Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part C 2009–10
for examination in 2010

Contents

1 Foreword

The synopses for Part C will be available in hard copy from reception in the Mathematical Institute in October 2009, and on the website at:

http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/

before the start of Michaelmas Term 2009.

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Assistant in the Mathematical Institute.

The list of courses here is provisional at this stage but it is unlikely to change. However we shall confirm the synopses in October. In the unlikely event that any course receives a very low registration we may offer this course as a reading course (this would include with some lectures but fewer classes).

Honour School of Mathematics
Extract from the Examination Conventions

Students staying on to take the four-year course will take the equivalent of 3 units from Part C in their fourth year. The equivalent of two units must be taken from the menu of "Mathematics Department Units" or offered as a dissertation on a mathematical topic. Up to one unit may be taken from the menu of "Other Units".

For the MMath in Mathematics in Trinity Term 2010 one of the classifications will be based on Part C alone. Each candidate will receive a numerical mark on each paper in each Part of the examination in the University standardised range 0-100, such that

- a First Class performance (on that paper) is indicated by a mark of 70 to 100;

- an Upper Second Class performance (on that paper) is indicated by a mark of 60 to 69;

- a Lower Second Class performance (on that paper) is indicated by a mark of 50 to 59

- a Third Class performance (on that paper) is indicated by a mark of 40 to 49;

- a Pass performance (on that paper) is indicated by a mark of 30 to 39;

- a Fail performance (on that paper) is indicated by a mark of 0 to 29.

The Examination Papers in Part C

Normally the number of questions on each paper is prescribed in the *Examination Regulations* or *Course Handbook* (including the *Lecture Synopses*).

Each whole unit paper in Part C will be of three hours duration and will consist of two sections; the first section covering the first half of the whole unit's syllabus (the sixteen hours lectured in MT), and the second section covering the second half of the whole unit's syllabus (the sixteen hours lectured in HT). Each section will consist of three questions,

each marked out of 25. Candidates may submit answers to as many questions as they wish: the best two from each section will count for the total mark for the paper.

Each half unit paper in Part C will be of one and a half hours duration and consist of three questions, each marked out of 25. Candidates may submit answers to as many questions as they wish: the best two answers will count for the total mark for the paper. These papers will cover either the syllabus of a course which has been designated as stand-alone half units, or in some cases the first or second half of the syllabus of a whole unit (in which case its questions will be identical to the corresponding section of the whole unit paper).

In all papers the questions set should give a reasonable coverage of the syllabus.

Marking of Papers in Part C

For the Mathematics Department papers, each question will initially be marked out of 25 and marking schemes for the questions will aim to ensure that the following qualitative criteria hold:

- 20-25 marks. A completely, or almost completely, correct answer, showing excellent understanding of the concepts and skill in carrying through the arguments and/or calculations.

- 13-19 marks. A good though not complete answer, showing understanding of the concepts and competence in handling the arguments and/or calculations. Such an answer might consist of an excellent answer to a substantial part of the question, or a good answer to the whole question which nevertheless shows some flaws in calculation or in understanding or in both.

Marking of Dissertations

Marks for other mathematical and non-mathematical papers will be reported to the Mathematics Examiners by the relevant assessing panel in University Standardised Mark (USM) form.

All dissertations are independently marked by at least two assessors. The examiner responsible for dissertations will oversee the reconciliation of marks. Discussion between all assessors should focus on any reasons for differing marks and thereby agree a reconciled mark. If agreement is not possible an additional assessor should be appointed.

Analysis of Marks in Part C

The Board of Examiners in Part C will assign USMs for whole-unit and half-unit papers taken in Part C and may recalibrate the raw marks to arrive at USMs reported to candidates. The whole-unit papers are designed so that the raw marks sum to 100, however, Examiners will take into account the relative difficulty of papers when assigning USMs. In order to achieve this, Examiners may use information on candidates' performances on the earlier Parts of the examination when recalibrating the raw marks. They may also use other

statistics to check that the USMs assigned fairly reflect the students' performances on a paper.

The USMs awarded to a candidate for papers in Part C will be used to arrive at a classification for Part C of the MMath.

