UNIVERSITY OF OXFORD
Mathematical Institute

HONOUR SCHOOL OF MATHEMATICS & PHILOSOPHY

SUPPLEMENT TO THE UNDERGRADUATE HANDBOOK – 2014 Matriculation

SYNOPSES OF LECTURE COURSES

Part B
2016-17
For examination in 2017

These synopses can be found at:
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses

Issued October 2016

Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics & Philosophy
Syllabus and Synopses for Part B 2016–2017
for examination in 2017

Contents

1 Foreword

This Supplement to the Mathematics Course Handbook specifies the Mathematics courses available for Part B in Mathematics & Philosophy in the 2017 examination. It should be read in conjunction with the Handbook for Mathematics & Philosophy for the academic year 2016–2017, to be issued in Michaelmas Term. The Handbook contains in particular information on the format and rubrics for written examination papers in Mathematics, and the classification rules applicable to Part B.

See the current edition of the *Examination Regulations* for the full regulations governing the examinations.

1.1 Part B of the Honour School of Mathematics & Philosophy

The following is reproduced from the *Examination Regulations* applicable to the 2017 examinations.

The examination for Part B shall consist of units in Mathematics and subjects in Philosophy. The schedule of units in *Mathematics* shall be published in Mathematics and Philosophy Synopses of lecture courses supplement to the Mathematics Course Handbook by the beginning of the Michaelmas Full Term in the academic year of the examination concerned. The schedule shall be in two parts: Schedule 1 (standard units) and Schedule 2 (additional units). In *Philosophy* the subjects shall be subjects 101–118, 120, 122, 124, 125, 127 and 199 from the list given in *Special Regulations for All Honour Schools Including Philosophy*. Each subject in Philosophy other than a Thesis shall be examined in one 3-hour paper. Each candidate shall offer

(i) Four units of *Mathematics* from Schedule 1, two of which shall be B1.1 *Logic* and B1.2 *Set Theory*,

(ii) three subjects in *Philosophy* from 101-118, 120, 122, 124, 125 and 127 of which two must be 122 and **either** 101 **or** 102, and

(iii) **either** two further units in *Mathematics* drawn from Schedules 1 and 2 combined **or** one further subject in *Philosophy* from subjects 101-118, 120, 124, 125, 127 and 199: *Thesis*.

Note that the units listed under Schedule 2 are not available to those who wish to offer a total of four Philosophy subjects.

Further information on the units in Mathematics is given below.

1.2 "Units" and methods of examination

Most courses in Mathematics are assessed by examination. Most subjects offered have a 'weight' of a unit, and will be examined in an examination paper of $1\frac{3}{4}$ hours duration. Each unit paper will contain **3** questions.

1.3 The Schedules of Mathematics units for Mathematics & Philosophy

All units in Mathematics are drawn from the list of options for Mathematics Part B.

Schedule 1 comprises those Mathematics Department courses for which the core and options in Mathematics & Philosophy Part A provide the requisite background.

Schedule 2 contains an Extended Essay option and certain further courses from Mathematics Part B appropriate for the Joint School.

In addition you may apply for special approval to be examined in Mathematics Department units not included under Schedule 1; any such subject approved will be treated as falling under Schedule 2. For the procedure for seeking approval, see Subsection 1.4 below.

For the 2017 examination, the Schedules are as follows. (N.B. All topics listed are units unless otherwise stated).

Schedule 1

B1.1 Logic (Compulsory)

B1.2 Set Theory (Compulsory)

B2.1 Introduction to Representation Theory

B2.2 Commutative Algebra

B3.1 Galois Theory

B3.2 Geometry of Surfaces

B3.3 Algebraic Curves

B3.4 Algebraic Number Theory

B3.5 Topology and Groups

B4.1 Banach Spaces

B4.2 Hilbert spaces

B8.1 Martingales Through Measure Theory

B8.2 Continuous Martingales and Stochastic Calculus

B8.4 Communication Theory

B8.5 Graph Theory

SB3a Applied Probability

Schedule 2 (additional units)

BEE "Mathematical" Extended Essay	double unit
BO1.1 History of Mathematics	double unit
BOE "Other Mathematical" Extended Essay	double unit
OCS1 Lambda Calculus and Types	
OCS2 Computational Complexity	
OCS3 Knowledge Representation and Reasoning	
OCS4 Computer-aided Formal Verification	
BN1 Mathematics Education and Undergraduate Ambassadors' Scheme	double unit
Consisting of:	
BN1.1 Mathematics Education (may also be taken as a single unit)	
BN1.2 Undergraduate Ambassadors' Scheme	

And also

Any other unit course from the list of Mathematics Department units in Part B for which special approval has been granted.

1.4 Procedure for seeking approval of additional options where this is required

You may, if you have the support of your Mathematics tutor, apply to the Chairman of the Joint Committee for Mathematics and Philosophy for approval of one or more other options from the list of Mathematics Department units for Part B. This list can be found in the Supplement to the Mathematics Course Handbook giving syllabuses and synopses for courses in Mathematics Part B and at the end of this Supplement.

Applications for special approval must be made through the candidate's college and sent to the Chairman of the Joint Committee for Mathematics and Philosophy, c/o Academic Administrator, Mathematical Institute, to arrive by **Friday of Week 5 of Michaelmas Term**. Be sure to consult your college tutors if you are considering asking for approval to offer one of these additional options.

Given that each of these additional options, which are all in applied mathematics, presume facility with some or other results and techniques covered in first, second or third year Mathematics courses not taken by Mathematics & Philosophy candidates, such applications will be exceptional. You should also be aware that there may be a clash of lectures for specially approved options and those listed in Schedules 1 and 2 and with lectures in Philosophy; see the section in The Mathematics Part B Synopses on lecture clashes.

1.5 Registration for Part B courses 2016–2017

CLASSES Students will have to register in advance for the courses they wish to take. Students will have to register by Friday of Week 10 of Trinity Term 2016 using the online registration system which can be accessed at https://www.maths.ox.ac.uk/courses/registration/. Students will then be asked to sign up for classes at the start of Michaelmas Term 2016. Further information about this will be sent via email before the start of term.

Students who register for a course or courses for which there is a quota should consider registering for an additional course (by way of a "reserve choice") in case they do not receive a place on the course with the quota. They may also have to give the reasons why they wish to take a course which has a quota, and provide the name of a tutor who can provide a supporting statement for them should the quota be exceeded. Where this is necessary students will be contacted by email after they have registered. In the event that the quota for a course is exceeded, the Mathematics Teaching Committee will decide who may have a place on the course on the basis of the supporting statements from the student and tutor, and all relevant students will be notified of the decision by email. In the case of the "Undergraduate Ambassadors' Scheme" students may have to attend a short interview in Week 0, Michaelmas Term.

