's Adventures in Wonderland* that she demanded: "Send me the next book that Mr Carroll produces".

The next book duly arrived. It was entitled *An Elementary Treatise on Determinants with their Application to Simultaneous Linear Equations and Algebraical Geometry.*

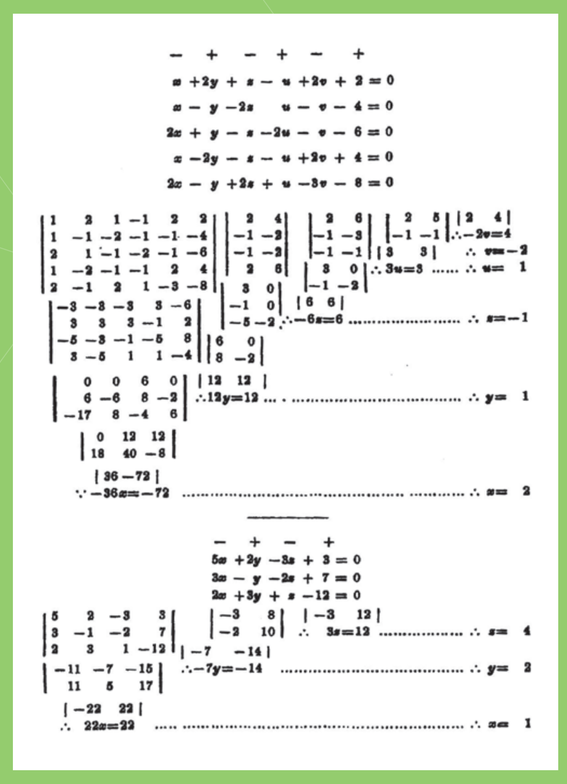
The Queen was not amused.

The theory of determinants was much in vogue in the 1850s and 1860s, and featured in the University examinations. It arises in the context of analytic geometry, in which algebraic methods are used to obtain results in geometry.

Unfortunately, Dodgson's book was not a success, possibly due to a lack of distribution to key mathematicians, while his terminology and notation were too cumbersome and his over-formal approach made the book difficult to read.

However, it was the book in which a certain well-known undergraduate result (the Kronecker–Capelli theorem) first appeared in print, and it also contained a significant new method, his *condensation method*, in which large determinants (used to solve large systems of simultaneous equations) are replaced by many smaller ones that can be evaluated easily.

Dodgson showed his condensation method to his friend 'Bat' Price, who presented it on his behalf at a Royal Society meeting, and Dodgson's paper on the subject was subsequently published in the Society's *Proceedings*.



Dodgson's condensation method

C L Dodgson (Lewis Carroll) (1832–98)

Logic and Voting

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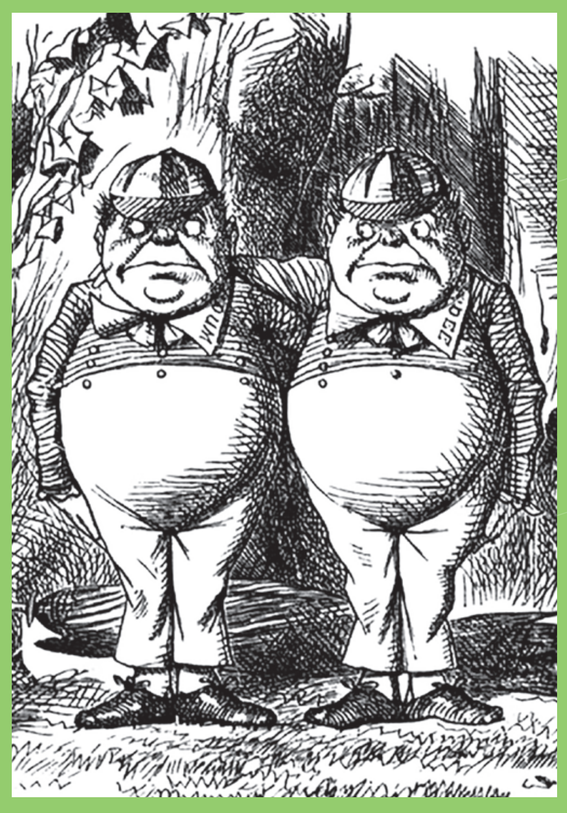
Dodgson's most influential contributions were to mathematical logic and the theory of voting.