AvUSM-Part C = Average USM in Part C (rounded up to whole number);

- First Class:
  $AvUSM - \text{Part C} \geq 70$

- Upper Second Class:
  $70 > AvUSM - \text{Part C} \geq 60$

- Lower Second Class:
  $60 > AvUSM - \text{Part C} \geq 50$

- Third Class:
  $50 > AvUSM - \text{Part C} \geq 40$

[Note: Half-unit papers count as half a paper when determining the average USM.]

Candidates leaving after four years who satisfy the Examiners will be awarded an MMath in Mathematics, with two associated classifications; for example: MMath in Mathematics: Years 2 and 3 together - First Class; Year 4 - First Class

A 'Pass' will not be awarded for Year 4. Candidates achieving:

$$AvUSM - \text{Part C} < 40,$$

may supplicate for a BA based on their performance in Parts A and B.

Class Descriptors

The average USM ranges used in the classifications reflect the following descriptions:

- First Class: the candidate shows excellent skills in reasoning, deductive logic and problem-solving. He/she demonstrates an excellent knowledge of the material, and is able to use that innovatively in unfamiliar contexts.

- Upper Second Class: the candidate shows good or very good skills in reasoning, deductive logic and problem-solving. He/she demonstrates a good or very good knowledge of much of the material.

- Lower Second Class: the candidate shows adequate basic skills in reasoning, deductive logic and problem-solving. He/she demonstrates a sound knowledge of much of the material.

- Third Class: the candidate shows reasonable understanding of at least part of the basic material and some skills in reasoning, deductive logic and problem-solving.

- Pass: the candidate shows some limited grasp of basic material demonstrated by the equivalent of an average of one meaningful attempt at a question on each unit of study. A stronger performance on some papers may compensate for a weaker performance on others.

- Fail: little evidence of competence in the topics examined; the work is likely to show major misunderstanding and confusion, coupled with inaccurate calculations; the answers to questions attempted are likely to be fragmentary only.

"Units" and "Half-Units" and methods of examination

Most Mathematics Department lecture courses are available as half-units, the exceptions being:

1. C7.4 Theoretical Physics - this is available as a whole-unit only.

2. C11.1b Probabilistic Combinatorics, where C11.1a Graph Theory is an essential prerequisite.

Half-units are examined in an examination paper of $1\frac{1}{2}$ hours, unless the synopsis states otherwise; whole-units will be examined in a 3 hour exam unless the synopsis states otherwise.

All Computer Science half-units will be examined by a paper of $1\frac{1}{2}$ hours in length.

All the units and half-units described in this booklet are "M–Level".

Registration

Classes

Students will have to register in advance for the classes they wish to take. Students will have to register by Friday of Week 9 of Trinity Term 2009 using the online system which can be accessed at http://www.maths.ox.ac.uk. Please note that students will need to login using OU Webauth. It will then be possible for students to select the units and half-units they wish to take from the drop-down menu. Further guidance on how to use the online system can be found at:
http://www.maths.ox.ac.uk/help/faqs/undergrads/course-registration.

In the case of unexpected excessive demand for intercollegiate class places for an option, the lecturer may apply for a quota to be imposed. Students should check their e-mail regularly over the summer, as those affected by the introduction of such a quota will be contacted by email, and presented with an opportunity to make a case for a place on a quota and to name an alternative option.

Note on Intercollegiate Classes

Where undergraduate registrations for lecture courses fall below 5, classes will not run as part of the intercollegiate scheme but will be arranged informally by the lecturer.