2 Schedule

2.1 B1.1: Logic — Prof. Jochen Koenigsmann — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: None

Overview

To give a rigorous mathematical treatment of the fundamental ideas and results of logic that is suitable for the non-specialist mathematicians and will provide a sound basis for more advanced study. Cohesion is achieved by focusing on the Completeness Theorems and the relationship between provability and truth. Consideration of some implications of the Compactness Theorem gives a flavour of the further development of model theory. To give a concrete deductive system for predicate calculus and prove the Completeness Theorem, including easy applications in basic model theory.

Learning Outcomes

Students will be able to use the formal language of propositional and predicate calculus and be familiar with their deductive systems and related theorems. For example, they will know and be able to use the soundness, completeness and compactness theorems for deductive systems for predicate calculus.

Synopsis

The notation, meaning and use of propositional and predicate calculus. The formal language of propositional calculus: truth functions; conjunctive and disjunctive normal form; tautologies and logical consequence. The formal language of predicate calculus: satisfaction, truth, validity, logical consequence.

Deductive system for propositional calculus: proofs and theorems, proofs from hypotheses, the Deduction Theorem; Soundness Theorem. Maximal consistent sets of formulae; completeness; constructive proof of completeness.

Statement of Soundness and Completeness Theorems for a deductive system for predicate calculus; derivation of the Compactness Theorem; simple applications of the Compactness Theorem.

A deductive system for predicate calculus; proofs and theorems; prenex form. Proof of Completeness Theorem. Existence of countable models, the downward Löwenheim–Skolem Theorem.

Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part I)* (Oxford University Press, 2001), sections 1, 3, 4.
2. A. G. Hamilton, *Logic for Mathematicians* (2nd edition, Cambridge University Press, 1988), pp.1–69, pp.73–76 (for statement of Completeness (Adequacy)Theorem), pp.99–103 (for the Compactness Theorem).
3. W. B. Enderton, *A Mathematical Introduction to Logic* (Academic Press, 1972), pp.101–144.
4. D. Goldrei, *Propositional and Predicate Calculus: A model of argument* (Springer, 2005).
5. A. Prestel and C. N. Delzell, *Mathematical Logic and Model Theory* (Springer, 2010).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 8.

2.2 B1.2: Set Theory — Prof. Jonathan Pila — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: There are no formal prerequisites, but familiarity with some basic mathematical objects and notions such as: the rational and real number fields; the idea of surjective, injective and bijective functions, inverse functions, order relations; the notion of a continuous function of a real variable, sequences, series, and convergence, and the definitions of basic abstract structures such as fields, vector spaces, and groups (all covered in Mathematics I and II in Prelims) will be helpful at points.

Overview

To introduce sets and their properties as a unified way of treating mathematical structures, including encoding of basic mathematical objects using set theoretic language. To emphasize the difference between intuitive collections and formal sets. To introduce and discuss the notion of the infinite, the ordinals and cardinality. The Axiom of Choice and its equivalents are presented as a tool.

Learning Outcomes

Students will have a sound knowledge of set theoretic language and be able to use it to codify mathematical objects. They will have an appreciation of the notion of infinity and arithmetic of the cardinals and ordinals. They will have developed a deep understanding of the Axiom of Choice, Zorn's Lemma and well-ordering principle, and have begun to appreciate the implications.

Synopsis

What is a set? Introduction to the basic axioms of set theory. Ordered pairs, cartesian products, relations and functions. Axiom of Infinity and the construction of the natural numbers; induction and the Recursion Theorem.

Cardinality; the notions of finite and countable and uncountable sets; Cantor's Theorem on power sets. The Tarski Fixed Point Theorem. The Schröder–Bernstein Theorem.

Isomorphism of ordered sets; well-orders. Transfinite induction; transfinite recursion [informal treatment only].

Comparability of well-orders.

The Axiom of Choice, Zorn's Lemma, the Well-ordering Principle; comparability of cardinals. Equivalence of WO, CC, AC and ZL. Ordinals. Arithmetic of cardinals and ordinals; in [ZFC].

Reading

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).

2. H. B. Enderton, *Elements of Set Theory* (Academic Press, 1978).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 7.1–7.5.

2. R. Rucker, *Infinity and the Mind: The Science and Philosophy of the Infinite* (Princeton University Press, 1995). An accessible introduction to set theory.

3. J. W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite* (Princton University Press, 1990). For some background, you may find JW Dauben's biography of Cantor interesting.

4. M. D. Potter, *Set Theory and its Philosophy: A Critical Introduction* (Oxford University Press, 2004). An interestingly different way of establishing Set Theory, together with some discussion of the history and philosophy of the subject.

5. W. Sierpinski, *Cardinal and Ordinal Numbers* (Polish Scientific Publishers, 1965). More about the arithmetic of transfinite numbers.

6. J. Stillwell, *Roads to Infinity* (CRC Press, 2010).

2.3 B2.1: Introduction to Representation Theory — Prof. Nikolay Nikolov — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Rings and Modules is essential. Group Theory is recommended.

Overview

This course gives an introduction to the representation theory of finite groups and finite dimensional algebras. Representation theory is a fundamental tool for studying symmetry by means of linear algebra: it is studied in a way in which a given group or algebra may act on vector spaces, giving rise to the notion of a representation.

A large part of the course will deal with the structure theory of semisimple algebras and their modules (representations). We will prove the Jordan-Hölder Theorem for modules. Moreover, we will prove that any finite-dimensional semisimple algebra is isomorphic to a product of matrix rings (Wedderburn's Theorem over $\mathbb{C}$).

In the later part of the course we apply the developed material to group algebras, and classify when group algebras are semisimple (Maschke's Theorem). All of this material will be applied to the study of characters and representations of finite groups.

Learning Outcomes

They will know in particular simple modules and semisimple algebras and they will be familiar with examples. They will appreciate important results in the course such as the Jordan-Hölder Theorem, Schur's Lemma, and the Wedderburn Theorem. They will be familiar with the classification of semisimple algebras over $\mathbb{C}$ and be able to apply this to representations and characters of finite groups.

Synopsis

Noncommutative rings, one- and two-sided ideals. Associative algebras (over fields). Main examples: matrix algebras, polynomial rings and quotients of polynomial rings. Group algebras, representations of groups.

Modules and their relationship with representations. Simple and semisimple modules, composition series of a module, Jordan-Hölder Theorem. Semisimple algebras. Schur's Lemma, the Wedderburn Theorem, Maschke's Theorem. Characters of complex representations. Orthogonality relations, finding character tables. Tensor product of modules. Induction and restriction of representations. Application: Burnside's $p^a q^b$ Theorem.

Reading

1. K. Erdmann, *B2 Algebras*, Mathematical Institute Notes (2007).

2. G. D. James and M. Liebeck, *Representations and Characters of Finite Groups* (2nd edition, Cambridge University Press, 2001).

