Moderations Syllabus and Synopses 2008–2009
for examination in 2009

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1 Foreword

Syllabus

The syllabus here is that referred to in the *Examination Regulations 2008*[1] and has been approved by the Mathematics Teaching Committee for examination in Trinity Term 2009.

Synopses

The synopses give some additional detail, and show how the material is split between the different lecture courses. They also include details of recommended reading.

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[1]**Special Regulations for the Honour Moderations in Mathematics**

A

The subjects of the examination shall be Mathematics and its applications. The syllabus and the number of papers shall be as prescribed by regulation from time to time by the Mathematical and Physical Sciences Board. Candidates must offer all the papers in the same academic year, and also offer a practical assessment which has not been taken into account in a previous sitting of Honour Moderations.

B

1. Each candidate shall offer four papers as follows:
   A. Pure Mathematics I
   B. Pure Mathematics II
   C. Applied Mathematics I
   D. Applied Mathematics II
2. The syllabus for each paper will be published by the Mathematical Institute in a handbook for candidates by the beginning of the Michaelmas Full Term in the academic year of the examination, after consultation with the Mathematics Teaching Committee. Each paper will contain questions of a straightforward character.
3. The Chairman of Mathematics, or a deputy, shall make available to the moderators evidence showing the extent to which each candidate has pursued an adequate course of practical work. In assessing a candidate's performance in the examination, the moderators shall take this evidence into account. Where there are deadlines for handing in practical work, these will be published in a handbook for candidates by the beginning of the Michaelmas Full Term in the academic year of the examination.
4. Candidates are usually required to submit such practical work electronically. Details will be given in the handbook for the practical course. Any candidate who is unable, for some reason, to submit practical work electronically must apply to the Academic Administrator, Mathematical Institute, for permission to submit the work in paper form. Such application must reach the Academic Administrator two weeks before the deadline for submitting the work.
5. No candidate shall be declared worthy of Honours who is not deemed worthy of Honours on the basis of papers A–D and no candidate shall be awarded a Pass who has not shown competence on these papers.
6. The use of calculators is generally not permitted but certain kinds may be allowed for certain papers. Specifications of which papers and which types of calculators are permitted for those exceptional papers will be announced by the examiners in the Hilary Term preceding the examination.

Practical Work

The requirement in the *Examination Regulations* to pursue an adequate course of practical work will be satisfied by following the Maple course and submitting two Maple projects. Details about submission of these projects will be given in the Maple handbook.

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Assistant in the Mathematical Institute.

2 Syllabus

2.1 Pure Mathematics I

Naïve treatment of sets and mappings. Definition of a countable set. Relations, equivalence relations and partitions.

Vector spaces over $\mathbb{R}$ and more generally over a field F; subspaces. The span of a (finite) set of vectors; spanning sets; examples. Finite dimensionality.

Linear dependence and linear independence. Definition of bases; reduction of a spanning set and extension of a linearly independent set to a basis; proof that all bases have the same size. Dimension of the space. Co-ordinates with respect to a basis.

Linear transformations from one (real) vector space to another. Rectangular matrices over $\mathbb{R}$ and F. Algebra of matrices. The matrix representation of a linear transformation with respect to fixed bases; change of basis and co-ordinate systems. Composition of transformations and product of matrices.

Elementary row operations on matrices; echelon form and row-reduction. Invariance of the row space under row operations; row rank. Applications to finding bases of vector spaces.

Sums and intersections of subspaces; formula for the dimension of the sum.

The image and kernel of a linear transformation. The Rank-Nullity Theorem. Applications.

Matrix representation of a system of linear equations. Significance of image, kernel, rank and nullity for systems of linear equations. Solution by Gaussian elimination. Bases of solution space of homogeneous equations.

Invertible matrices; use of row operations to decide invertibility and to calculate inverse. Column space and column rank. Equality of row rank and column rank.

Permutations, composition of permutations. Cycles, standard cycle-product notation. The parity of a permutation; calculation of parity from cycle structure.

Determinants of square matrices; properties of the determinant function. Computation of determinant by reduction to row echelon form. Relationship of determinant to invertibility. Determinant of a linear transformation of a vector space to itself.

Eigenvalues of linear transformations of a vector space to itself. The characteristic polynomial of a square matrix; the characteristic polynomial of a linear transformation of a vector space to itself. The linear independence of a set of eigenvectors associated with distinct eigenvalues; diagonalisability of matrices.

Division algorithm and Euclid's algorithm for integers. Polynomials with real or complex coefficients, factorization; Division algorithm and Euclid's algorithm for polynomials.

Axioms for a group. Examples, including the symmetric groups. Subgroups, intersections of subgroups. The order of a subgroup. The order of an element. Cyclic subgroups. Cosets and Lagrange's Theorem. Straightforward examples. The structure of cyclic groups; orders of elements in cyclic groups.

Axioms for a Ring. Definition of a Field and an Integral Domain.

Homomorphisms of groups. The (First) Isomorphism Theorem.

Ring homomorphisms and their kernels; First Isomorphism Theorem for commutative rings.

Euclidean Geometry in two and three dimensions approached by vectors and co-ordinates. Vector algebra. Equations of planes, lines and circles. Rotations, reflections, isometries. Parametric representation of curves, tangents, conics (normal form only) focus, directrix. Simple surfaces: spheres, right circular cones.

2.2 Pure Mathematics II

Real numbers: arithmetic, ordering, suprema, infima; real numbers as a complete ordered field. The real numbers are uncountable.

The complex number system. De Moivre's Theorem, polar form, the triangle inequality.

Sequences of (real or complex) numbers. Limits of sequences of numbers; the algebra of limits. Order notation.

Subsequences; every subsequence of a convergent sequence converges to the same limit. Bolzano–Weierstrass Theorem. Cauchy's convergence principle. Limit point of a subset of the line or plane.

Series of (real or complex) numbers. Convergence of series. Simple examples to include geometric progressions and power series. Alternating series test, absolute convergence, comparison test, ratio test, integral test.

Power series, radius of convergence, important examples including exponential, sine and cosine series.

Continuous functions of a single real or complex variable. The algebra of continuous functions. A continuous real-valued function on a closed bounded interval is bounded, achieves its bounds and is uniformly continuous. Intermediate Value Theorem. Inverse Function Theorem for continuous strictly monotonic functions.

Sequences and series of functions. The uniform limit of a sequence of continuous functions is continuous. Weierstrass's M-test. Continuity of functions defined by power series.

Definition of derivative of a function of a single real variable. The algebra of differentiable functions. Rolle's Theorem. Mean Value Theorem. L'Hôpital's rule. Taylor's expansion with remainder in Lagrange's form. Binomial theorem with arbitrary index.

Step functions and their integrals. The integral of a continuous function on a closed bounded interval defined as the supremum of the integral of the step functions it dominates. Properties of the integral including linearity and the interchange of integral and limit for a uniform limit of continuous functions on a bounded interval. The Mean Value Theorem for Integrals. The Fundamental Theorem of Calculus; integration by parts and substitution.

Term-by-term differentiation of a (real) power series (interchanging limit and derivative for a series of functions where the derivatives converge uniformly); relationships between exponential, trigonometric functions and hyperbolic functions.