Lectures

Some combinations of subjects are not advised and lectures may clash. Details are given below. We will use the information on your registration forms to aim to keep clashes to a minimum. However, because of the large number of options available in Part C some clashes are inevitable, and we must aim to accommodate the maximum number of student preferences.
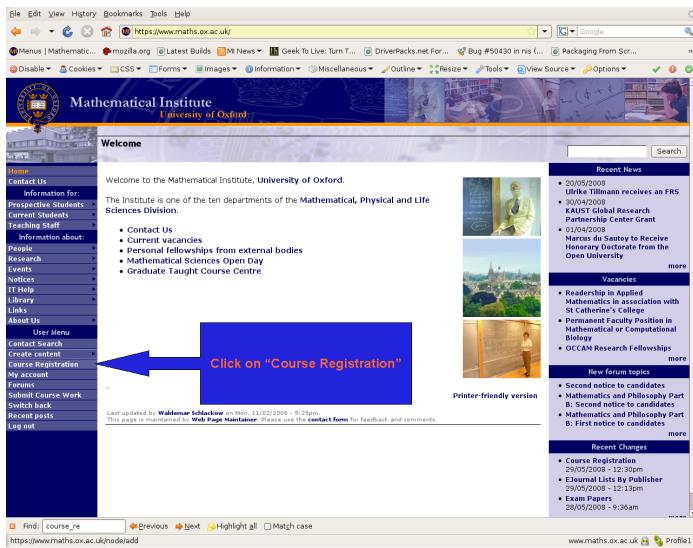
Figure 1: Logging on to the registration system.
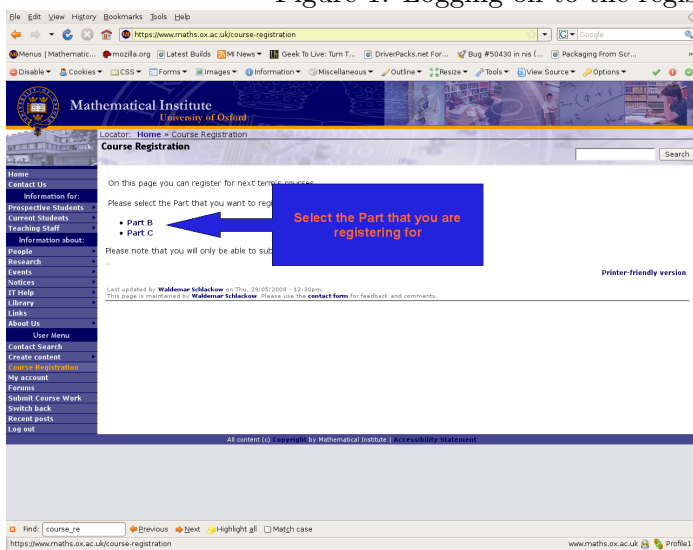
Figure 2: Select Part C.
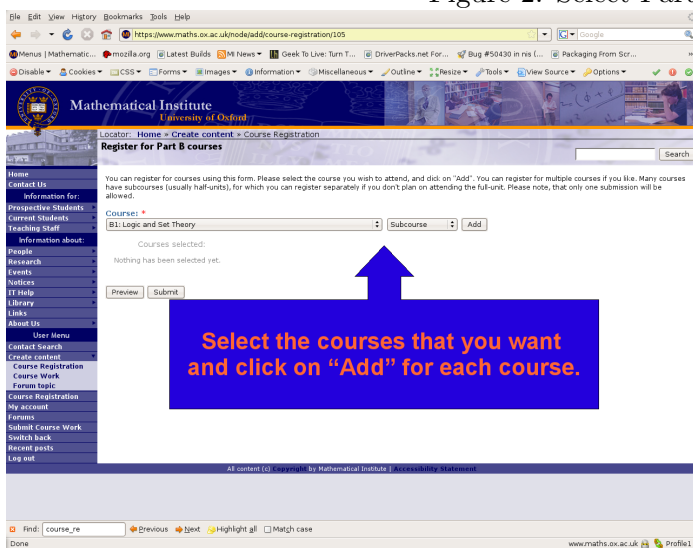
Figure 3: Select courses from the menu.

Lecture Timetabling in Part C, 2009–10

The Teaching Committee has agreed that the following clashes be allowed.

C1.1 Model Theory & Godel's Incompleteness Theorems	may clash with	C6.1 Solid Mechanics
C1.2 Analytic Topology Axiomatic Set Theory		C6.2 Elasticity and Plasticity
C2.1 Lie Algebras & Representation Theory of Symmetric Groups		C6.3 Perturbation Methods & Applied Complex Variables
C3.1 Topology and Groups & Algebraic Topology		C8.1 Mathematics and the Environment & Mathematical Physiology
C9.1 Analytic Number Theory & Elliptic Curves		All Statistics options
		C12.1 Numerical Linear Algebra and Continuous Optimization
		C12.2 Approximation Theory and Finite Element Methods