Further Reading

1. J. L. Alperin and R. B. Bell, *Groups and Representations,* Graduate Texts in Mathematics 162 (Springer-Verlag, 1995).
2. P. M. Cohn, *Classic Algebra* (Wiley & Sons, 2000). (Several books by this author available.)
3. C. W. Curtis, and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras* (Wiley & Sons, 1962).
4. L. Dornhoff, *Group Representation Theory* (Marcel Dekker Inc., New York, 1972).
5. I. M. Isaacs, *Character Theory of Finite Groups* (AMS Chelsea Publishing, American Mathematical Society, Providence, Rhode Island, 2006).
6. J.-P. Serre, *Linear Representations of Finite Groups,* Graduate Texts in Mathematics 42 (Springer-Verlag, 1977).

2.4 B2.2: Commutative Algebra — Prof. Nikolay Nikolov — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites Rings and Modules is essential. Representation Theory and Galois Theory are recommended.

Overview

Amongst the most familiar objects in mathematics are the ring of integers and the polynomial rings over fields. These play a fundamental role in number theory and in algebraic geometry, respectively. The course explores the basic properties of such rings.

Synopsis

Modules, ideals, prime ideals, maximal ideals.
Noetherian rings; Hilbert basis theorem. Minimal primes.
Localization.
Polynomial rings and algebraic sets. Weak Nullstellensatz.
Nilradical and Jacobson radical; strong Nullstellensatz.
Integral extensions. Prime ideals in integral extensions.
Noether Normalization Lemma.
Krull dimension; dimension of an affine algebra.
Noetherian rings of small dimension, Dedekind domains.

Reading

1. M. F. Atiyah and I. G. MacDonald: *Introduction to Commutative Algebra*, (Addison-Wesley, 1969).
2. D. Eisenbud: *Commutative Algebra with a view towards Algebraic Geometry*, (Springer GTM, 1995).

2.5 B3.1: Galois Theory — Dr Giacomo Micheli — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Rings and Modules is essential and Group Theory is recommended. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

Overview

The course starts with a review of second-year ring theory with a particular emphasis on polynomial rings, and a discussion of general integral domains and fields of fractions. This is followed by the classical theory of Galois field extensions, culminating in some of the classical theorems in the subject: the insolubility of the general quintic and impossibility of certain ruler and compass constructions considered by the Ancient Greeks.

Learning Outcomes

Understanding of the relation between symmetries of roots of a polynomial and its solubility in terms of simple algebraic formulae; working knowledge of interesting group actions in a nontrivial context; working knowledge, with applications, of a nontrivial notion of finite group theory (soluble groups); understanding of the relation between algebraic properties of field extensions and geometric problems such as doubling the cube and squaring the circle.

Synopsis

Review of polynomial rings, factorisation, integral domains. Reminder that any nonzero homomorphism of fields is injective. Fields of fractions.

Review of group actions on sets, Gauss' Lemma and Eisenstein's criterion for irreducibility of polynomials, field extensions, degrees, the tower law. Symmetric polynomials.

Separable extensions. Splitting fields and normal extensions. The theorem of the primitive element. The existence and uniqueness of algebraic closure (proofs not examinable).

Groups of automorphisms, fixed fields. The fundamental theorem of Galois theory.

Examples: Kummer extensions, cyclotomic extensions, finite fields and the Frobenius automorphism. Techniques for calculating Galois groups.

Soluble groups. Solubility by radicals, solubility of polynomials of degree at most 4, insolubility of the general quintic, impossibility of some ruler and compass constructions.

Reading

1. J. Rotman, *Galois Theory* (Springer-Verlag, NY Inc, 2001/1990).
2. I. Stewart, *Galois Theory* (Chapman and Hall, 2003/1989)
3. D.J.H. Garling, *A Course in Galois Theory* (Cambridge University Press I.N., 1987).
4. Herstein, *Topics in Algebra* (Wiley, 1975)

2.6 B3.2: Geometry of Surfaces — Prof. Alexander Ritter — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Topology. Introduction to Manifolds would be useful but not essential. Also, B3.2 is helpful, but not essential, for B3.3 (Algebraic Curves).

Overview

Different ways of thinking about surfaces (also called two-dimensional manifolds) are introduced in this course: first topological surfaces and then surfaces with extra structures which allow us to make sense of differentiable functions ('smooth surfaces'), holomorphic functions ('Riemann surfaces') and the measurement of lengths and areas ('Riemannian 2-manifolds').

These geometric structures interact in a fundamental way with the topology of the surfaces. A striking example of this is given by the Euler number, which is a manifestly topological quantity, but can be related to the total curvature, which at first glance depends on the geometry of the surface.

The course ends with an introduction to hyperbolic surfaces modelled on the hyperbolic plane, which gives us an example of a non-Euclidean geometry (that is, a geometry which meets all of Euclid's axioms except the axiom of parallels).

Learning Outcomes

Students will be able to implement the classification of surfaces for simple constructions of topological surfaces such as planar models and connected sums; be able to relate the Euler characteristic to branching data for simple maps of Riemann surfaces; be able to describe the definition and use of Gaussian curvature; know the geodesics and isometries of the hyperbolic plane and their use in geometrical constructions.

Synopsis

The concept of a topological surface (or 2-manifold); examples, including polygons with pairs of sides identified. Orientation and the Euler characteristic. Classification theorem for compact surfaces (the proof will not be examined).

Riemann surfaces; examples, including the Riemann sphere, the quotient of the complex numbers by a lattice, and double coverings of the Riemann sphere. Holomorphic maps of Riemann surfaces and the Riemann–Hurwitz formula. Elliptic functions.

Smooth surfaces in Euclidean three-space and their first fundamental forms. The concept of a Riemannian 2-manifold; isometries; Gaussian curvature.

Geodesics. The Gauss–Bonnet Theorem (statement of local version and deduction of global version). Critical points of real-valued functions on compact surfaces.

The hyperbolic plane, its isometries and geodesics. Compact hyperbolic surfaces as Riemann surfaces and as surfaces of constant negative curvature.

Reading

1. A. Pressley, *Elementary Differential Geometry,* Springer Undergraduate Mathematics Series (Springer-Verlag, 2001). (Chapters 4–8 and 10–11.)
2. G. B. Segal, *Geometry of Surfaces*, Mathematical Institute Notes (1989).
3. R. Earl, *The Local Theory of Curves and Surfaces*, Mathematical Institute Notes (1999).
4. J. McCleary, *Geometry from a Differentiable Viewpoint* (Cambridge, 1997).

Further Reading

1. P. A. Firby and C. E. Gardiner, *Surface Topology* (Ellis Horwood, 1991) (Chapters 1–4 and 7).
2. F. Kirwan, *Complex Algebraic Curves,* Student Texts 23 (London Mathematical Society, Cambridge, 1992) (Chapter 5.2 only).
3. B. O'Neill, *Elementary Differential Geometry* (Academic Press, 1997).
4. M. P. do Carmo, *Differential Geometry of Curves and Surfaces* (Dover, 2016)

2.7 B3.3: Algebraic Curves — Prof. Dominic Joyce — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Topology. Introduction to Manifolds would be useful but not essential. Projective Geometry is recommended. Also, B3.2 (Geometry of Surfaces) is helpful, but not essential.