The geometry of the complex numbers. The Argand diagram. Complex numbers as a method of studying plane geometry, roots of unity, cocyclic points, Ptolemy's Theorem, equations of lines and circles (Apollonius's Theorem). Möbius transformations acting on the extended complex plane. Möbius transformations take lines and circles to lines and circles, Stereographic projection. The Riemann sphere and the conformal map to the extended complex plane. The geometry of the sphere: great circles, triangles (their six angles, angle-excess and area formula). Platonic solids.

2.3 Applied Mathematics I

Standard integrals, integration by parts. General linear homogeneous ordinary differential equations: integrating factor for first-order linear ordinary differential equations, second solution for second-order linear ordinary differential equations when one solution is known. First- and second-order linear ordinary differential equations with constant coefficients. General solution of linear inhomogeneous ordinary differential equation as particular solution, plus solution of homogeneous equation. Simple examples of finding particular integrals by guesswork.

Systems of linear, coupled, first-order ordinary differential equations.

Calculation of determinants, eigenvalues and eigenvectors.

Functions of several real variables: continuity, partial derivatives, chain rule, critical points, change of variable, calculation of very simple plane areas using double integrals, evaluation by change of variable.

Laplace's equation and the heat equation (all in two independent variables), applications. D'Alembert's solution of the wave equation and applications. Characteristic diagrams (excluding reflection and transmission). Transformations in the independent variables. Solution by separation of variables.

Newton's laws. Free and forced linear oscillations. Simple oscillatory systems with two degrees of freedom, natural frequencies. Two-dimensional motion, projectiles. Use of polar coordinates, circular motion. Central forces, differential equation for the particle path. Inverse square law, planetary orbits.

Energy and potential for one-dimensional motion. Equivalent ideas for central force problems and three-dimensional problems with axial symmetry.

Examples of stability and instability in physical situations, via linearised equations. Simple ideas of phase space, stable and unstable fixed points, periodic orbits. Informal introduction to chaos.

Sample space as the set of all possible outcomes, events and probability function. Permutations and combinations, examples using counting methods, sampling with or without replacement. Algebra of events. Conditional probability, partitions of sample space, theorem of total probability, Bayes's theorem, independence. Examples with statistical implications.

Random variable. Probability mass function. Discrete distributions: Bernoulli, binomial, Poisson, geometric, situations in which these distributions arise. Expectation: mean and variance. Probability generating functions, use in calculating expectations. Bivariate discrete distribution, conditional and marginal distributions. Extensions to many random variables. Independence for discrete random variables. Random walks (finite state space only). Solution of quadratic difference equations with applications to random walks.

Expectations of functions of more than one random variable. Random sample. Conditional expectation, application of theorem of total probability to expectation of a random variable.

Sums of independent random variables. Examples from well-known distributions.

Continuous random variables. Cumulative distribution function for both discrete and continuous random variables. Probability density function. Examples: uniform, exponential, gamma, normal. Practical examples. Expectation. Cumulative distribution function and probability density function for a function of a single continuous random variable. Simple examples of joint distributions of two or more continuous random variables, independence, expectation (mean and variance of sums of independent, identically distributed random variables).

2.4 Applied Mathematics II

C^n functions. Statement of conditions for equality of mixed partial derivatives. Statement of Taylor's theorem for a function of two variables. Critical points including degenerate cases. Gradient vector; normal to surface informal (geometrical) treatment of Lagrange multipliers.

Fourier series. Periodic, odd and even functions. Calculation of sine and cosine series. Simple applications (concentrating on the calculation of Fourier coefficients and the use of Fourier series).

Derivation of (i) the wave equation of a string, (ii) the heat equation in one and two dimensions (box argument only). Examples of solutions and their interpretation. Boundary conditions.

Use of Fourier series to solve the wave equation.

Uniqueness theorems for the wave equation, heat equation and Laplace's equation (all in two independent variables). Energy.

Integrals along curves in the plane. Green's Theorem in the plane (informal proof only).

Div, grad and curl in Euclidean coordinates. Evaluation of line, surface and volume integrals. Stokes' Theorem and the Divergence Theorem in two and three variables (proof excluded).

Rederivation of models of continuity of flow: the heat equation from the Divergence Theorem.

Gravity as a conservative force. Gauss's Theorem. The equivalence of Poisson's equation and the inverse-square law.

Random sample, concept of a statistic and its distribution, sample mean and sample variance.

Concept of likelihood, examples of likelihood for simple distributions. Estimation for a single unknown parameter by maximising likelihood. Examples drawn from: Bernoulli, binomial, geometric, Poisson, exponential (parametrised by mean), normal (mean only, variance known). Data to include simple surveys, opinion polls, archaeological studies. Properties of estimators: unbiasedness, Mean Squared Error $=((\text{bias})^2 +$ variance). Statement of Central Limit Theorem (excluding proof). Confidence intervals using Central Limit Theorem. Simple straight line fit, $Y_t = a + bx_t + \epsilon_t$, with ϵ_t normal independent errors of zero mean and common known variance; estimators for a, b by maximising likelihood using partial differentiation, unbiasedness and calculation of variance as linear sums of Y_t. (No confidence intervals). Examples (use of scatter plots to show suitability of linear regression).

3 Synopses of Lectures

3.1 Introduction to Pure Mathematics

3.1.1 Introduction to Pure Mathematics — Prof. Vaughan-Lee — 5 MT

There will be 5 introductory lectures in the first week of Michaelmas term.

Overview

The purpose of these introductory lectures is to establish some of the basic notation of mathematics, introduce the elements of (naïve) set theory and the nature of formal proof.

Learning Outcomes

Students will have the:

(i) Ability to describe, manipulate, and prove results about sets and functions using standard mathematical notation.

(ii) Ability to follow and to construct proofs by Mathematical Induction (including strong induction, minimal counterexample)

(iii) (Naïve) understanding of finite, countable and uncountable sets.

Synopsis

Sets: examples including the natural numbers, the integers, the rational numbers, the real numbers; inclusion, union, intersection, power set, ordered pairs and cartesian product of sets.

The well-ordering property of the natural numbers. Induction as a method of proof, including a proof of the binomial theorem with non-negative integral coefficients.

Maps: composition, restriction, 1-1ness, ontoness, invertible maps, images and preimages.

Definition of a countable set. The countability of the rational numbers.

Reading

1. G. Smith, *Introductory Mathematics: Algebra and Analysis* (Springer-Verlag, 1998), Chapters 1 and 2.

2. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis* (Wiley,Third Edition, 2000), Chapter 1 and Appendices A and B.

3. C. Plumpton, E. Shipton, R. L. Perry, *Proof* (MacMillan, 1984).

3.1.2 Introduction to Complex Numbers — Dr Szendroi — 2 MT

This course will run in the first week of Michaelmas Term.

Overview

This course aims to give all students a common background in complex numbers.

Learning Outcomes

Students will be able to

(i) manipulate complex numbers with confidence;

(ii) understand geometrically their representation on the Argand diagram including the nth roots of unity;

(iii) know the polar representation form and be able to apply it.

Synopsis

Basic arithmetic of complex numbers, the Argand diagram; modulus and argument of a complex number. Statement of the Fundamental Theorem of Algebra. Roots of unity. De Moivre's theorem. Simple transformations in the complex plane. Polar form $re^{i\theta}$, with applications.