Logic



Tweedledum: I know what you're thinking about, but it isn't so, nohow.
Tweedledee: Contrariwise, if it was so, it might be; and if it were so, it would be: but as it isn't, it ain't. That's logic.

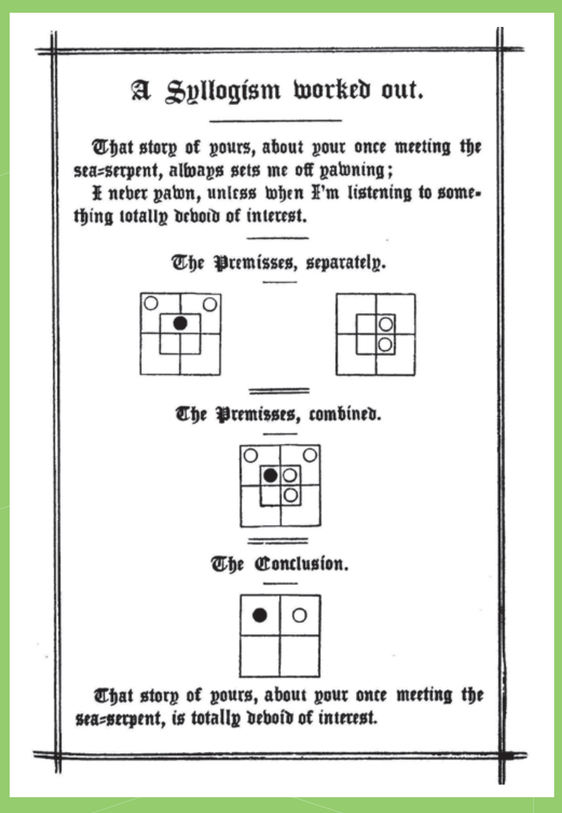
Towards the end of his life, Dodgson wrote extensively about logic. He was particularly interested in presenting symbolic logic as an entertainment for children to develop their powers of logical thought, and as a topic of serious study for adults.

Much of his early work on logic was concerned with *syllogisms*, consisting of a couple of statements called premises that lead to a conclusion.

These can be traced back to the 4th century BC, when Aristotle presented the two premises *All men are mortal* and *Socrates is a man* and concluded that *Socrates is mortal.*

Dodgson's were more entertaining; for example:
A prudent man shuns hyenas
No banker is imprudent
Conclusion: *No banker fails to shun hyenas*

No bald creature needs a hairbrush
No lizards have hair
Conclusion: *No lizard needs a hairbrush*



A Syllogism worked out.

That story of yours, about your once meeting the sea-serpent, always sets me off yawning;
I never yawn, unless when I'm listening to something totally devoid of interest.

The Premisses, separately.

The Premisses, combined.

The Conclusion.

That story of yours, about your once meeting the sea-serpent, is totally devoid of interest.

Dodgson believed that symbolic logic could be understood by his many child-friends, and devised *The Game of Logic* to help them sort out syllogisms. His method used red and grey counters that were placed on sections of a board to represent statements of the forms 'Some X are Y' and 'No X are Y'.

He then extended his examples to more than two premises that could be sorted out with his counters – for example:
No kitten that loves fish is unteachable.
No kitten without a tail will play with a gorilla.
Kittens with whiskers always love fish.
No teachable kitten has green eyes.
No kittens have tails unless they have whiskers.
Conclusion: *No kitten with green eyes will play with a gorilla.*

One of his more complicated examples included no fewer than 50 premises.