Overview

A real algebraic curve is a subset of the plane defined by a polynomial equation $p(x, y) = 0$. The intersection properties of a pair of curves are much better behaved if we extend this picture in two ways: the first is to use polynomials with complex coefficients, the second to extend the curve into the projective plane. In this course projective algebraic curves are studied, using ideas from algebra, from the geometry of surfaces and from complex analysis.

Learning Outcomes

Students will know the concepts of projective space and curves in the projective plane. They will appreciate the notion of nonsingularity and know some basic features of intersection theory. They will view nonsingular algebraic curves as examples of Riemann surfaces, and be familiar with divisors, meromorphic functions and differentials.

Synopsis

Projective spaces, homogeneous coordinates, projective transformations.

Algebraic curves in the complex projective plane. Irreducibility, singular and nonsingular points, tangent lines.

Bezout's Theorem (the proof will not be examined). Points of inflection, and normal form of a nonsingular cubic.

Nonsingular algebraic curves as Riemann surfaces. Meromorphic functions, divisors, linear equivalence. Differentials and canonical divisors. The group law on a nonsingular cubic.

The Riemann–Roch Theorem (the proof will not be examined). The geometric genus. Applications.

Reading

1. F. Kirwan, *Complex Algebraic Curves,* Student Texts 23 (London Mathematical Society, Cambridge, 1992), Chapters 2–6.
2. W. Fulton, *Algebraic Curves,* 3rd ed., downloadable at www.math.lsa.umich.edu/∼wfulton

2.8 B3.4: Algebraic Number Theory — Prof. Minhyong Kim — 16 HT

Level: H-level **Method of Assessment:** Written examination

Weight: Unit

Prerequisites: Rings and Modules and Number Theory. B3.1 Galois Theory is an essential pre-requisite.

Recommended Prerequisites: All second-year algebra and arithmetic. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

Overview

An introduction to algebraic number theory. The aim is to describe the properties of number fields, but particular emphasis in examples will be placed on quadratic fields, where it is easy to calculate explicitly the properties of some of the objects being considered. In such fields the familiar unique factorisation enjoyed by the integers may fail, and a key objective of the course is to introduce the class group which measures the failure of this property.

Learning Outcomes

Students will learn about the arithmetic of algebraic number fields. They will learn to prove theorems about integral bases, and about unique factorisation into ideals. They will learn to calculate class numbers, and to use the theory to solve simple Diophantine equations.

Synopsis

1. field extensions, minimum polynomial, algebraic numbers, conjugates, discriminants, Gaussian integers, algebraic integers, integral basis
2. examples: quadratic fields
3. norm of an algebraic number
4. existence of factorisation
5. factorisation in $\mathbb{Q}(\sqrt{d})$
6. ideals, $\mathbb{Z}$-basis, maximal ideals, prime ideals
7. unique factorisation theorem of ideals
8. relationship between factorisation of number and of ideals
9. norm of an ideal
10. ideal classes
11. statement of Minkowski convex body theorem
12. finiteness of class number
13. computations of class number to go on example sheets

Reading

1. I. Stewart and D. Tall, *Algebraic Number Theory and Fermat's Last Theorem.* (Third Edition, Peters, 2002).

Further Reading

1. D. Marcus, *Number Fields* (Springer-Verlag, New York–Heidelberg, 1977). ISBN 0-387-90279-1.

2.9 B3.5: Topology and Groups — Prof. Marc Lackenby — 16 MT

Level: H-level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites Part A Topology is essential and Group Theory is recommended.

Overview

This course introduces the important link between topology and group theory. On the one hand, associated to each space, there is a group, known as its fundamental group. This can be used to solve topological problems using algebraic methods. On the other hand, many results about groups are best proved and understood using topology. For example, presentations of groups, where the group is defined using generators and relations, have a topological interpretation. The endpoint of the course is the Nielsen–Shreier Theorem, an important, purely algebraic result, which is proved using topological techniques.

Learning Outcomes

Students will develop a sound understanding of simplicial complexes, cell complexes and their fundamental groups. They will be able to use algebraic methods to analyse topological spaces. They will also be able to address questions about groups using topological techniques.

Synopsis

Homotopic mappings, homotopy equivalence. Simplicial complexes. Simplicial approximation theorem.

The fundamental group of a space. The fundamental group of a circle. Application: the fundamental theorem of algebra. The fundamental groups of spheres.

Free groups. Existence and uniqueness of reduced representatives of group elements. The fundamental group of a graph.

Groups defined by generators and relations (with examples). Tietze transformations.

The free product of two groups. Amalgamated free products.

The Seifert–van Kampen Theorem.

Cell complexes. The fundamental group of a cell complex (with examples). The realization of any finitely presented group as the fundamental group of a finite cell complex.

Covering spaces. Liftings of paths and homotopies. A covering map induces an injection between fundamental groups. The use of covering spaces to determine fundamental groups: the circle again, and real projective n-space. The correspondence between covering spaces and subgroups of the fundamental group. Regular covering spaces and normal subgroups.

Cayley graphs of a group. The relationship between the universal cover of a cell complex, and the Cayley graph of its fundamental group. The Cayley 2-complex of a group.

The Nielsen–Schreier Theorem (every subgroup of a finitely generated free group is free) proved using covering spaces.

Reading

1. John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

Additional Reading

1. D. Cohen, *Combinatorial Group Theory: A Topological Approach,* Student Texts 14 (London Mathematical Society, 1989), Chapters 1–7.
2. A. Hatcher, *Algebraic Topology* (CUP, 2001), Chapter. 1.
3. M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chapters. 1–7, 12, 17 .
4. D. L. Johnson, *Presentations of Groups,* Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chapters. 1–5, 10,13.
5. W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chapters. 1–4.

2.10 B4.1: Banach Spaces — Prof. Hilary Priestley — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Integration is recommended; the only concepts which will be used are the convergence theorems and the theorems of Fubini and Tonelli, and the notions of measurable functions and null sets. No knowledge is needed of outer measure, or of any particular construction of the integral, or of any proofs.

Learning Outcomes

Students will have a firm knowledge of real and complex normed vector spaces, with their geometric and topological properties. They will be familiar with the notions of completeness, separability and density, will know the properties of a Banach space and important examples, and will be able to prove results relating to the Hahn–Banach Theorem. They will have developed an understanding of the theory of bounded linear operators on a Banach space.

Synopses

Real and complex normed vector spaces, their geometry and topology. Completeness. Banach spaces, examples (ℓ^p, ℓ^∞, L^p, $C(K)$, spaces of differentiable functions). Finite-dimensional normed spaces; equivalence of norms and completeness.

Density. Stone-Weierstrass Theorem. Separable spaces; separability of subspaces.

Bounded linear operators, examples (including integral operators). Continuous linear functionals. Dual spaces. Hahn–Banach Theorem (proof for real separable spaces only); applications, including density of subspaces and embedding of a normed space into its second dual. Adjoint operators.

