Oxbridge PDE Conference 23 & 24 March 2014
Titles and Abstracts

Grégoire Allaire (L'Ecole Polytechnique)

Geometrical constraints in the level set method for shape and topology optimization

In the context of structural optimization via a level-set method we propose a framework to handle geometric constraints related to a notion of local thickness. The local thickness is calculated using the signed distance function to the shape. We formulate global constraints using integral functionals and compute their shape derivatives. We discuss different strategies and possible approximations to handle the geometric constraints. We implement our approach in two and three space dimensions for a model of linearized elasticity. As can be expected, the resulting optimized shapes are strongly dependent on the initial guesses and on the specific treatment of the constraints since, in particular, some topological changes may be prevented by those constraints.
This is a joint work with F. Jouve and G. Michailidis.

Yoshinori Morimoto (University of Kyoto)

Recent topics on the non-cutoff Boltzmann equation

The Boltzmann equation is an integro-differential equation whose integral kernel has the singularity in the collisional angle between two particles, in physically important models. It is known that many natural properties of the Boltzmann equation are preserved in spite of Grad's angular cutoff approximation to avoid this singularity. However, there is one important property lost after this cutoff approximation, that is, the collision integral operator without angular cutoff intrinsically behaves like the differential operator, and hence it has been expected that the Boltzmann equation enjoys the regularizing effect of solutions, such as the heat equation and the Kolmogorov equation, corresponding to the spatially homogeneous case and the inhomogeneous one, respectively. In this talk, the smoothing effect and the time asymptotic state of solutions to the Cauchy problem with the measure-valued initial data will be firstly considered for the spatially homogeneous Boltzmann equation of the Maxwellian molecule type, and, secondly, the global well-posedness under the perturbation frame work around the equilibrium will be discussed in spatially critical Besov spaces for the Boltzmann equation of hard potentials. The former part of this talk is based on joint works with T. Yang, S. Wang and H. Zhao, and the latter is with S. Sakamoto.

Jan Sbierski (University of Cambridge)

On the Existence of a Maximal Cauchy Development for the Einstein Equations - a Dezornification

In 1969, Choquet-Bruhat and Geroch showed that there exists a unique maximal Cauchy development of given initial data for the Einstein equations. Their proof, however, has the unsatisfactory feature that it relies crucially on the axiom of choice in the form of Zorn's lemma. In particular, their proof ensures the existence of the maximal development without actually constructing it.
In this talk, we present a proof of the existence of a maximal Cauchy development which avoids the use of Zorn's lemma and, moreover, provides an explicit construction of the maximal development.

Siran Li (University of Oxford)

A generalised Div-Curl Lemma and its application to isometric immersions of higher dimensional Riemannian Manifolds

In this talk we sketch a proof for a generalised version of the Div-Curl Lemma. As a central tool in the weak convergence methods of nonlinear PDEs, the classical Div-Curl Lemma guarantees the convergence in distribution of inner products of two weakly convergent sequences of vector fields in R3, one with compactly confined divergence and the other with compactly confined curl. We generalise this result to suitable differential forms on arbitrary n-dimensional Riemannian manifolds, based on the Hodge Theory for the Laplace-Beltrami operator. Notably, our proof is global and geometric in nature. Moreover, we apply our result to the analysis of the Gauss-Ricci-Codazzi Equations for n-dimensional manifolds, which has implications to the isometric immersion problem. This project is carried out under the supervision of, and in collaboration with, Prof. Gui-Qiang G. Chen.

Panu Lahti (Aalto University)

On some pointwise properties of functions of bounded variation on metric spaces

During the past decade, properties of functions of bounded variation, or BV functions, have been studied on metric measure spaces. The standard set of assumptions in this setting are that the space is complete, equipped with a doubling Radon measure, and supports a Poincaré inequality. In the Euclidean setting, it is known that in its jump set, a BV function "jumps" across hyperplanes from its lower approximate limit to its upper approximate limit. We show that on a metric measure space, an analogous result holds if hyperplanes are replaced by level sets of the function.