Reading

1. R A Earl, *Bridging course material on complex numbers*
2. D W Jordan & P Smith, *Mathematical Techniques* (OUP, 2002), Ch. 6.

3.1.3 Reasoning and Proofs — Dr Knight — 3 MT

This course will run in the second week of Michaelmas term.

Overview

Prior to their arrival, undergraduates are encouraged to read Professor Batty's study guide "How do undergraduates do Mathematics?" The aim of this course is to provide an initial treatment of this material to better prepare students for the Mathematics programme.

This course builds on the first course of Introduction to Pure Mathematics exposing students to further examples and applications.

Learning Outcomes

Students will begin to develop

(i) a reflective approach to "How to do Mathematics," through theory and practice;

(ii) sound reasoning skills;

(iii) the ability to identify the essential parts of a proof technique, to construct proofs, setting them out clearly, concisely and accurately.

Synopsis

Beginning University Mathematics : Studying the theory, problem solving, writing mathematics. Examples from Applied Mathematics. [1 lecture]

Formulation of Mathematical statements with examples. Including hypotheses, conclusions, if, only if, if and only if, and, or. Quantifiers - for all, there exists. [1 lecture]

Proofs - standard techniques with examples. Counter examples. Constructing proofs. Experimentation, followed by making the proof precise. Example of proof by contradiction and induction. [1 lecture]

Reading

1. C. J. K. Batty, *How do undergraduates do Mathematics?* (Mathematical Institute Study Guide, 1994).
2. R. B. J. T. Allenby, *Numbers and Proof* (Modular Mathematics Series), (Arnold, 1997).
3. R. A. Earl, *Bridging Material on Induction.*
4. G. Pólya. *How to solve it : a new aspect of mathematical method* (Penguin, 1990).

3.2 Pure Mathematics I

3.2.1 Linear Algebra I — Dr Henke — 14 MT

Overview

Linear algebra pervades and is fundamental to geometry (from which it originally arose), algebra, analysis, applied mathematics, statistics - indeed most of mathematics. Vector spaces will usually be discussed over the field of real numbers or complex numbers and also as a general field.

Learning Outcomes

Students will

(i) understand the general concepts of a vector space, a subspace, linear dependence and independence, spanning sets and bases.

(ii) have an understanding of matrices and of their applications to the algorithmic solution of systems of linear equations and to their representation of linear transformations of vector spaces.

Synopsis

Algebra of matrices.

Vector spaces over the real numbers and more generally over field F; subspaces. Linear dependence and linear independence. The span of a (finite) set of vectors; spanning sets. Examples.

Definition of bases; reduction of a spanning set and extension of a linearly independent set to a basis; proof that all bases have the same size. Dimension of the space. Co-ordinates with respect to a basis.

Sums and intersections of subspaces; formula for the dimension of the sum.

Linear transformations from one (real) vector space to another. The image and kernel of a linear transformation. The Rank-Nullity Theorem. Applications.

The matrix representation of a linear transformation with respect to fixed bases; change of basis and co-ordinate systems. Composition of transformations and product of matrices.

Elementary row operations on matrices; echelon form and row-reduction. Matrix representation of a system of linear equations. Invariance of the row space under row operations; row rank.

Significance of image, kernel, rank and nullity for systems of linear equations including geometrical examples. Solution by Gaussian elimination. Bases of solution space of homogeneous equations. Applications to finding bases of vector spaces.

Invertible matrices; use of row operations to decide invertibility and to calculate inverse.

Column space and column rank. Equality of row rank and column rank.

Reading List

1. C. W. Curtis, *Linear Algebra - An Introductory Approach* (Springer, 4th edition, reprinted 1994).

2. R. B. J. T. Allenby, *Linear Algebra* (Arnold, 1995).

3. T. S. Blyth and E. F. Robertson, *Basic Linear Algebra* (Springer, 1998).

4. D. A. Towers, *A Guide to Linear Algebra* (Macmillan, 1988).

5. D. T. Finkbeiner, *Elements of Linear Algebra* (Freeman, 1972). [Out of print, but available in many libraries]

6. B. Seymour Lipschutz, Marc Lipson, *Linear Algebra* (McGraw Hill, Third Edition 2001).

7. R. B. J. T. Allenby, *Rings, Fields and Groups* (Edward Arnold, Second Edition, 1999). [Out of print, but available in many libraries also via Amazon.]

3.2.2 Linear Algebra II — Dr Curnock — 8HT

Learning Outcomes

Students will

(i) have an understanding of matrices, and their representation of linear transformations of vector spaces, including change of basis.

(ii) understand the elementary properties of determinants.

(iii) understand the elementary parts of eigenvalue theory with some of its applications.

Synopsis

Review of a matrix of a linear transformation with respect to bases, and change of bases. [1 lecture]

Permutations of a finite set, composition of permutations. Cycles and standard cycle-product notation. The result that a permutation is a product of transpositions. The parity of a permutation; goodness of the definition; how to calculate parity from cycle structure.

Determinants of square matrices; properties of the determinant function; determinants and the scalar triple product.[2 lectures]

Computation of determinant by reduction to row echelon form.

Proof that a square matrix is invertible if and only if its determinant is non-zero.

Determinant of a linear transformation of a vector space to itself. [$2\frac{1}{2}$ lectures]

Eigenvalues of linear transformations of a vector space to itself. The characteristic polynomial of a square matrix; the characteristic polynomial of a linear transformation of a vector space to itself. The linear independence of a set of eigenvectors associated with distinct eigenvalues; diagonalisability of matrices. [$2\frac{1}{2}$ lectures]

Reading List

1. C. W. Curtis, *Linear Algebra - An Introductory Approach* (Springer, 4th edition, reprinted 1994).

2. R. B. J. T. Allenby, *Linear Algebra* (Arnold, 1995).

3. T. S. Blyth and E. F. Robertson, *Basic Linear Algebra* (Springer, 1998).

4. D. A. Towers, *A Guide to Linear Algebra* (Macmillan, 1988).

5. D. T. Finkbeiner, *Elements of Linear Algebra* (Freeman, 1972). [Out of print, but available in many libraries]

6. B. Seymour Lipschutz, Marc Lipson, *Linear Algebra* (McGraw Hill, Third Edition 2001).

For permutations:

7. R. B. J. T. Allenby, *Rings, Fields and Groups* (Edward Arnold, Second Edition,1999). [Out of print, but available in many libraries also via Amazon]

8. W.B.S. Stewart, *Abstract Algebra* (Mathematical Institute Lecture Notes, 1994).

3.2.3 An Introduction to Groups, Rings, and Fields — Prof. Priestley — 8 HT and 8 TT

Overview

Abstract algebra evolved in the twentieth century out of nineteenth century discoveries in algebra, number theory and geometry. It is a highly developed example of the power of generalisation and axiomatisation in mathematics. The first objective is to develop some concepts of set theory as a useful language for abstract mathematics. Next we aim to introduce the student to some of the basic techniques of algebra. Thirdly, we aim to introduce group theory through axioms and examples both because groups are typical and famous examples of algebraic structures, and because of their use in the measurement of symmetry. Finally we begin the study of fields and rings which are important in the study of polynomials and their zeros.

