REPORT ON EXAMINATIONS

M.Sc. in Mathematical Modelling and Scientific Computing 2022-23

Part I

A. Statistics

- **Numbers and percentages in each class**

	Number				Percentage			
	2022/23	2021/22	2020/21	2019/20	2022/23	2021/22	2020/21	2019/20
Distinction	9	7	8	11	38	39	32	50
Merit	6	7	7	5	25	39	28	22
Pass	8	2	10	4	33	11	40	18
Fail	1	1	0	2	4	5.5	0	10
Incomplete	0	1	0	0	0	5.5	0	0

- **Vivas**
  The 24 candidates who submitted dissertations were examined by *viva voce*.

- **Marking of scripts**
  Written examinations were sat in Weeks 0 of Hilary and Trinity Terms 2023. Scripts were single-marked by assessors followed by a script check carried out by the Course Director. Finalisation of marks by the examiners took place during an examiners' meeting in week 3 of each term. Special topics and case studies were double-marked by assessors. In cases where marks varied over the pass/fail borderline, or the difference in marks was greater than ten, the assessors were asked to meet and reconcile their marks. All marks were approved by the examiners during the meetings held in week 7 of Hilary Term and week 7 of Trinity Term, as well as at the final examiners' meeting, before being released to the candidates. All dissertations were read and marked by at least two examiners; marks were approved by all examiners at the final examiners' meeting and by confidential correspondence.

B. Changes in examining methods etc. which the examiners would wish the faculty/department and the divisional board to consider

The written examinations will be in-person, closed-book exams in the academic year 2023-24. Students will no longer be permitted to take one A4 sheet of notes into the exam.

C. How candidates are made aware of conventions

The conventions are posted on the course website and electronic copies are circulated to the students. The Course Director discusses the conventions with the candidates and the candidates are reminded of them by email on several occasions during the year. The candidates are notified via email about any changes to the examination conventions and amended conventions are uploaded to the course website.

Part II

A. General comments on the examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. In addition, the internal examiners would like to express their gratitude to Prof Katerina Kaouri for carrying out her duties as external examiner in a constructive and supportive way during the year, and for valuable input at the final examiners' meeting.

Setting and checking of papers

Following established practice, the questions for each paper were initially set by the course lecturer, with a qualified person involved as checker before the first drafts of the questions were presented to the Chair of Examiners and the External Examiner. The course lecturers also acted as assessors, marking the questions on their course(s).

Determination of University Standardised Marks

The examiners followed established practice in determining the University standardized marks (USMs) reported to candidates for the written examinations. The algorithm converts raw marks to USMs for each paper separately. For each paper, the algorithm sets up a map R → U (R = raw, U = USM) which is piecewise linear. The graph of the map consists of three line segments which join the points (0,0), (P,50), (D,70) and (100,100). The values of P and D are chosen so that the resulting USMs are in line with the mark descriptors in the Examination Conventions. Particular attention is paid to the scripts that lie around class borderlines after the mapping has been applied. The values of P and D for each of the four written examinations in 2022-23 is given in the table below.

Paper	P	D
A1	42	67
A2	49	70
B1	49	69
B2	45	70

B. Equal opportunities issues and sex breakdown

The breakdown of results by gender is given in the tables below. This data is based on the sex recorded against students' records.

Class	Number							
	2022-23		2021-22		2020-21		2019-20	
	Female	Male	Female	Male	Female	Male	Female	Male
Distinction	2	7	2	5	2	6	2	9
Merit	2	4	0	6	3	4	2	3
Pass	4	4	0	2	4	6	3	1
Fail	1	0	0	1	0	0	1	1
Incomplete	0	0	0	2	0	0	0	0
Total	9	15	2	16	9	16	8	14

Class	Percentage							
	2022-23		2021-22		2020-21		2019-20	
	Female	Male	Female	Male	Female	Male	Female	Male
Distinction	22.2	46.7	100	31.25	22.2	37.5	25	64.3
Merit	22.2	26.7	0	37.5	33.3	25.0	25	21.4
Pass	44.4	26.7	0	12.5	44.4	37.5	37.5	7.1
Fail	11.1	0	0	6.25	0	0	12.5	7.1
Incomplete	0	0	0	12.5	0	0	0	0
Total	99.9	100.1	100	100	99.9	100	100	99.9

C. Candidates' performance in each part of the examination

This course administers examinations internally in January and April, with each student sitting 4 papers. Each of the two sets of examinations is split into Paper A (Mathematical Methods) and Paper B (Numerical Analysis). Both sets of examinations went smoothly this year, with a good distribution of marks between failure and distinction ranges.