Spectrum and resolvent. Spectral mapping theorem for polynomials.

Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chapters 2, 4, 5.
2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chapters 2, 4.2174.3, 4.5, 7.1177.4.

2.11 B4.2: Hilbert Spaces — Prof. Zhongmin Qian — 16 HT

Level: H-level **Method of Assessment:** Written examination

Weight: Unit

Prerequisites: B4.1 Banach Spaces is an essential pre-requisite.

Recommended Prerequisites: A good working knowledge of Part A Core Analysis (both metric spaces and complex analysis) is expected. Part A Integration is desirable, but the only concepts which will be used are the convergence theorems and the theorems of Fubini and Tonelli, and the notions of measurable functions and null sets. No knowledge is needed of outer measure, or of any particular construction of the integral, or of any proofs.]

Learning Outcomes

Students will appreciate the role of completeness through the Baire category theorem and its consequences for operators on Banach spaces. They will have a demonstrable knowledge of the properties of a Hilbert space, including orthogonal complements, orthonormal sets, complete orthonormal sets together with related identities and inequalities. They will be familiar with the theory of linear operators on a Hilbert space, including adjoint operators, self-adjoint and unitary operators with their spectra. They will know the L^2-theory of Fourier series and be aware of the classical theory of Fourier series and other orthogonal expansions.

Synopses

Hilbert spaces; examples including L^2-spaces. Orthogonality, orthogonal complement, closed subspaces, projection theorem. Riesz Representation Theorem.

Linear operators on Hilbert space, adjoint operators. Self-adjoint operators, orthogonal projections, unitary operators.

Baire Category Theorem and its consequences for operators on Banach spaces (Uniform Boundedness, Open Mapping, Inverse Mapping and Closed Graph Theorems). Strong convergence of sequences of operators.

Spectral theory in Hilbert spaces, in particular spectra of self-adjoint and unitary operators.

Orthonormal sets, Pythagoras, Bessels inequality. Complete orthonormal sets, Parseval.

L^2-theory of Fourier series, including completeness of the trigonometric system. Discussion of classical theory of Fourier series (including statement of pointwise convergence for piecewise differentiable functions, and exposition of failure for some continuous functions). Examples of other orthogonal expansions (Legendre, Laguerre, Hermite etc.).

Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chapters 3, 4.4, 6.

2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chapters 3, 4.7–4.9, 4.12–4.13, 9.1–9.2.

3. N. Young, *An Introduction to Hilbert Space* (Cambridge University Press, 1988), Chs 1177.

Further Reading

1. E.M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration & Hilbert Spaces* (Princeton Lectures in Analysis III, 2005), Chapter 4.

2.12 B8.1: Martingales Through Measure Theory — Prof Zhongmin Qian — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Integration is a prerequisite, so that the corresponding material will be assumed to be known. Part A Probability is a prerequisite.

Overview

Probability theory arises in the modelling of a variety of systems where the understanding of the "unknown" plays a key role, such as population genetics in biology, market evolution in financial mathematics, and learning features in game theory. It is also very useful in various areas of mathematics, including number theory and partial differential equations. The course introduces the basic mathematical framework underlying its rigorous analysis, and is therefore meant to provide some of the tools which will be used in more advanced courses in probability.

The first part of the course provides a review of measure theory from Integration Part A, and develops a deeper framework for its study. Then we proceed to develop notions of conditional expectation, martingales, and to show limit results for the behaviour of these martingales which apply in a variety of contexts.

Learning Outcomes

The students will learn about measure theory, random variables, independence, expectation and conditional expectation, product measures and discrete-parameter martingales.

Synopsis

A branching-process example. Review of σ-algebras, measure spaces. Uniqueness of extension of π-systems and Carathéodory's Extension Theorem [both without proof], monotone-convergence properties of measures, $\limsup$ and $\liminf$ of a sequence of events, Fatou's Lemma, reverse Fatou Lemma, first Borel–Cantelli Lemma.

Random variables and their distribution functions, σ-algebras generated by a collection of random variables. Product spaces. Independence of events, random variables and σ-algebras, π-systems criterion for independence, second Borel–Cantelli Lemma. The tail σ-algebra, Kolomogorov's 0–1 Law. Convergence in measure and convergence almost everywhere.

Integration and expectation, review of elementary properties of the integral and L^p spaces [from Part A Integration for the Lebesgue measure on $\mathbb{R}$]. Scheffé's Lemma, Jensen's inequality. The Radon–Nikodym Theorem [without proof]. Existence and uniqueness of conditional expectation, elementary properties. Relationship to orthogonal projection in L^2.

Filtrations, martingales, stopping times, discrete stochastic integrals, Doob's Optional-Stopping Theorem, Doob's Upcrossing Lemma and "Forward" Convergence Theorem, martingales bounded in L^2, Doob decomposition, Doob's submartingale inequalities.

Uniform integrability and L^1 convergence, backwards martingales and Kolmogorov's Strong Law of Large Numbers.

Examples and applications, including branching processes.

Reading

1. D. Williams, *Probability with Martingales*, Cambridge University Press, 1995.
2. Lecture Notes for the course.

Further Reading

1. Z. Brzeźniak and T. Zastawniak, Basic stochastic processes. A course through exercises. Springer Undergraduate Mathematics Series. (Springer-Verlag London, Ltd., 1999) [more elementary than D. Williams' book, but can provide with a complementary first reading].
2. M. Capinski and E. Kopp, *Measure, integral and probability*, Springer Undergraduate Mathematics Series. (Springer-Verlag London, Ltd., second edition, 2004).
3. R. Durrett, *Probability: Theory and Examples.* (Second Edition Duxbury Press, Wadsworth Publishing Company, 1996).
4. A. Etheridge, *A Course in Financial Calculus*, (Cambridge University Press, 2002).
5. J. Neveu, *Discrete-parameter Martingales.* (North-Holland, Amsterdam, 1975).
6. S. I. Resnick, *A Probability Path*, (Birkhäuser, 1999).

2.13 B8.2: Continuous Martingales and Stochastic Calculus — Prof. Jan Obłój — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: B8.1 Martingales through Measure Theory is a prerequisite. Consequently, Part A Integration and Part A Probability are also prerequisites.

Overview

Stochastic processes - random phenomena evolving in time - are encountered in many disciplines from biology, through geology to finance. This course focuses on mathematics needed to describe stochastic processes evolving continuously in time and introduces the basic tools of stochastic calculus which are at the cornerstone of modern probability theory. The motivating example of a stochastic process is Brownian motion, also called the Wiener process - a mathematical object initially proposed by Bachelier and Einstein, which originally modelled displacement of a pollen particle in a fluid. The paths of Brownian motion, or of any continuous martingale, are of infinite variation (they are in fact nowhere differentiable and have non-zero quadratic variation) and one of the aims of the course is to define a theory of integration along such paths equipped with a suitable integration by parts formula (Itô formula).