Learning Outcomes

Students will have developed an abstract approach to reasoning about number systems, their arithmetic structures and properties. They will be familiar with the axioms of a group, ring, and field together with examples. They will have a detailed knowledge of the ring of integers and the ring of polynomials and the Euclidean Algorithm in each case. They will have developed an appreciation of the isomorphism theorem for groups and rings.

Synopsis

HT (8 lectures)

Review of algebraic structures encountered in the course so far: $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$; $\mathbb{R}^n$ and $M_{m,n}(\mathbb{R})$.

Concept of a binary operation. Axioms for a group. Examples from above, including invertible $n \times n$ matrices under multiplication; note that a vector space is an Abelian group under $+$.

Axioms for a ring (not assumed commutative with identity). Optional additions: · commutative; ∃ identity for multiplication. Examples. Definition of a field and an integral domain, with preliminary examples. Fact that a field is an ID. [2 lectures]

The integers as an example of an integral domain which is not a field. Division algorithm and Euclidean algorithm. Polynomials with real or complex coefficients; ring structure, degree of a polynomial; Division algorithm, Euclid's algorithm; the Remainder Theorem. Statement of Fundamental Theorem of Arithmetic. [$2\frac{1}{2}$ lectures]

Relations, equivalence relations, partitions; examples, including integers mod n.

Review of permutations of a finite set: S_n as a group. Decomposition of a permutation into a product of disjoint cycles. [2 lectures]

Groups in general, with additional (straightforward) examples. Subgroups, with examples. Products of groups. [$1\frac{1}{2}$ lectures]

TT (8 Lectures)

Brief review of HT course.

Cyclic groups; order of an element; fact that a subgroup of a cyclic group is cyclic. Cosets and Lagrange's Theorem; simple examples.

Homomorphisms of groups with motivating examples. Kernels, normal subgroups, The 1st Isomorphism Theorem. More on S_n: conjugacy in S_n and relationship to normality. [6 lectures]

Ring homomorphisms and their kernels; examples. Ideals in commutative rings, 1st Isomorphism Theorem for commutative rings; the example of integers mod n. [2 lectures]

Remark : The number of lectures is an indicative guide.

Reading List

1. R. B. J. T. Allenby, *Rings, Fields and Groups* (Edward Arnold, Second Edition, 1999). [Out of print, but available in many libraries also via Amazon]

2. I. N. Herstein, *An Introduction to Abstract Algebra* (Wiley, Third edition, 1996), ISBN - 978-0-471-36879-3.

Alternative reading

1. N. L. Biggs, *Discrete Mathematics* (OUP, revised edition).

2. Peter J. Cameron, *Introduction to Algebra* (OUP 1998), §§1.1, 1.2, 1.3, 1.4, 3.1, 3.2, 3.3, 3.4.

3. John B. Fraleigh, *A First Course in Abstract Algebra* (Pearson, Seventh edition, 2002).

4. W. Keith Nicholson, *Introduction to Abstract Algebra* (John Wiley, Second edition, 1999), §§1.2, 1.3, 1.4, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10.

5. Joseph J. Rotman, *A First Course in Abstract Algebra* (Prentice-Hall, Second edition, 2000), §§1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 3.3, 3.5.

6. G. C. Smith, *Introductory Mathematics: Algebra and Analysis* (Springer Undergraduate Mathematics Series, 1998), Chapters 1, 2, 5.

3.2.4 Geometry I — Prof. Joyce — 7 MT

Overview

These lectures give an introduction to elementary ideas in the geometry of Euclidean space through vectors.

Learning Outcomes

By the end of the course students will have a detailed knowledge of Euclidean geometry in two and three dimensions.

Synopsis

Euclidean geometry in two and three dimensions approached by vectors and co-ordinates. Vector addition and scalar multiplication. The scalar product, equations of planes, lines and circles. The vector product in three dimensions. Scalar triple products and vector triple products, vector algebra. Rotations, reflections, isometries. Parametric representation of curves, tangents; conics (normal form only), focus and directrix. Simple surfaces: spheres, right circular cones.

Reading

Main text

1. J. Roe, *Elementary Geometry* OUP (1993), Chapters 1, 2.2, 3.4, 4, 7.1, 7.2, 8.1-8.3.

Alternative reading

1. D. A. Brannan, M. F. Esplen and J. J. Gray, *Geometry* (CUP, 1999), Chapter 1.
2. P. M. Cohn, *Solid Geometry* (Routledge and Kegan Paul, 1961), Chapters 1-3.
3. R. Fenn, *Geometry* (Springer, 2001), Chapters 2, 5, 7.
4. J. E. Marsden and A. J. Tromka, *Vector Calculus* (McGraw-Hill, fourth edition, 1996), Chapters 1, 2.
5. J. Silvester, *Geometry, Ancient and Modern* (OUP, 2001), Chapters 3, 5, 6, 7.

3.3 Pure Mathematics II

Students will already be aware of the importance and power of calculus. The aim of Analysis I, II and III is to put precision into its formulation and to make students more secure in their understanding and use. The lectures on analysis come in three parts, beginning in the first term with the properties of the real numbers and studying limits through sequences and series. The second part is concerned with continuity and differentiation, and the third part with integration. Power series and the functions they define provide a unifying theme.

3.3.1 Analysis I: Sequences and Series — Dr Earl — 14 MT

Overview

In these lectures we study the real and complex numbers and study their properties, particularly completeness; define and study limits of sequences, convergence of series, and power series.

Learning Outcomes

Student will have

(i) An ability to work within an axiomatic framework.

(ii) A detailed understanding of how Cauchy's criterion for the convergence of real and complex sequences and series follows from the completeness axiom for $\mathbb{R}$, and the ability to explain the steps in standard mathematical notation.

(iii) Knowledge of some simple techniques for testing the convergence of sequences and series, and confidence in applying them.

(iv) Familiarity with a variety of well-known sequences and series, with a developing intuition about the behaviour of new ones.

(v) An understanding of how the elementary functions can be defined by power series, with an ability to deduce some of their easier properties.

Synopsis

Real numbers: arithmetic, ordering, suprema, infima; the real numbers as a complete ordered field. The reals are uncountable. The complex number system. The triangle inequality.

Sequences of real or complex numbers. Definition of a limit of a sequence of numbers. Limits and inequalities. The algebra of limits. Order notation: O, o.

Subsequences; a proof that every subsequence of a convergent sequence converges to the same limit; bounded monotone sequences converge. Bolzano-Weierstrass Theorem. Limit point of a set. Cauchy's convergence principle.

Series of real or complex numbers. Convergence of series. Simple examples to include geometric progressions and some power series. Absolute convergence, comparison test, ratio test, integral test. Alternating series test.

Power series, radius of convergence; important examples to include the exponential, cosine and sine series.

Reading – Main texts

1. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis* (Wiley, Third Edition, 2000), Chapters 2, 3, 9.1, 9.2.
2. R. P. Burn, *Numbers and Functions, Steps into Analysis* (Cambridge University Press, 2000), Chapters 2-6. [This is a book of problems and answers, a DIY course in analysis.]
3. J. M. Howie, *Real Analysis* (Springer Undergraduate Texts in Mathematics Series), ISBN 1-85233-314-6.

Alternative reading

The first five books take a slightly gentler approach to the material in the syllabus, whereas the last two cover it in greater depth and contain some more advanced material.