Dodgson also produced two logical paradoxes that have been praised by logicians and philosophers such as Bertrand Russell. Unfortunately, he died suddenly before he could publish all his logical writings, and they were lost for over 70 years. Otherwise, he might have been remembered as the best British logician between George Boole and Russell.

Theory of voting

Through College elections Dodgson became interested in the theory of voting. He was always concerned to achieve fairness – both to the winner and to minority candidates – and he designed ingenious examples to show how several widely used voting systems are flawed.

As an example, consider the simple majority, or first-past-the-post system:
"*Suppose that there are eleven voters and four candidates, a, b, c, d, and that each voter arranges the candidates in a column in order of preference:*

a	*a*	*a*	*b*	*b*	*b*	*b*	*c*	*c*	*c*	*d*
c	*c*	*c*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*
d	*d*	*d*	*c*	*c*	*c*	*c*	*d*	*d*	*d*	*c*
b	*b*	*b*	*d*	*d*	*d*	*d*	*b*	*b*	*b*	*b*

Then a is considered best by three voters and second best by the rest, and surely ought to be elected. But by the above method, b is the winner, even though he is considered worst by seven of the voters."

Dodgson also made various recommendations concerning Parliamentary elections. Some of these were eventually adopted, such as the rule that no results should be announced until all the polling stations had closed. Others, such as his various methods for proportional representation, were not.

Dodgson had intended to write a book on voting patterns, but this never materialised, causing the Oxford philosopher Michael Dummett (himself, an expert on the subject) to remark, many years later:
"It is a matter of the deepest regret that Dodgson never completed the book that he planned to write on the subject. Such were the lucidity of exposition and his mastery of the topic that it seems possible that, had he ever published it, the political theory of Britain would have been significantly different."

An intriguing thought indeed!"

C. L. Dodgson (Lewis Carroll)

Oxford Mathematics

C L Dodgson (Lewis Carroll) (1832–98)

Dodgson at play

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Mathematical language pervades Dodgson's popular writings for children. Also included here are some of his puzzles for children and adults alike.

Mathematics in his writings

In *Alice's Adventures in Wonderland*, the Mock Turtle reminisces:

"I only took the regular course…
Reeling and writhing to begin with.
And then the different branches of arithmetic – ambition, distraction, uglification and derision.
[And how many hours a day did you do lessons?]
Ten hours the first day, nine hours the next, and so on… That's the reason they're called lessons – because they lessen from day to day."

[I don't know. I lost count.]
She can't do addition. Can you do subtraction? Take nine from eight.
[Nine from eight I can't, you know, but…]
She can't do subtraction.
Can you do division? Divide a loaf by a knife…
[I suppose –]
Bread and butter of course.
She can't do sums a bit!"

Other examples are more sophisticated, involving ideas of geometry, logic and gravity, and *Sylvie and Bruno Concluded* includes a scene in which Dodgson shows how to construct a projective plane by sewing together three pocket handkerchiefs to make 'Fortunatus's purse' which has no outside or inside and so contains all the wealth of the world inside it.

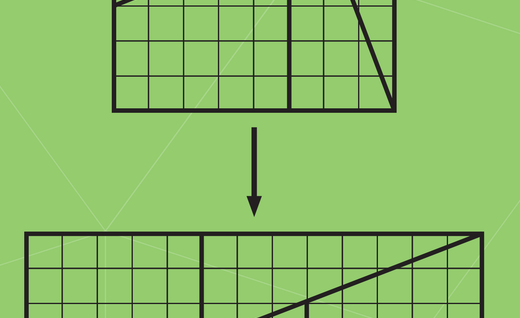
And in *Through the Looking-Glass* the White Queen and Red Queen test Alice on whether she should become a Queen:
Can you do addition? What's one and one and one and one and one and one and one and one and one and one?

Recreational puzzles

Dodgson enjoyed showing mathematical puzzles to his child-friends and to adults. Here is a selection of them: answers are given at the end of the column.