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A1	23	50.30	17.09	55.30	16.71
A2	23	64.13	11.95	64.48	11.58
B1	23	59.13	14.87	60.04	14.90
B2	23	58.22	18.81	60.57	17.08

The tables that follow give the question statistics for each paper. Examiners' comments for all papers can be found at the end of this document.

Paper A1: Mathematical Methods I

Question	Mean mark	StDev	Number of attempts	
			Used	Unused
Q1	11.12	5.49	15	2
Q2	8.86	4.60	12	2
Q3	9.64	4.25	20	2
Q4	23.00	1.00	2	0
Q5	16.48	5.31	23	0
Q6	11.09	6.14	20	2

Paper A2: Mathematical Methods II

Question	Mean mark	StDev	Number of attempts	
			Used	Unused
Q1	14.52	3.83	21	0
Q2	17.62	3.91	21	0
Q3	16.00	5.77	15	1
Q4	20.54	3.57	13	0

Q5	13.38	5.22	7	1
Q6	12.56	5.27	15	1

Paper B1: Numerical Solution of Partial Differential Equations and Numerical Linear Algebra

Question	Mean mark	StDev	Number of attempts	
			Used	Unused
Q1	15.16	3.75	19	0
Q2	13.53	5.56	13	2
Q3	11.80	4.51	9	1
Q4	14.68	4.97	18	1
Q5	18.81	3.79	21	0
Q6	10.55	5.55	10	1

Paper B2: Numerical Linear Algebra and Continuous Optimisation

Question	Mean mark	StDev	Number of attempts	
			Used	Unused
Q1	15.95	5.88	19	0
Q2	17.33	3.40	9	0
Q3	13.05	5.59	20	1
Q4	16.21	5.92	14	0
Q5	12.64	7.04	13	1
Q6	13.56	4.47	15	1

Performances on the special topics and dissertations also ranged from fail to distinction level. No student failed the case studies in Mathematical Modelling or Scientific Computing, where a high proportion of distinction grades were seen. 17 of 24 (71%) of both case studies resulted in Distinction grades.
Grades for the special topics ranged from pass through to distinction. Of the 71 special topics submitted this academic year, 30 (42.25%) attained a distinction grade, and 33 (46.5%) attained a merit. One special topic mark was carried over from the prior academic year at a failing grade.

D. Distribution of special topics

Of the 23 topics listed this year, 6 failed to attract any students.

Special Topic Course	**Passed**	**Failed**
Approximation of Functions	2	1
Differentiable Manifolds*	1	0
Elasticity and Plasticity	1	0
Finite Element Methods for Partial Differential Equations	5	0
Further Mathematical Biology	7	0

Integer Programming	3	0
Introduction to Quantum Information*	1	0
Machine Learning*	1	0
Mathematical Geoscience	2	0
Mathematical Mechanical Biology	2	0
Mathematical Models of Financial Derivatives	5	0
Mathematical Physiology	5	0
Networks	7	0
Optimisation for Data Science	2	0
Python in Scientific Computing	18	0
Random Matrix Theory*	1	0
Stochastic Differential Equations	1	0
Stochastic Modelling of Biological Processes	2	0
Topics in Fluid Mechanics	1	0
Viscous Flow	2	0
Waves and Compressible Flow	2	0

Courses labelled * were offered by special approval.

E. Names of members of the board of examiners

Examiners:
Prof. R. Baker (Chair)
Prof. V. Nanda
Prof. Y. Nakatsukasa
Prof. P. Howell (for Final Exam Board)
Prof. S. J. Chapman (until Final Exam Board)
Prof. K. Kaouri (External Examiner)

Assessors:
Dr F. Aznaran
Dr G. Benham
Prof. C. Breward
Prof. H. Byrne
Dr Z. Cai
Prof. A. Cartea
Prof. C. Cartis
Dr G. Cazassus
Prof. S. Cohen
Prof. P. Dellar
Dr M. Dvoriashyna
Prof. A. Ekert
Prof. R. Erban
Prof. P. Farrell
Dr B. Fehrman