1. Mary Hart, *A Guide to Analysis* (MacMillan, 1990), Chapter 2.
2. J. C. Burkill, *A First Course In Mathematical Analysis* (CUP, 1962), Chapters 1, 2 and 5.
3. K. G. Binmore, *Mathematical Analysis, A Straightforward Approach* (Cambridge University Press, second edition, 1990), Chapters 1-6.
4. Victor Bryant, *Yet Another Introduction to Analysis* (Cambridge University Press, 1990), Chapters 1 and 2.
5. G. Smith, *Introductory Mathematics: Algebra and Analysis* (Springer-Verlag, 1998), Chapter 3 (introducing complex numbers).
6. Michael Spivak, *Calculus* (Benjamin, 1967), Parts I, IV, and V (for a construction of the real numbers).
7. Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner, *Elementary Analysis* (Prentice Hall, 2001), Chapters 1-4.

3.3.2 Analysis II: Continuity and Differentiability — Dr Stewart — 16 HT

Overview

In this term's lectures we study continuity of functions of a real or complex variable, and differentiability of functions of a real variable.

Learning Outcomes

At the end of the course students will be able to apply limiting properties to describe and prove continuity and differentiability conditions for real and complex functions. They will be able to prove important theorems, such as the I.V.T., Rolle's and Mean Value, and will begin the study of power series and their convergence.

Synopsis

Definition of the function limit. Examples and counter examples to illustrate when $\lim_{x\to a} f(x) = f(a)$ (and when it doesn't). Definition of continuity of functions on subsets of $\mathbb{R}$ and $\mathbb{C}$ in terms of ϵ and δ. The algebra of continuous functions; examples, including polynomials. Continuous functions on closed bounded intervals: boundedness, maxima and minima, uniform continuity. Intermediate Value Theorem. Inverse Function Theorem for continuous strictly monotone functions. Sequences and series of functions. The uniform limit of a sequence of continuous functions is continuous. Weierstrass's M-test for uniformly convergent series of functions. Continuity of functions defined by power series. Definition of the derivative of a function of a real variable. Algebra of derivatives, examples to include polynomials and inverse functions. The derivative of a function defined by a power series is given by the derived series (proof not examinable). Vanishing of the derivative at a local maximum or minimum. Rolle's Theorem. Mean Value Theorem with simple applications: constant and monotone functions. Cauchy's (generalized) Mean Value Theorem and L'Hôpital's formula. Taylor's Theorem with remainder in Lagrange's form; examples of Taylor's Theorem to include the binomial expansion with arbitrary index.

Reading – Main texts

1. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis* (Wiley, Third Edition, 2000), Chapters 4-8.

2. R. P. Burn, *Numbers and Functions, Steps into Analysis* (Cambridge University Press, 2000). [This is a book of problems and answers, a DIY course in analysis]. Chapters 6-9, 12.

3. Walter Rudin, *Principles of Mathematical Analysis* (McGraw-Hill, 3rd edition, 1976). Chapters 4,5,7.

4. J. M. Howie, *Real Analysis* (Springer Undergraduate Texts in Mathematics Series), ISBN 1-85233-314-6.

Alternative reading

1. Mary Hart, *A Guide to Analysis* (MacMillan, 1990), Chapters 4,5.

2. J. C. Burkill, *A First Course in Mathematical Analysis* (CUP, 1962), Chapters 3, 4, and 6.

3. K. G. Binmore, *Mathematical Analysis* A Straightforward Approach, (CUP, second edition, 1990), Chapters 7-12, 14-16.

4. Victor Bryant, *Yet Another Introduction to Analysis* (CUP, 1990), Chapters 3 and 4

5. M. Spivak, *Calculus* (Publish or Perish, 3rd Edition, 1994), Part III.

6. Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner, *Elementary Analysis* (Prentice Hall, 2001), Chapters 5-10.

3.3.3 Analysis III: Integration — Prof Haydon — 8 TT

Overview

In these lectures we define a simple integral and study its properties; prove the Mean Value Theorem for Integrals and the Fundamental Theorem of Calculus. This gives us the tools to justify term-by-term differentiation of power series and deduce the elementary properties of the trigonometric functions.

Learning Outcomes

At the end of the course students will be familiar with the construction of an integral from fundamental principles including important theorems. They will know when it is possible to integrate or differentiate term-by-term and be able to apply this to, for example, trigonometric series.

Synopsis

Step functions, their integral, basic properties. The application of uniform continuity to approximate continuous functions above and below by step functions. The integral of a continuous function on a closed bounded interval. Elementary properties of the integral of a continuous function: positivity, linearity, subdivision of the interval.

The Mean Value Theorem for Integrals. The Fundamental Theorem of Calculus; linearity of the integral, integration by parts and substitution.

The interchange of integral and limit for a uniform limit of continuous functions on a bounded interval. Term-by-term integration and differentiation of a (real) power series (interchanging limit and derivative for a series of functions where the derivatives converge uniformly); examples to include the derivation of the main relationships between exponential, trigonometric functions and hyperbolic functions.

Reading

1. T. Lyons *Lecture Notes.*

2. J. Roe, *Integration* Mathematical Institute Notes (1994).

3. H. A. Priestley, *Introduction to Integration* (Oxford Science Publications, 1997), Chapters 1–8. [These chapters commence with a useful summary of background 'cont and diff' and go on to cover not only the integration but also the material on power series.]

4. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis* (Wiley, Third Edition, 2000), Chapter 8.

3.3.4 Geometry II — Dr Giansiracusa — 8 TT

Overview

The aim of these last 8 lectures in geometry is to introduce students to some of the most elegant and fundamental results in the geometry of the Euclidean plane and the sphere.

Learning Outcomes

By the end of the course students will be able to use Möbius transforms and will be able to relate geometry and the complex plane and know the important theorems in the subject. They will know the Platonic solids and their properties and relate these to why there are precisely five.

Synopsis

The geometry of the complex numbers. The Argand diagram. Complex numbers as a methods of studying plane geometry, examples to include roots of unity, cocyclic points, Ptolemy's Theorem. Equations of lines and circles (Apollonius's Theorem), Möbius transformations acting on the extended complex plane, stereographic projection. Möbius transformations take lines and circles to lines and circles, The Riemann sphere and the conformal map to the extended complex plane.

The geometry of the sphere: great circles, triangles (their six angles, angle-excess and area formula).

Platonic solids and the corresponding subdivision of the sphere noting Euler's formula.

Reading

1. Roger Fenn, *Geometry* (Springer, 2001), Chapters 4 and 8.
2. J. Roe, *Elementary Geometry* (Oxford Science Publications, 1992), Section 6.5.
3. Liang-shin Hahn, *Complex Numbers and Geometry* (The Mathematical Association of America, 1994).
4. George A. Jennings, *Modern Geometry and Applications* (Springer, 1994), Chapter 2.
5. David A. Brannan, Matthew F. Esplen, Jeremy Gray, *Geometry* (Cambridge University Press, 1999), Chapter 7.
6. Elmer G. Rees, *Notes on Geometry* (Springer-Verlag, 1983), pp 23-28.