Mathematics vs. Computing		
C6.1 Solid Mechanics	may clash with	CCS1a Categories, Proofs and Processes
C6.2 Elasticity and Plasticity		
C6.3 Perturbation Methods and Applied Complex Variables		
C3.1 Lie Groups and Differentiable Manifolds		

Other and Non-mathematical subjects		
All Statistics options	may clash with	All Philosophy papers

2 Mathematics Department Units

C1.1: Gödel's Incompleteness Theorems and Model Theory

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2460), or can be taken as either a half-unit in C1.1a (OSS paper code 2A60) or a half-unit in C1.1b (OSS paper code 2B60).

C1.1a: Gödel's Incompleteness Theorems — Dr Isaacson — 16MT

Recommended Prerequisites

This course presupposes basic knowledge of first-order predicate logic up to and including soundness and completeness theorems for a formal system of first-order predicate logic (B1 Logic).

Overview

The starting point is Gödel's mathematical sharpening of Hilbert's insight that manipulating symbols and expressions of a formal language has the same formal character as arithmetical operations on natural numbers. This allows the construction for any consistent formal system containing basic arithmetic of a 'diagonal' sentence in the language of that system which is true but not provable in the system. By further study we are able to establish the intrinsic meaning of such a sentence. These techniques lead to a mathematical theory of formal provability which generalizes the earlier results. We end with results that further sharpen understanding of formal provability.

Learning Outcomes

Understanding of arithmetization of formal syntax and its use to establish incompleteness of formal systems; the meaning of undecidable diagonal sentences; a mathematical theory of formal provability; precise limits to formal provability and ways of knowing that an unprovable sentence is true.

Synopsis

Gödel numbering of a formal language; the diagonal lemma. Expressibility in a formal language. The arithmetical undefinability of truth in arithmetic. Formal systems of arithmetic; arithmetical proof predicates. Σ_0-completeness and Σ_1-completeness. The arithmetical hierarchy. ω-consistency and 1-consistency; the first Gödel incompleteness theorem. Separability; the Rosser incompleteness theorem. Adequacy conditions for a provability predicate. The second Gödel incompleteness theorem; Löb's theorem. Provable Σ_1-completeness. Provability logic; the fixed point theorem. The ω-rule. The Bernays arithmetized completeness theorem; formally undecidable Δ_2^0-sentences of arithmetic.

Reading

1. Raymond M. Smullyan, *Gödel's Incompleteness Theorems* (Oxford University Press, 1992).

Further Reading

1. George S. Boolos and Richard C. Jeffrey, *Computability and Logic* (3rd edition, Cambridge University Press, 1989), Chs 15, 16, 27 (pp 170–190, 268-284).

2. George Boolos, *The Logic of Provability* (Cambridge University Press, 1993), pp. 44-49.

C1.1b: Model Theory — Dr Koenigsmann — 16HT

Recommended Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems. Also a familiarity with (at least the statement of) the Compactness Theorem would also be desirable.

Overview

The course deepens a student's understanding of the notion of a mathematical structure and of the logical formalism that underlies every mathematical theory, taking B1 Logic as a starting point. Various examples emphasise the connection between logical notions and practical mathematics.

The concepts of completeness and categoricity will be studied and some more advanced technical notions, up to elements of modern stability theory, will be introduced.

Learning Outcomes

Students will have developed an in depth knowledge of the notion of an algebraic mathematical structure and of its logical theory, taking B1 Logic as a starting point. They will have an understanding of the concepts of completeness and categoricity and more advanced technical notions.

Synopsis

Structures. The first-order language for structures. The Compactness Theorem for first-order logic. Elementary embeddings. Löwenheim–Skolem theorems. Preservation theorems for substructures.

Categoricity for first-order theories. Types and saturation. Omitting types. The Ryll–Nardzewski theorem characterizing aleph-zero categorical theories. Theories with few types.