Dr K. Gillow
Prof. A Goriely
Prof. I. Griffiths
Prof. P. Grindrod
Prof. M. Gubinelli
Prof. R. Hauser
Prof. I. Hewitt
Dr K. Hu
Prof. D. Joyce
Prof. J. Keating
Prof. R. Lambiotte
Prof. P. Maini
Prof. I. Moroz
Prof. D. Moulton
Prof. A. Münch
Prof. J. Oliver
Dr J. Panovska-Griffiths
Prof. C. Reisinger
Prof. E. Süli
Prof. L.N. Trefethen
Prof. S. Waters
Prof. A. Wathen

F. Examiners’ Comments

Paper A1: Mathematical Methods I

Examiners’ Comments

The exam outcome was somewhat disappointing. The style of the questions was somewhat different from previous years, with more parts that required an insight or had a bit more surprising elements that in previous years. However, there was quite a bit of material that was standard and had been sat in similar forms in previous exams.

Q1 was the second most popular question. (a) was generally well done by most candidates who attempted this question, but many struggled with (b). Some made use of preliminary invariance information to stipulate u=t^(c/a) f(xi), which was not exactly the intention, but was generally accepted. In part c, several students got alpha, beta right and the ODE/BVP but only very few made progress with the solution.

Q2 was done by a less than half the candidates. Many got (a) right, though there were some ambiguities and incomplete answers (such as introducing dx_-/dt for the characteristic speed at u=u_- for the left state of the shock, without defining this quantity. Surprisingly, most students struggled with (b) and I cannot remember a single complete solution here. One problem was to realise that the solution pertaining to the g(x)=x piece in the initial data (3) is delimited by the two shocks and not by the characteristic going through x=½ and x=-½. Part c was attempted by very few students and not done successfully.

Q3 was the question most the question most similar to questions in exam papers and previous exams and done by almost all students. However, students still struggled. While most got the PDEs for x_tau, y_tau etc right, solving them often got wrong due to algebraic mistakes. If students continued with their wrong solutions in subsequent parts this was accepted and typically marked down only if the problem was severely simplified by the ‘wrong’ intermediate solution. Students got the initial data right for p_0, q_0 in (b) but sometimes failed to write down the explicit form for x,y, for which some marks were taken away. Nobody finished c. Some marks were given for partial answers or even explanations of how to proceed.

Q4 was only done by two students, and they gave reasonable, if not always correct or complete, answers.

Q5 was done well, with many students scoring full marks on parts (a) and (b) (average mark: 16.5/25). Part (c) was more challenging, with many students failing to recognise that a weighting factor was needed.

Q6: the students found this question more challenging than Q5 (average mark: 11/25), with many failing to use the approach outlined in part (a) and/or unable to accurately determine the Greens function in part (b). Full credit was given to candidates who stated the correct boundary conditions for the Greens function in part (a), even if they had not followed the proposed method. Similarly, in part (b), credit was given to students who did not use the results from part (a) to determine the particular integral in the solution to the stated problem.

Assessors' Report Form

Report on: Nonlinear Systems

Section A – Confidential Report for Examiners

There were no alterations to the mark scheme. No mistakes were found on the paper. Raw marks should be a reasonable approximation to USM.

Section B - Comments on the paper

Question 1

This was a popular question. Each part of the question was answered correctly by at least one student (so no part was too hard), and yet there was only one mark in the 20s. Most of the mistakes were silly algebraic mistakes, especially in part b(i). I was surprised how few students got all the attracting sets correct in part (iv).

Question 2

This question was also very popular. In general students seem to have found it slightly easier that Q1, perhaps because it followed a more familiar format. Again most marks were dropped because of algebraic mistakes—most students approached the problem in the right way. Nobody managed to draw the final bifurcation diagram, even those who correctly identified all the steady states in part (a)

Further Partial Differential Equations 2023
Examination commentary

Question 5
The level of physical understanding of this question was appropriate. Candidates were let down by algebraic mistakes and by not attempting parts of the question.

(a) Candidates who attempted this question got all of this correct.
(b) All candidates understood the concept of this part. Some candidates lost marks for algebraic mistakes.
(c,d) Algebraic mistakes led around half of candidates making an incorrect interpretation of superheating.
(e) Half of the candidates got this right; half made little or no attempt.
(f) Many candidates who attempted this achieved a good score. Most did not attempt this part.

Question 6
This question was somewhat challenging algebraically by many candidates, which caused them to lose marks. Most did not attempt the latter parts of the question, which were less algebraically challenging.