3.4 Applied Mathematics I

3.4.1 Calculus of One Variable — Dr Gaffney — 6 MT

Overview

These lectures are designed to give students a gentle introduction to applied mathematics in their first term at Oxford, allowing time for both students and tutors to work on developing and polishing the skills necessary for the course. It will have an 'A-level' feel to it, helping in the transition from school to university. The emphasis will be on developing skills and familiarity with ideas using straightforward examples.

Background reading:

D. W. Jordan & P. Smith, *Mathematical Techniques* (OUP, 3rd Edition, 2003), Chapters 1–4, 14–17.

Learning Outcomes

At the end of the course students will be able to solve a range of ODEs and linear systems of first order ODEs.

Synopsis

Standard integrals, integration by parts.
Definition of order of an ODE - example of separation of variables. General linear homogeneous ODEs: integrating factor for first order linear ODEs, second solution when one solution known for second order linear ODEs. First and second order linear ODEs with constant coefficients. General solution of linear inhomogeneous ODE as particular solution plus solution of homogeneous equation. Simple examples of finding particular integrals by guesswork.
(Jordan & Smith, Chapters 18, 19, 22; Kreyszig, Sections 1.1–1.3, 1.6, 2.1–2.3, 2.8.)

Systems of linear coupled first order ODEs. Calculation of determinants, eigenvalues and eigenvectors and their use in the solution of linear coupled first order ODEs.
(Kreyszig, Sections 3.0–3.3.)

Reading

1. D. W. Jordan & P. Smith, *Mathematical Techniques* (OUP, 3rd Edition, 2003).
2. Erwin Kreyszig, *Advanced Engineering Mathematics* (Wiley, 8th Edition, 1999).

3.4.2 Calculus of Two or More Variables — Dr Gaffney — 10 MT

Synopsis

10 lectures Michaelmas Term continuing on from ODEs

Introduction to partial derivatives. Chain rule, change of variable; examples to include plane polar coordinates. Examples of solving some simple partial differential equations (e.g. $f_{xy} = 0, yf_x = xf_y$). Jacobians for two variable systems, calculations of areas including basic examples of double integrals. Gradient vector; normal to surface, directional derivative. Critical points and classification using directional derivatives (non-degenerate case only). Laplace's and Poisson's equation, including change of variable to plane polar coordinates and circularly symmetric solutions. The wave equation in two variables, including derivation of general solution.
(Jordan & Smith, Chapters 27-29, 31. Kreyszig appendix 3.2, Sections 8.8, 8.9, 9.3, 9.5, 9.6, 11.1, 11.2. Bourne & Kendall, Sections 4.1-4.5.)

Reading

1. D. W. Jordan & P. Smith, *Mathematical Techniques* (OUP, 3rd Edition, 2003).
2. Erwin Kreyszig, *Advanced Engineering Mathematics* (Wiley, 8th Edition, 1999).
3. D. E. Bourne & P. C. Kendall, *Vector Analysis and Cartesian Tensors* (Stanley Thornes, 1992).
4. David Acheson, *From Calculus to Chaos: An Introduction to Dynamics* (OUP, 1997).
5. G. F. Carrier & C. E. Pearson, *Partial Differential Equations — Theory and Technique* (Academic Press, 1988). [Advanced — leads on to 2nd year PDEs]

3.4.3 Dynamics — Dr Acheson — 16 MT

Overview

The subject of dynamics is about how things change with time. A major theme is the modelling of a physical system by differential equations, and one of the highlights involves using the law of gravitation to account for the motion of planets.

Learning Outcomes

Students will be familiar with the laws of motion, including circular and planetary motion. They will know how forces are used and be introduced to stability and chaos in a physical system.

Synopsis

Newton's laws. Free and forced linear oscillations. Simple oscillatory systems with two degrees of freedom, natural frequencies. Two dimensional motion, projectiles. Use of polar coordinates, circular motion. Central forces, differential equation for the particle path. Inverse square law, planetary orbits.

Energy and potential for one dimensional motion. Equivalent ideas for central force problems and three dimensional problems with axial symmetry.

Examples of stability and instability in physical situations, via linearized equations. Simple ideas of phase space, stable and unstable fixed points, periodic orbits. Informal introduction to chaos.

Reading

David Acheson, *From Calculus to Chaos: an Introduction to Dynamics* (OUP, 1997), Chapters 1,5,6,10,11.

Further reading

1. M. W. McCall, *Classical Mechanics: A Modern Introduction* (Wiley, 2001), Chapters 1–4, 7.
2. M. Lunn, *A First Course in Mechanics* (OUP, 1991), Chapters 1–3, (up to 3.4).

3.4.4 Probability I — Dr Laws — 8 MT

Overview

An understanding of random phenomena is becoming increasingly important in today's world within social and political sciences, finance, life sciences and many other fields. The aim of this introduction to probability is to develop the concept of chance in a mathematical framework. Discrete random variables are introduced, with examples involving most of the common distributions.

Learning Outcomes

Students should have a knowledge and understanding of basic probability concepts, including conditional probability. They should know what is meant by a random variable, and have met the common distributions and their probability mass functions. They should understand the concepts of expectation and variance of a random variable. A key concept is that of independence which will be introduced for events and random variables.

Synopsis

Motivation, relative frequency, chance. (What do we mean by a 1 in 4 chance?) Sample space as the set of all possible outcomes—examples. Events and the probability function. Permutations and combinations, examples using counting methods, sampling with or without replacement. Algebra of events. Conditional probability, partitions of sample space, theorem of total probability, Bayes's Theorem, independence.

Random variable. Probability mass function. Discrete distributions: Bernoulli, binomial, Poisson, geometric, situations in which these distributions arise. Expectation: mean and variance. Probability generating functions, use in calculating expectations. Bivariate discrete distribution, conditional and marginal distributions. Extensions to many random variables. Independence for discrete random variables. Conditional expectation. Solution of linear and quadratic difference equations with applications to random walks.

Main reading

1. D. Stirzaker, *Elementary Probability* (CUP, 1994), Chapters 1–4, 5.1–5.6, 6.1–6.3, 7.1, 7.2, 7.4, 8.1, 8.3, 8.5 (excluding the joint generating function).
2. D. Stirzaker, *Probability and Random Variables: A Beginner's Guide* (CUP, 1999).

Further reading

1. J. Pitman, *Probability* (Springer-Verlag, 1993).
2. S. Ross, *A First Course In Probability* (Prentice-Hall, 1994).
3. G. R. Grimmett and D. J. A. Welsh, *Probability: An Introduction* (OUP, 1986), Chapters 1–4, 5.1–5.4, 5.6, 6.1, 6.2, 6.3 (parts of), 7.1–7.3, 10.4.

3.4.5 Probability II — Dr Marchini — 8HT

Learning Outcomes

Students should have a knowledge and understanding of basic probability concepts, including conditional probability. They should know what is meant by a random variable, and have met the common distributions, and their probability density functions. They should understand the concepts of expectation and variance of a random variable. A key concept is that of independence which will be introduced for events and random variables. The emphasis on this course is a continuation of discrete variables studied in Probability I, followed by continuous random variables, with examples involving the common distributions.

Synopsis

Random walks (finite state space only). Expectations of functions of more than one random variable. Random sample. Conditional expectation, application of theorem of total probability to expectation of a random variable. Sums of independent random variables. Examples from well-known distributions.