Learning Outcomes

The students will develop an understanding of Brownian motion and continuous martingales in continuous time. They will became familiar with stochastic calculus and in particular be able to use Itô's formula.

Synopsis

An introduction to stochastic processes in continuous time. Brownian motion - definition, construction and basic properties, regularity of paths. Filtrations and stopping times, first hitting times. Brownian motion - martingale and strong Markov properties, reflection principle. Martingales - definitions, regularisation and convergence theorems, optional sampling theorem, maximal and Doob's L^p inequalities. Quadratic variation, local martingales, semimartingales. Recall of Stieltjes integral. Stochastic integration and Itô's formula with applications.

Reading

There are a large number of textbooks which cover the course material with a varying degree of detail/rigour. Precise references for reading from two excellent reference books will be given. These are:

1. D. Revuz and M. Yor, "Continuous martingales and Brownian motion", Springer (Revised 3^{rd} ed.), 2001. Selected pages from Chapters 0–4: *exact pages covering each lecture will be indicated in the course materials.*

2. I. Karatzas and S. Shreve, "Brownian motion and stochastic calculus", Springer (2^{nd} ed.), 1991. Selected pages from Chapters 1–3: *exact pages covering each lecture will be indicated in the course materials.*

Further Reading

Further helpful references include:

1. R. Durrett, "Stochastic Calculus: A practical introduction", CRC Press, 1996. Sections 1.1 – 2.10.

2. F. Klebaner, "Introduction to Stochastic Calculus with Applications", 3rd edition, Imperial College Press, 2012. Chapters 1, 2, 3.1–3.11, 4.1–4.5, 7.1–7.8, 8.1–8.7.

3. J. M. Steele, "Stochastic Calculus and Financial Applications", Springer, 2010. Chapters 3 – 8.

4. B. Oksendal, "Stochastic Differential Equations: An introduction with applications", 6th edition, Springer (Universitext), 2007. Chapters 1 – 3.

5. S. Shreve, "Stochastic calculus for finance", Vol 2: Continuous-time models, Springer Finance, Springer-Verlag, New York, 2004. Chapters 3 – 4.

2.14 B8.4: Communication Theory — Prof. Harald Oberhauser — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Probability would be helpful, but not essential.

Overview

The aim of the course is to investigate methods for the communication of information from a sender, along a channel of some kind, to a receiver. If errors are not a concern we are interested in codes that yield fast communication; if the channel is noisy we are interested in achieving both speed and reliability. A key concept is that of information as reduction in uncertainty. The highlight of the course is Shannon's Noisy Coding Theorem.

Learning Outcomes

(i) Know what the various forms of entropy are, and be able to manipulate them.

(ii) Know what data compression and source coding are, and be able to do it.

(iii) Know what channel coding and channel capacity are, and be able to use that.

Synopsis

Uncertainty (entropy); conditional uncertainty; information. Chain rules; relative entropy; Gibbs' inequality; asymptotic equipartition and typical sequences. Instantaneous and uniquely decipherable codes; the noiseless coding theorem for discrete memoryless sources; constructing compact codes.

The discrete memoryless channel; decoding rules; the capacity of a channel. The noisy coding theorem for discrete memoryless sources and binary symmetric channels.

Extensions to more general sources and channels.

Reading

1. D. MacKay, *Information Theory, Inference, and Learning Algorithms* (Cambridge, 2003). [Can be seen at: http://www.inference.phy.cam.ac.uk/mackay/itila. Do not infringe the copyright!]

2. T. Cover and J. Thomas, *Elements of Information Theory* (Wiley, 1991), Chapters 1–5, 8.

Further Reading

1. R. B. Ash, *Information Theory* (Dover, 1990).

2. D. J. A. Welsh, *Codes and Cryptography* (Oxford University Press, 1988), Chapters 1–3, 5.

3. G. Jones and J. M. Jones, *Information and Coding Theory* (Springer, 2000), Chapters 1–5.

4. Y. Suhov & M. Kelbert, *Information Theory and Coding by Example* (Cambridge University Press, not yet published - available at the end of 2013), Relevant examples.

2.15 B8.5: Graph Theory — Prof. Oliver Riordan — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Graph Theory is recommended.

Overview

Graphs (abstract networks) are among the simplest mathematical structures, but nevertheless have a very rich and well-developed structural theory. Since graphs arise naturally in many contexts within and outside mathematics, Graph Theory is an important area of mathematics, and also has many applications in other fields such as computer science.

The main aim of the course is to introduce the fundamental ideas of Graph Theory, and some of the basic techniques of combinatorics.

Learning Outcomes

The student will have developed a basic understanding of the properties of graphs, and an appreciation of the combinatorial methods used to analyze discrete structures.

Synopsis

Introduction: basic definitions and examples. Trees and their characterization. Euler circuits; long paths and cycles. Vertex colourings: Brooks17' theorem, chromatic polynomial. Edge colourings: Vizing's theorem. Planar graphs, including Euler's formula, dual graphs. Maximum flow - minimum cut theorem: applications including Menger's theorem and Hall's theorem. Tutte's theorem on matchings. Extremal Problems: Turan's theorem, Zarankiewicz problem, Erdős-Stone theorem.

Reading

1. B. Bollobas, *Modern Graph Theory*, Graduate Texts in Mathematics 184 (Springer-Verlag, 1998)

Further Reading

1. J. A. Bondy and U. S. R. Murty, *Graph Theory: An Advanced Course*, Graduate Texts in Mathematics 244 (SpringerVerlag, 2007).
2. R. Diestel, *Graph Theory*, Graduate Texts in Mathematics 173 (third edition, Springer-Verlag, 2005).
3. D. West, *Introduction to Graph Theory*, Second edition, (PrenticeHall, 2001).

2.16 SB3a: Applied Probability — Prof. Paul Chleboun — 16 MT

[Teaching responsibility of the Department of Statistics. Please note, this course is offered from the schedule of Mathematics Department Units]

Level: H-Level **Method of Assessment:** Written examination.

Weight: Unit

The double-unit (SB3a and SB3b) has been designed so that a student obtaining at least an upper second class mark on the double unit can expect to gain exemption from the Institute of Actuaries' paper CT4, which is a compulsory paper in their cycle of professional actuarial examinations. The first unit, clearly, and also the second unit, apply much more widely than just to insurance models.

Recommended Prerequisites: Part A Probability.

Overview

This course is intended to show the power and range of probability by considering real examples in which probabilistic modelling is inescapable and useful. Theory will be developed as required to deal with the examples.

Synopsis

Poisson processes and birth processes. Continuous-time Markov chains. Transition rates, jump chains and holding times. Forward and backward equations. Class structure, hitting times and absorption probabilities. Recurrence and transience. Invariant distributions and limiting behaviour. Time reversal. Renewal theory. Limit theorems: strong law of large numbers, strong law and central limit theorem of renewal theory, elementary renewal theorem, renewal theorem, key renewal theorem. Excess life, inspection paradox.