Jan Kristensen (University of Oxford)

The Morse-Sard theorem, generalized Luzin property and level sets for Sobolev functions

Many classical results from multivariate calculus can be generalized to suitable Sobolev functions that need not even be everywhere differentiable. In this talk we discuss some new results that have been obtained in joint work with Jean Bourgain (Princeton) and Mikhail Korobkov (Novosibirsk).

Amit Einav (University of Cambridge)
Between Functional Inequalities and Cercignani's Conjecture
One of the most influential equations in the kinetic theory of gases is the so-called Boltzmann equation, describing the time evolution of the probability density of a particle in dilute gas. While widely used, and intuitive, the Boltzmann equation has no formal validation from Newtonian laws, in macroscopic time scales.
In 1956 Marc Kac presented an attempt to solve this problem in particular settings of the spatially homogeneous Boltzmann equation. Kac considered a stochastic linear model of N indistinguishable particles, with one-dimensional velocities, that undergo a random binary collision process. Under the property of 'chaoticity' Kac managed to show that when one takes the number of particles to infinity, the limit of the first marginal of the N-particle distribution function satisfies a caricature of the Boltzmann equation, the so-called Boltzmann-Kac equation. Kac hoped that using this mean field approach will lead to new results in the convergence to equilibrium of the limit equation using the simpler, yet dimension dependent, linear ODE.
In our talk we will introduce Kac's model and discuss possible trends to equilibrium. We will then explore the connection between functional inequalities and the rate of convergence, and describe recent research in the field, in connection to the model.

Anton Mühlemann (University of Oxford)
From Calculus of Variations to Steel
Typical energy functionals in the modelling of martensitic phase transformations (Ball and James, 1987 and 1992) are not quasiconvex and thus do in general not attain their minimum. A remedy is to consider minimisers in the enlarged space of gradient Young measures (GYM). The GYM efficiently describes the main features of the microstructure such as the volume fractions and the macroscopic deformation gradient. This theory has been very successful in predicting the main features of interfaces between twinned martensite and austenite. Because of the relatively large strains involved steel seemed to be out of the scope of this theory. However, backed by some successful predictions, we propose that the same mechanics holds in this case and thereby provide an example where an interface between a second order laminate and the austenite seems preferable.
This is joint work with K. Koumatos.

Sara Merino Aceituno (University of Cambridge)
Anomalous energy transport in one-dimensional crystals One-dimensional solid crystals are modelled through a chain of oscillators called Fermi-Pasta-Ulam chain. When atoms in the crystal are assumed to interact through a quartic potential, we prove that the macroscopic equation describing energy transport corresponds to a fractional heat equation rather than the standard heat equation. The proof uses an intermediate step between the discrete chain model and the macroscopic one; the phonon-Boltzmann equation, which describes vibrations through the chain. (This is a joint work with Dr Mellet from the University of Maryland).

Angkana Rüland (University of Oxford)
Carleman estimates for the variable coefficient thin obstacle problem
In this talk I present a new strategy of addressing the (variable coefficient) thin obstacle problem. Using Carleman estimates as a central tool, I indicate how to obtain semi-continuity of the vanishing order, uniform upper growth bounds for solutions and sufficient compactness properties to carry out a blow-up procedure. Furthermore, along appropriate subsequences, the blow-up limits are homogeneous, which leads to an almost optimal regularity result. As this method is very robust, it is possible to work in the setting of $W^{1,p}$, $p>n+1$, regular coefficients.
This is based on joint work with Herbert Koch and Wenhui Shi.

Emanuele Spadaro (Max Planck Institute)
An epiperimetric inequality for the thin obstacle problem
We discuss some recent results about the thin obstacle problem consisting in minimizing the Dirichlet energy among functions with positive trace on a hyperplane. In particular, our focus is on the rate of converge of the solutions to the thin obstacle problem to their blowup limits near certain points of the free boundary via an "epiperimetric inequality" a la Reifenberg inspired by the pioneering work by G. Weiss on the classical obstacle problem.
This a joint work with M. Focardi of the University of Florence.

Clément Mouhot (University of Cambridge)
Commutating the mean-field and classical limits of quantum mechanics

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