Continuous random variables, motivation. Cumulative distribution functions for both discrete and continuous random variables. Probability density function – analogy with mass and density of matter. Examples: uniform, exponential, gamma, normal. Practical examples. Expectation. Cdf and pdf for function of a single continuous random variable. Simple examples of joint distributions of two or more continuous random variables; independence, expectation (mean and variance of sums of independent, identically distributed random variables).

Main reading

1. D. Stirzaker, *Elementary Probability* (CUP, 1994), Chapters 1–4, 5.1–5.6, 6.1–6.3, 7.1, 7.2, 7.4, 8.1, 8.3, 8.5 (excluding the joint generating function).

2. D. Stirzaker, *Probability and Random Variables: A Beginner's Guide* (CUP, 1999).

Further reading

1. J. Pitman, *Probability* (Springer-Verlag, 1993).

2. S. Ross, *A First Course In Probability* (Prentice-Hall, 1994).

3. G. R. Grimmett and D. J. A. Welsh, *Probability: An Introduction* (OUP, 1986), Chapters 1–4, 5.1–5.4, 5.6, 6.1, 6.2, 6.3 (parts of), 7.1–7.3, 10.4.

3.5 Applied Mathematics II

3.5.1 Fourier Series and Two Variable Calculus — Dr Capdeboscq — 16 HT

Overview

The first quarter of these lectures introduce students to Fourier series, concentrating on their practical application rather than proofs of convergence. The second half deals with scalar functions of two independent variables. Students will learn how to evaluate area and line integrals and how they are related via Green's Theorem. They will also be introduced to the ideas of continuity and differentiability of two-variable functions and shown how to classify critical points using Taylor's Theorem.

Learning Outcomes

Students will be familiar with Fourier series and their applications and be notionally aware of their convergence. They will know how to evaluate double integrals and integrals along curves in the plane and how these are linked using Green's theorem. They will have a deeper appreciation of partial derivatives and their applications, for example in Taylor's Theorem and optimisation.

Synopsis

Fourier series. Periodic, odd and even functions. Calculation of sine and cosine series. Simple applications, concentrating on imparting familiarity with the calculation of Fourier coefficients and the use of Fourier series.

Issues of convergence addressed through example and special cases. *[No attempt to state or prove convergence theorems, though students should be aware that convergence is linked to smoothness, referring to Maple exercises, e.g., exploring the Gibbs phenomenon.]*

(Jordan & Smith, 3rd Edition, Chapter 26. Kreyszig, Sections 10.1-10.4.)

Informal definition of double integrals. Evaluation by change of variable. Simple applications. Cylindrical and spherical polar coordinates. Elementary surface integrals. (Jordan & Smith, 2nd Edition Chapters 31, 32 or 3rd Edition Chapters 32 and 33. Kreyszig, Sections 9.3, 9.5, 9.6.)

Integrals along curves in the plane. Green's Theorem in the plane *(informal proof only)*. (Jordan & Smith, 3rd Edition Chapter 33 . Kreyszig, Sections 9.1, 9.4.)

Definitions of continuity, of partial derivatives and the gradient vector in terms of limits. C^n functions. Conditions for equality of mixed partial derivatives. Taylor's Theorem for a function of two variables (statement only). Revision of critical points and classification using Taylor's Theorem. (Examples to include degenerate cases.) Informal (geometrical) treatment of Lagrange multipliers. (J. C. Burkill, Chapter 8.)

Reading

Fourier Series

1. D. W. Jordan and P. Smith, *Mathematical Techniques* (Oxford University Press, 3rd Edition 2003), Chapter 26.

2. Erwin Kreyszig, *Advanced Engineering Mathematics* (Wiley, 8th Edition, 1999), Sections 10.1-10.4.

Multiple Integrals

1. D. W. Jordan and P. Smith, *Mathematical Techniques* (Oxford University Press, 3rd Edition, 2003),Chapters 31, 32 or 3rd Edition, Chapters 32 and 33.

2. Erwin Kreyszig, *Advanced Engineering Mathematics* (Wiley, 8th Edition,1999), Sections 9.1, 9.3, 9.4, 9.5, 9.6.

Continuity, Partial Differentiation, Taylor Series, Critical Points:

1. J. C. Burkill, *A First Course in Mathematical Analysis* (CUP, Reprinted 1991), Chapter 8.

2. D. W. Jordan and P. Smith, *Mathematical Techniques* (Oxford University Press, 3rd Edition, 2003), Chapter 28.

3. Erwin Kreyszig, *Advanced Engineering Mathematics* (Wiley, 8th Edition, 1999), Section 8.8.

3.5.2 Partial Differential Equations in Two Dimensions and Applications — Dr Day — 16 HT

Overview

In these lectures, students will be shown how the heat equation, the wave equation and Laplace's equation arise in physical models. They will learn basic techniques for solving each of these equations in two independent variables, and will be introduced to elementary uniqueness theorems.

Learning Outcomes

Students will know how to derive the heat, wave and Laplace's equations in 2 independent variables and to solve them. They will begin the study of uniqueness of solution of these important PDEs.

Synopsis

Introductory lecture in descriptive mode on particular differential equations and how they arise. Derivation of (i) the wave equation of a string, (ii) the heat equation in one and two dimensions *(box argument only)*. Examples of solutions and their interpretation. Boundary conditions.

(Kreyszig,Sections 11.1, 11.2. Carrier & Pearson, Sections 1.1, 1.2, 3.1, 3.2, 4.1, 4.2. Strauss, Sections 1.3, 1.4.)

Use of Fourier series to solve the wave equation, Laplace's equation and the heat equation *(all with two independent variables)*. Applications. D'Alembert's solution of the wave equation and applications. Characteristic diagrams *(excluding reflection and transmission)*. Transformations in the independent variables. Solution by separation of variables.

(Kreyszig, Sections 11.3–11.5. Carrier & Pearson, Sections 1.3–1.8, 3.3–3.6, 4.4, 4.5. Strauss, Sections 2.1, 5.1, 5.2, 6.2, chapter 4.)

Uniqueness theorems for the wave equation, heat equation and Laplace's equation *(all in two independent variables)*. Energy.
(Carrier & Pearson, Sections 1.9, 4.3. Strauss, Sections 2.2, 2.3, 6.1.)

Reading

1. Erwin Kreyszig, *Advanced Engineering Mathematics* (Wiley, 8th Edition, 1999).
2. G. F. Carrier & C. E. Pearson, *Partial Differential Equations — Theory and Technique* (Academic Press, 1988).
3. W. A. Strauss, *Partial Differential Equations: An Introduction* (Wiley, 1992).

3.5.3 Calculus in Three Dimensions and Applications — Dr Earl — 16 TT

Overview

In these lectures, students will be introduced to three-dimensional vector calculus. They will be shown how to evaluate volume, surface and line integrals in three dimensions and how they are related via the divergence Theorem and Stokes' Theorem. The theory will be applied to problems in physics.

Background reading

Erwin Kreyszig, *Advanced Engineering Mathematics* (Wiley, 8th Edition, 1999), Sections 8.1–8.6, 8.8;
D. E. Bourne & P. C. Kendall, *Vector Analysis and Cartesian Tensors* (Stanley Thornes, 1992), Chapters 1–3, Sections 4.1–4.3.

Synopsis

Div, grad and curl in Euclidean coordinates.
(Kreyszig, Sections 8.9–8.11. Bourne & Kendall, Sections 4.4–4.9.)