Applications in areas such as: queues and queueing networks - M/M/s queue, Erlang's formula, queues in tandem and networks of queues, M/G/1 and G/M/1 queues; insurance ruin models; applications in applied sciences.

Reading

1. J. R. Norris, *Markov Chains* (Cambridge University Press, 1997).

2. G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes* (3rd edition, Oxford University Press, 2001).

3. G. R. Grimmett and D. R. Stirzaker, *One Thousand Exercises in Probability* (Oxford University Press, 2001).

4. S. M. Ross, *Introduction to Probability Models* (4th edition, Academic Press, 1989).

5. D. R. Stirzaker: *Elementary Probability* (2nd edition, Cambridge University Press, 2003).

3 Schedule 2 (additional units)

3.1 BEE "Mathematical" Extended Essay

Level: H-level **Method of Assessment:** Written extended essay.

Weight: Double unit (7,500 words).

An essay on a mathematical topic may be offered for examination at Part B as a double unit. It is equivalent to a 32-hour lecture course. Generally, students will have 8 hours of supervision distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Students considering offering an essay should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects

Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their essay. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room S0.20, Mathematical Institute and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. Note that a BEE essay must have a substantial mathematical content. The application form is available at:
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects

Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission

THREE copies of your essay, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part B, Examination Schools, Oxford, to arrive no later than **12noon on Monday of week 10, Hilary Term 2017**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics*.

3.2 BO1.1: History of Mathematics — Dr Christopher Hollings — 16 lectures in MT and reading course of 8 seminars in HT

Level: H-level

Assessment: 2-hour written examination paper for the MT lectures and 3000-word essay for the reading course.

Weight: Double unit.

Recommended prerequisites: None.

Quota: The maximum number of students that can be accepted will be 20. Students should note, however, that numbers are unlikely to reach this level, and so there is little danger of not being accepted onto the course.

Learning outcomes

This course is designed to provide the historical background to some of the mathematics familiar to students from A-level and the first four terms of undergraduate study, and looks at a period from approximately the mid-sixteenth century to the end of the nineteenth century. The course will be delivered through 16 lectures in Michaelmas Term, and a reading course consisting of 8 seminars (equivalent to a further 16 lectures) in Hilary Term. Guidance will be given throughout on reading, note-taking, and essay-writing.

Students will gain:

- an understanding of university mathematics in its historical context;
- an enriched understanding of the mathematical content of the topics covered by the course

together with skills in:

- reading and analysing historical mathematical sources;
- reading and analysing secondary sources;
- efficient note-taking;
- essay-writing (from 1000 to 3000 words);
- construction of references and bibliographies;
- oral discussion and presentation.

Lectures

The Michaelmas Term lectures will cover the following material:

- Introduction: ancient mathematical knowledge and its transimission to early modern Europe; the development of symbolic notation up to the end of the sixteenth century.
- Seventeenth century: analytic geometry; the development of calculus; Newton's *Principia.*
- Eighteenth century: from calculus to analysis; functions, limits, continuity; equations and solvability.
- Nineteenth century: group theory and abstract algebra; the beginnings of modern analysis; rigorous definitions of real numbers; integration; complex analysis; set theory; linear algebra.

Classes to accompany the lectures will be held in Weeks 3, 5, 6, 7. For each class students will be expected to prepare one piece of written work (1000 words) and one discussion topic.

Reading course

The Hilary Term part of the course is run as a reading course during which we will study a selection of primary texts in some detail, using original sources and secondary literature. Details of the books to be read in HT 2017 will be decided and discussed towards the end of MT 2016. Students will be expected to write three essays (2000 words each) during the first six weeks of term. The course will then be examined by an essay of 3000 words to be completed during Weeks 7 to 9.

Recommended reading

Jacqueline Stedall, *Mathematics emerging: a sourcebook 1540–1900* (Oxford University Press, 2008).

Victor Katz, *A history of mathematics* (brief edition) (Pearson Addison Wesley, 2004), or:

Victor Katz, *A history of mathematics: an introduction* (third edition) (Pearson Addison Wesley, 2009).

Benjamin Wardhaugh, *How to read historical mathematics* (Princeton, 2010).

Jacqueline Stedall, *The history of mathematics: a very short introduction* (Oxford University Press, 2012).

Supplementary reading

John Fauvel and Jeremy Gray (eds), *The history of mathematics: a reader*, (Macmillan, 1987).

Further suggestions of additional reading on particular topics will be given throughout the lecture course.

Assessment

The Michaelmas Term material will be examined in a two-hour written paper at the end of Trinity Term. Candidates will be expected to answer two half-hour questions (commenting on extracts) and one one-hour question (essay). The paper will account for 50% of the marks for the course. The Reading Course will be examined by a 3000-word essay at the end of Hilary Term. The title will be set at the beginning of Week 7 and two copies of the project must be submitted to the Examination Schools by midday on Monday of Week 10. This essay will account for 50% of the marks for the course.

3.3 BOE "Other Mathematical" Extended Essay

Level: H-level **Method of Assessment:** Written essay.

Weight: Double unit (7,500 words).

An essay on a topic related to mathematics may be offered for examination at Part B as a double unit. It is equivalent to a 32-hour lecture course. Generally, students will have 8 hours of supervision distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Students considering offering an essay should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects.

Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their essay. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room S0.20, Mathematical Institute and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. The application form is available at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects.

Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission

THREE copies of your essay, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part B, Examination Schools, Oxford, to arrive no later than **12noon on Monday of week 9, Hilary Term 2017**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics*.

3.4 Computer Science: Units

There are other units that students in Part B Mathematics and Philosophy may take which are drawn from Part B of the Honour School of Mathematics and Computer Science. For full details of these units see the syllabus and synopses for Part B of the Honour School Mathematics and Computer Science, which are available on the Web at

http://www.cs.ox.ac.uk/teaching/mcs/PartB//.

The Computer Science units available are as follows:

- OCS1 Lambda Calculus and Types
- OCS2 Computational Complexity
- OCS3 Knowledge Representation and Reasoning
- OCS4 Computer-aided Formal Verification

3.5 BN1: Mathematics Education and Undergraduate Ambassadors Scheme

Level: H-level **Method of Assessment:** See individual synopses for each unit

Weight: Double-unit, or BN1.1 may be taken as a unit.

3.5.1 BN1.1 Mathematics Education — Dr Jenni Ingram and Dr Nick Andrews — MT

Level: H-level

Method of Assessment: Two examined written assignments and a short presentation.
1500 word written assignment (not examined) due: end of week 2

2500 word written assignment (examined) due: end of week 5
Presentation (examined) week 8
3000 word written assignment (examined) due: week 1 Hilary Term

Weight: Unit.

Quota: There will be a quota of approximately 20 students for this course.