(a) This was well completed by almost all candidates. More than half of the candidates did not write down *h* as requested in the question.
(b) This proved challenging for most candidates. Some found the solution that was independent of z that had already been found in part a. Many got the correct methodology but arrived at the incorrect answer due to algebraic mistakes. Marks were awarded for the correct idea.
(c) Most candidates attempted this but many struggled with the algebra associated with the chain rule.
(d) Almost all candidates who attempted this part managed to obtain a first-order differential equation although some did not obtain the correct solution.
(e) Most candidates did not attempt this part. However, those who did attempt this managed to obtain the answer.

MSc Assessors' Report Form

Report on: B1 Numerical Linear Algebra

Section A – Confidential Report for Examiners

There were no changes to the marking scheme.

Section B - Comments on the paper for inclusion in the Examiners' Report

Q1. 19 candidates attempted this question. Question (a-i) was answered correctly by most. Some added pertinently that the order of the singular values remain the same with the map $\sigma_i \leftarrow \sigma_i^3$; this was not necessary for a full mark, but a good point. Question (a-ii) can be solved using Courant-Fischer (and such solutions received a full mark), but this is somewhat backwards; if any thing (a-ii) would inspire Courant-Fischer. Direct calculations suffice for a solution. Some solved (a-iii) using Σ^{-1}, but as no assumption is given on rank(A), this problem should be solved without assuming Σ is invertible (1 mark off).

Almost all candidates attempted (b-i) by brute-force (making for a lengthy calculation), while it was expected to note that $A = BDDB^T$ where $\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ and D is diagonal, so $x^TAx = x^T(BD)(BD)^Tx = y^Ty$, where $y = (BD)^Tx$. For (b-ii) many identified the rank-1 approximation to be $A_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, but not many correctly computed $\|A_1 - A\|_2 = 2 \times 10^{-10}$. A number of candidates seemed to assume that B is orthonormal, which is incorrect.

(b-iii) was intended to be challenging, and most attempts received incomplete marks. Attempts to compute $\sigma_2(A)$ exactly would be far too tedious. Instead, it was intended to note that $A = QRD^2R^TQ$, so $\sigma_2(A) = \sigma_2(RD^2R^T) \geq \|RD^2R^Tx\|_2/\|x\|_2$ for any x by (a-ii), and set $x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Q2. This question was attempted by 15 candidates.

(a-i) was completed by most. Some solutions for (a-ii) had no justification. Some stated incorrectly that singular matrices do not have an LU factorisation in (a-iii). In (a-iv) many did not discuss the full-rankness of the submatrix after each elimination step.

A surprisingly large number failed to fully solve (b-i), which was a bookwork problem but perhaps not the easiest calculation. Many answered the $m < n$ case in (b-ii) to be $A = Q[R\ 0]$, which is incorrect. For a full mark in (b-iii), an argument is needed to show one can take $R = I$, and that sometimes $H_i = I$ is needed, which means that the Householder reflector is skipped (as no Householder reflector is equal to I).

MSc Assessors' Report Form

Report on: B1 Numerical Solution of Partial Differential Equations

Section A – Confidential Report for Examiners

There were no changes to the marking scheme.

Section B - Comments on the paper for inclusion in the Examiners' Report

Q3. The question was concerned with the finite difference approximation of a boundary-value problem for a second-order linear differential equation subject to homogeneous Dirichlet boundary conditions. There were ten attempts at the question, but only two were close to being complete. The majority of those who attempted the question had no difficulties showing the inequality in part (a) of the question based on integration by parts, but had difficulties replicating the analogous arguments in parts (b) and (c) of the question based on summation by parts.

Q4. Almost all candidates attempted this question concerned with the finite difference approximation of the elliptic boundary-value problem

$$-\Delta u + u + u^3 + u^5 = f(x, y) \qquad \text{for } (x, y) \in \Omega := (0, 1)^2,$$

subject to the homogeneous Dirichlet boundary condition $u|_{\partial\Omega} = 0$. The answers offered by those who attempted the first two parts of the question were mostly close to being complete. The majority of those who attempted part (c) of the question realised (using the hint provided) that $[u(x_i, y_j)]^3 - [U_{i,j}]^3$ and $[u(x_i, y_j)]^5 - [U_{i,j}]^5$ can be factorised as $e_{i,j} := u(x_i, y_j) - U_{i,j}$ multiplied by a nonnegative expression.