Volume, surface and line integrals. Stokes' Theorem and the Divergence Theorem (proofs excluded). Illustration by rederivation of models studied from Hilary Term, for example, the heat equation from the Divergence Theorem.
(Kreyszig, Sections 9.1, 9.5–9.9. Bourne & Kendall, Chapters 5, 6.)

Gravity as a conservative force. Gauss's Theorem. The equivalence of Poisson's equation and the inverse-square law.

Reading

1. Erwin Kreyszig, *Advanced Engineering Mathematics* (Wiley, 8th Edition, 1999).
2. D. E. Bourne & P. C. Kendall, *Vector Analysis and Cartesian Tensors* (Stanley Thornes, 1992).
3. Jerrold E. Marsden and Anthony J. Tromba, *Vector Calculus* (McGraw-Hill, Fourth Edition, 1996), chapters 7, 8.
4. H. M. Schey, *Div, grad and curl and all that* (W. W. Norton, Third Edition, 1996).

3.5.4 Statistics — Prof. Donnelly — 8 HT

Overview

The theme is the investigation of real data using the method of maximum likelihood to provide point estimation, given unknown parameters in the models. Maximum likelihood will be the central unifying approach. Examples will involve a distribution with a single unknown parameter, in cases for which the confidence intervals may be found by using the Central Limit Theorem (statement only). The culmination of the course will be the link of maximum likelihood technique to a simple straight line fit with normal errors.

Learning Outcomes

Students will have:

(i) an understanding of the concept of likelihood, and the use of the principle of maximum likelihood to find estimators;

(ii) an understanding that estimators are random variables, property unbiasedness and mean square error;

(iii) an understanding of confidence intervals and their construction including the use of the Central Limit Theorem;

(iv) an understanding of simple linear regression when the error variance is known.

Synopsis

Random sample, concept of a statistic and its distribution, sample mean as a measure of location and sample variance as a measure of spread.

Concept of likelihood; examples of likelihood for simple distributions. Estimation for a single unknown parameter by maximising likelihood. Examples drawn from: Bernoulli, Binomial, Geometric, Poisson, Exponential (parametrized by mean), Normal (mean only, variance known). Data to include simple surveys, opinion polls, archaeological studies, etc. Properties of estimators—unbiasedness, Mean Squared Error =((bias)^2+ variance). Statement of Central Limit Theorem (excluding proof). Confidence intervals using CLT. Simple straight line fit, $Y_t = a + bx_t + \epsilon_t$, with ϵ_t normal independent errors of zero mean and common known variance. Estimators for a, b by maximising likelihood using partial differentiation, unbiasedness and calculation of variance as linear sums of Y_t. (No confidence intervals). Examples (use scatter plots to show suitability of linear regression).

Reading

F. Daly, D. J. Hand, M. C. Jones, A. D. Lunn, K. J. McConway, *Elements of Statistics* (Addison Wesley, 1995). Chapters 1–5 give background including plots and summary statistics, Chapter 6 and parts of Chapter 7 are directly relevant.

Further reading

J. A. Rice, *Mathematical Statistics and Data Analysis* (Wadsworth and Brooks Cole, 1988).

3.6 Mathematics with Maple

3.6.1 Exploring Mathematics with Maple — Dr Wilkins — 16 MT and 16 HT

Overview

Mathematicians (like other professionals) use a wide range of generic computer packages: email, word-processors, web-browsers, spreadsheets, database managers and so on. Many, if not most, of the students on the Oxford Mathematics courses will have already used some of these packages, and are encouraged to use the facilities available centrally and in colleges to continue to develop their skills with these during their course.

The use by mathematicians of software developed for handling specific sorts of mathematical problems, especially numerical ones, is well-established; lecture courses in later years will, where appropriate, introduce students to these applications.

Increasingly, professional mathematicians use general purpose mathematical packages; sometimes these are called symbolic calculators, or algebraic manipulation packages. Such a package can be used as a super graphics calculator, as a scratchpad, or as a handbook of mathematical functions; its virtue is flexibility. Maple, used in this introductory course, is a good example of such a package.

Learning Outcomes

The aim of the course is to demonstrate the potential of general purpose mathematical packages; to allow students to gain familiarity with one of them (Maple); to provide a tool which can be used in the later years of the course.

By the end of Michaelmas term students should be able to

(i) edit, save, and use Maple worksheets;

(ii) manipulate expressions in Maple, and plot simple graphs using Maple;

(iii) write simple programs in Maple for solving problems in algebra, calculus, and applied mathematics.

By the end of Hilary term students should be able to

(i) use the **LinearAlgebra** package within Maple;

(ii) complete two or three small projects exploring some mathematical problems using Maple;

(iii) provide in a timely way reports on the projects in the form of commented Maple worksheets.

Synopsis

The Michaelmas term work consists of:

Using the workstation: accounts, passwords, logging in/out. Introduction to computer algebra systems: Maple; worksheets. Using Maple as a calculator. Manipulation of algebraic formulae. Sets, arrays, tables and lists. Solution of algebraic equations. Approximation. Linear Algebra in Maple. Calculus in Maple. Simple graphics. Elementary programming in Maple.

The Hilary term work is based on a menu of mathematical projects; the list will be printed in the second part of the Maple Course Manual.

Access to the system

Undergraduates use the workstations in the Statistics Department Computer Laboratory. For this you will need your university (Herald Webmail) account. Arrangements will be made to ensure that, as far as possible, you are allocated an account before MT lectures begin.

Students may also access the system through college or individual computers; for details of how to do this they should consult the computing support at their own college. A disk is provided for each college Senior Maths Tutor. The Maple package may also be installed and used on personally owned computers under the University's site licence. A disk with Maple for your own personal use will be made available to you through your college.

Teaching and Assessment

The course deliberately relies heavily on self-teaching through practical exercises. A manual for the course and examples to be worked through will be provided, with a variety of specimen worksheets. You will have access to help and advice from demonstrators.

You will be timetabled for 4 sessions of 2 hours each in the Statistics Department fortnightly in Michaelmas term. You will work alone on the projects in Hilary term, demonstrator sessions will be held daily for your assistance.

The Moderators in Mathematics are required, when assessing the performance of candidates, to take into account your work on the Maple course. For further information, see the section in the Undergraduate Handbook on examinations.

Plagiarism

The description of each HT project gives references to books covering the relevant mathematics; if you cannot understand some of this then you are free to consult your tutors or others about it, but not about the project itself. You may discuss with the demonstrators and others the techniques described in the Michaelmas Term Students' Guide, and the commands listed in the Hilary Term Students' Guide or found in the Maple Help pages. You may ask the Maple Coordinator to clarify any obscurity in the projects. The projects must be your own unaided work. You will be asked to make a declaration to that effect when you submit them. At the beginning of the year you will also be asked to sign a declaration stating that the work you will do and submit will be your own unaided work.

Reading

1. *Exploring Mathematics with Maple: Students' Guide*, (Mathematical Institute notes — available from reception).

2. A. Heck, *Introduction to Maple* (Springer, 3rd edition), ISBN 0-387-00230-8 (for college reference).

3. F. Wright, *Computing with Maple* (Chapman & Hall, 2002) (for college reference.)