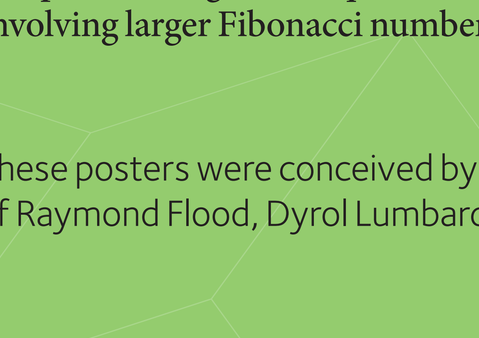
The monkey and the weight

A rope is supposed to be hung over a wheel fixed to the roof of a building; at one end of the rope a weight is fixed, which exactly counterbalances a monkey which is hanging on the other end. Suppose that the monkey begins to climb the rope, what will be the result?



The extra square

Start with an 8 × 8 grid of 64 squares and cut it into four pieces, as shown. If we rearrange the pieces, we obtain a 5 × 13 grid of 65 squares. Where did the extra square come from?



The numbers 5, 8 and 13 appearing here are all Fibonacci numbers, and Carroll showed how to extend this paradox to grids of squares involving larger Fibonacci numbers.

Brandy and water

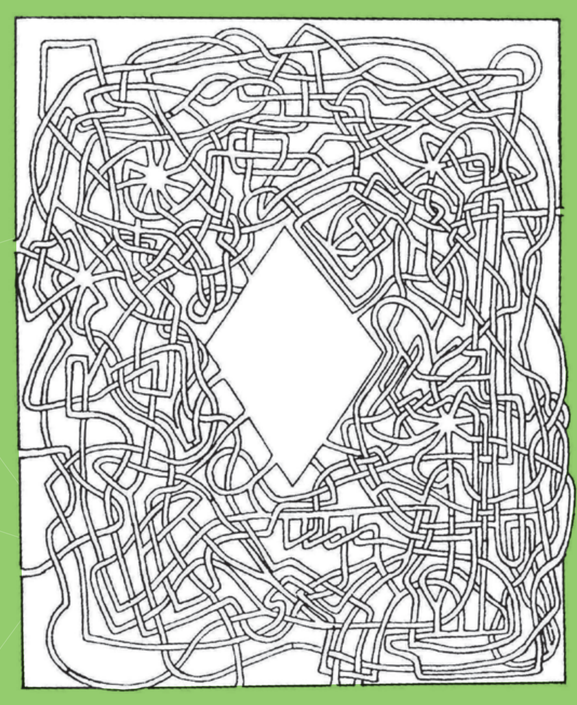
Take two tumblers, one of which contains 50 spoonfuls of pure brandy and the other 50 spoonfuls of pure water. Take from the first of these one spoonful of the brandy and transfer it into the second tumbler and stir it up. Then take a spoonful of the mixture and transfer it back to the first tumbler. If you consider the whole transaction, has more brandy been transferred from the first tumbler to the second, or more water from the second tumbler to the first?

The clock face

A clock face has all the hours indicated by the same mark, and both hands the same in length. It is opposite to a looking-glass. Find the time between 6 and 7 when the time as read direct and in the looking-glass shall be the same.

Three-dimensional maze

Possibly the earliest three-dimensional maze ever constructed was produced by Dodgson for his younger brothers and sisters.



Answers:

The monkey and the weight
The weight also moves up, in such a way that the monkey and the weight remain at the same level throughout.

The extra square
Careful drawing shows that the second diagram has a thin diamond-shaped gap in the middle with area one square.

Brandy and water
They are the same, since each tumbler ends up with 50 spoonfuls of liquid.

The clock face
27 $\frac{9}{13}$ minutes past 6 o'clock.

These posters were conceived by Robin Wilson, with the assistance of Raymond Flood, Dyrol Lumbard and Edward Wakeling.

Oxford Mathematics