Recommended Prerequisites: None

Overview

The Mathematics Education option will be a unit, run in Michaelmas Term. The course is appropriate for all students of the appropriate degree courses, whether or not they are interested in teaching subsequently. Final credit will be based on two examined written assignments (35 % each), one of which will be submitted at the start of Hilary Term, and a presentation (30%). Teaching will be 22 hours of contact time which will include lecture, seminar, class and tutorial formats as follows:

- A two-hour lecture/class per week. These will be interactive and involve discussion and other tasks as well as input from the lecturer.
- Four hours of tutorial workshops in small groups to review ideas from the course and preparation for written assignments.

Learning Outcomes

1. Understanding:
   - the psychology of learning mathematics;
   - the nature of mathematics and the curriculum;
   - relations between teaching and learning at primary, secondary and tertiary level;
   - the role of mathematics education in society;
   - issues associated with communicating mathematics.
2. Understanding connections between mathematics, education issues and the mathematical experience of learners.
3. The ability to express ideas about the study and learning of mathematics in writing, verbally, and in other forms of communication.

Synopsis

1. Introduction to mathematics education as a field of study.
   - Issues of mathematics education research: learner, content, teacher and policy; theory and practice. Introduction to course, use of library facilities and the expected forms of study.

2. Nature of mathematics as a human endeavour
    - Mathematics as a way of making sense of the world of experience, and ways of making sense of mathematics. Mathematical use of human powers, such as the power to imagine, notice patterns, classify, conjecture and generalise. Motivation as mix of disposition, desire, purpose, utility, experience.
3. Psychology of learning and doing mathematics
    - Psychology of templating (behaviourism) and of construing (constructivisms). Concept image and concept definition. Intuition and education. Human psyche comprising Awareness (cognition), Emotion (Affect), Behaviour (enaction) and Attention (will).
4. Mathematical Knowledge Needed Teaching: The Case of Multiple Representations
    - Verbal; symbolic; diagrammatic; dynamic representations.
5. Teaching and learning mathematics
    - Theories about teaching and learning. Relations between teaching and learning. Task design.
6. The discourse of mathematics
    - Theories about the discourse of mathematics and mathematics as a discourse. Practical implications on the teaching and learning of mathematics.
7. Mathematics education and society
    - Overview of interfaces between mathematics education and social issues. What issues are of concern and how these relate to the global political focus on mathematics?
8. Student presentations

Reading

Main Texts:

1. Tall, D. (1991) *Advanced Mathematical Thinking.* (Mathematics Education Library, 11). Dordrecht: Kluwer
2. Gates, P. (ed.) (2001) *Issues in Mathematics Teaching.* London: RoutledgeFalmer
3. Mason, J., Burton, L. & Stacey, K. (2010) *Thinking Mathematically.* Any edition by any publisher will do.
4. Polya, G. (1957) *How to Solve It.* Any edition by any publisher will do.
5. Davis, P. And Hersh, R. (1981) *The Mathematical Experience.* Any edition by any publisher will do.

6. Mason, J. & Johnston-Wilder, S. (2004) *Fundamental Constructs in Mathematics Education*, London: RoutledgeFalmer.

7. Carpenter, T., Dossey, J. & Koehler, J. (2004) *Classics in Mathematics Education Research*. Reston, VA: National Council of Teachers of Mathematics.

Important websites:

1. ncetm.org.uk

3.5.2 BN1.2 Undergraduate Ambassadors' Scheme — Dr Nick Andrews — mainly HT

[This course is not available as a unit; it must be taken alongside BN1.1 Mathematics Education]

Weight: Unit

Method of Assessment: Journal of activities, Oral presentation, Course report and project, Teacher report.

Quota: There will be a quota of approximately 10 students for this course.

Recommended Prerequisites: BN1.1 is an essential prerequisite.

Option available to Mathematics, Mathematics & Statistics, Mathematics & Philosophy students.

Learning Outcomes

The Undergraduate Ambassadors' Scheme (UAS) was begun by Simon Singh in 2002 to give university undergraduates a chance to experience assisting and, to some extent, teaching in schools, and to be credited for this. The option focuses on improving students' communication, presentation, cooperation and organizational skills and sensitivity to others' learning difficulties.

Course Description and Timing:

The Oxford UAS option, BN1.2, is a unit, mainly run in Hilary Term. A quota will be in place, of approximately 10 students, and so applicants for the UAS option will be asked to name a second alternative unit. The course is appropriate for all students, whether or not they are interested in teaching subsequently.

A student on the course will be assigned to a mathematics department in a local secondary school (in the Oxford, Wheatley, Abingdon, Didcot area) for half a day per week during Hilary Term. Students will be supervised in school by a teacher in the mathematics department. Students will be expected to keep a journal of their activities in school. These activities are likely to begin by assisting in the class, but may widen to include teaching the whole class for a part of a period, or working separately with a smaller group of the class. Students will be required at one point to give a presentation to one of their school classes relating to a topic from university mathematics. Students will also run a small project in school based on some aspect of mathematics education with advice from the course co-ordinator and school teachers. Final credit will be based on the journal (20%), the presentation (30%), an end of course report (approximately 3000 words) including details of the project (35%), together with a report from the teacher (15%).

A key attribute for the course is the ability to communicate mathematics, and a reference will be sought from college tutors in support of this. Should the quota for the course be exceeded, short interviews will take place on Thursday or Friday of 0th week in Michaelmas term to select students for this course. In the interview (of roughly 15 minutes) the student will discuss how they might choose to introduce a piece of university mathematics to A level students. Students will be chosen on the basis of their ability to communicate mathematics and to account for the educational needs of others. Applicants will be quickly notified of the decision.

Those on the course will also need to complete a DBS form, or to have done so already. By the end of Michaelmas term students will have been assigned to a school and have made a first, introductory, visit to meet their teacher supervisor. The course will begin properly in Hilary term with students attending school for half a day each week. Funds are available to cover travel expenses. Support classes will be provided throughout Hilary for feedback and to discuss issues such as the planning of the project. The deadline for the journal and report will be noon on Monday of 1st week of Trinity term.

Any further questions on the UAS option should be passed on to the option's co-ordinator, via (nicholas.andrews@education.ox.ac.uk.).

Reading List

Tickly, C., Watson, A. and Morgan, C. (2004) Teaching School Subjects: Mathematics. Abingdon: Routledge Falmer.

3.6 List of Mathematics Department units available only if special approval is granted

For details of these courses, and prerequisites for them, please consult the Supplement to the Mathematics Course Handbook, Syllabus and Synopses for Mathematics Part B 2016–2017, for examination in 2017,
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses

B5.1: Stochastic Modelling of Biological Processes

B5.2: Applied Partial Differential Equations

B5.3: Viscous Flow

B5.4: Waves and Compressible Flow

B5.5: Further Mathematical Biology

B5.6: Nonlinear Systems

B6.1: Numerical Solution of Differential Equations I

B6.2: Numerical Solution of Differential Equations II

B6.3: Integer Programming

B7.1: Classical Mechanics

B7.2: Electromagnetism

B7.3: Further Quantum Theory

B8.3: Mathematical Models of Financial Derivatives