Q5. The question was conserned with the stability analysis of the implicit and explicit Euler finite difference approximations of the initial-value problem

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = a\frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad t > 0,$$

subject to the initial condition $u(x, 0) = u_0(x)$, in the discrete ℓ^2 norm via Fourier analysis. This was a popular question and was attempted by all. The first three parts of the question were generally very well done by all candidates. The final part of the question was completed by one candidate only, but there were a number of almost complete answers to part (d) of the question.

Q6. The question was concerned with the finite difference approximation of an initial-boundary-value problem for the first-order nonlinear hyperbolic PDE

$$\frac{\partial u}{\partial t} + (1 + u^2)\frac{\partial u}{\partial x} = 0$$

posed on the half-line $x \in [0, \infty)$ subject to the homogeneous boundary-condition $u(0, t) = 0$. There were eleven attempts at the question, but only one candidate was able to successfully answer all parts of the question.

Paper B2: Numerical Linear Algebra and Continuous Optimisation

Q1 This was a popular question with 19 attempts.
(a) (i) This was mostly fine.
(ii) Mostly ok, but orthogonal invariance of the 2-norm needs to be noted.
(iii) Also fine.
(iv) Not many attempts identified the correct polynomial ((2-z)/2)^k, although it was shown in lecture. Quite a few assumed the eigenvalues are real, and attempted an analysis using Chebyshev polynomials (the analysis gets harder, and such attempts were not penalized much).
b-(i) Most attempts were solid, but for a full mark one needs to explain both inclusion directions. (ii) surprisingly few got this right.
(iii) This was to be connected to (a-iv). Many more attempts were successful than (a-iv); solutions assuming that are fine even if a-iv was incomplete.

Q2 This was meant to be problem that tells a coherent story, and a nontrivial but relatively small extension of lecture notes. It was not very popular though, with only nine attempts.
(a) (i) Mostly fine. QR-based solution is acceptable, as is the normal equation.
(ii) Fewer correct answers than expected.
(b)-(i) This is straightforward, but some missed the point.
(ii) Rather few identified the correct vector v.
(iii) Not so difficult given the build-up, but not many received a full mark.

Q3 A very popular question with 21 attempts. It starts with (a), which is a series of bookwork problems with linesearch, but the proof is somewhat long.
(b-ii) was a nonstandard question, though discussed in lecture.
(b-iii) was intended to be challenging; most attempts assumed f is quadratic. This was not the intention (and the statement holds generally), though such attempts received half credit.

Q4 14 attempts made, and many made very good progress. Many lacked explanation for (a-i,ii). Some, though very few, noted that (iv) is equivalent to a step Newton's method, so the solution is in some sense immediate (and in any case should be used as a sanity check).
In (b-ii) it is crucial to mention the ability to update the *inverse* of the matrix, not just that low-rank matrices are easy to store or multiply. (b-iii) The BFGS update is actually not hard to derive under mild assumptions; some seemed to use lengthy arguments to get it.

Q5 14 attempts. (a) Most candidates got (i) correctly. Some tried a perturbation approach, which is more cumbersome than it could be. Most also solved (ii), with some few informal arguments seen.
(iii-iv) on optimality of convex optimization problems is standard but important and were answered reasonably well. The assumptions and facts used need to be highlighted when used.

(b-i) This seemed to have been a tricky question; even though (as discussed in lecture) it is relatively straightforward if we plug in the solution x_* in the Lagrangian.
(ii) This was unsurprisingly a difficult problem (though the calculations are not long), with the key fact being the Lagrangian function is convex wrt x at the optimal multipliers, and x_* satisfies the KKT conditions there. A several attempts received a full mark.

Q6 15 attempts. (i) Many failed to explain fully where the trust-region constraint comes from; the model (e.g. Taylor expansion) ceases to be a good model away from 0.
(ii) This was mostly fine.
(iii) This appeared to be challenging. The key fact is that the secular equation has a pole at the rightmost eigenvalue.

(b-i) Not many attempts made a convincing explanation as to why there are m eigenvalues tending to infinity.
(ii) This was mostly fine, but many failed to fully explain how ill-conditioning poses isses computationally. (inaccurate solution, and possibly slow convergence when an iterative method is used).
(iii) This was mostly fine but the derivations need to be justified.