Reading

1. D. Marker, *Model Theory: An Introduction* (Springer, 2002).

2. W. Hodges, *Shorter Model Theory* (CUP, 1997).

3. J. Bridge, *Beginning Model Theory* (OUP, 1977). (Out of print but can be found in libraries.)

Further reading

1. All topics discussed (and much more) can also be found in W. Hodges, *Model Theory* (CUP, 1993).

C1.2: Analytic Topology and Axiomatic Set Theory

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2461), or can be taken as either a half-unit in C1.2a (OSS paper code 2A61) or a half-unit in C1.2b (OSS paper code 2B61).

C1.2a: Analytic Topology — Dr Suabedissen — 16MT

We find it necessary to run Analytic Topology in MT. Classes for Analytic Topology will be run both in MT and HT, so that students who find themselves overburdened in MT will have the opportunity to attend classes on Analytic Topology in HT.

Recommended Prerequisites

Part A Topology; a basic knowledge of Set Theory, including cardinal arithmetic, ordinals and the Axiom of Choice, will also be useful.

Overview

The aim of the course is to present a range of major theory and theorems, both important and elegant in themselves and with important applications within topology and to mathematics as a whole. Central to the course is the general theory of compactness and Tychonoff's theorem, one of the most important in all mathematics (with applications across mathematics and in mathematical logic) and computer science.

Synopsis

Bases and initial topologies (including pointwise convergence and the Tychonoff product topology). Separation axioms, continuous and semicontinuous functions, Urysohn's lemma. Separable, Lindelöf and second countable spaces. Urysohn's metrization theorem. Filters and ultrafilters. Tychonoff's theorem. Compactifications, in particular the Alexandroff One-Point Compactification and the Stone–Čech Compactification. Connectedness and local connectedness. Components and quasi-components. Totally disconnected compact spaces, Boolean algebras and Stone spaces. Paracompactness (brief treatment).

Reading

1. S. Willard, *General Topology* (Addison–Wesley, 1970), Chs. 1–8.

2. N. Bourbaki, *General Topology* (Springer–Verlag, 1989), Ch. 1.

C1.2b: Axiomatic Set Theory — Prof. Zilber — 16HT

Recommended Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems, together with a course on basic set theory, including cardinals and ordinals, the Axiom of Choice and the Well Ordering Principle.

Overview

Inner models and consistency proofs lie at the heart of modern Set Theory, historically as well as in terms of importance. In this course we shall introduce the first and most important of inner models, Gödel's constructible universe, and use it to derive some fundamental consistency results.

Synopsis

A review of the axioms of ZF set theory. The recursion theorem for the set of natural numbers and for the class of ordinals. The Cumulative Hierarchy of sets and the consistency of the Axiom of Foundation as an example of the method of inner models. Levy's Reflection Principle. Gödel's inner model of constructible sets and the consistency of the Axiom of Constructibility ($V = L$). The fact that $V = L$ implies the Axiom of Choice. Some advanced cardinal arithmetic. The fact that $V = L$ implies the Generalized Continuum Hypothesis.

Reading

For the review of ZF set theory:

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).

For course topics (and much more):

1. K. Kunen, *Set Theory: An Introduction to Independence Proofs* (North Holland, 1983) (now in paperback). Review: Chapter 1. Course topics: Chapters 3, 4, 5, 6 (excluding section 5).

Further Reading

1. K. Hrbacek and T. Jech, *Introduction to Set Theory* (3rd edition, M Dekker, 1999).

C2.1: Lie Algebras and Representation Theory of Symmetric Groups

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2462), or can be taken as either a half-unit in C2.1a (OSS paper code 2A62) or a half-unit in C2.1b (OSS paper code 2B62).

C2.1a: Lie Algebras — Prof. J. Wilson — 16MT

Recommended Prerequisites

Part B course B2a. A thorough knowledge of linear algebra and the second year algebra courses; in particular familiarity with group actions, quotient rings and vector spaces, with isomorphism theorems and with inner product spaces will be assumed. Some familiarity with the Jordan–Hölder theorem and the general ideas of representation theory will be an advantage.

Overview

Lie Algebras are mathematical objects which, besides being of interest in their own right, elucidate problems in several areas in mathematics. The classification of the finite-dimensional complex Lie algebras is a beautiful piece of applied linear algebra. The aims of this course are to introduce Lie algebras, develop some of the techniques for studying them, and describe parts of the classification mentioned above, especially the parts concerning root systems and Dynkin diagrams.

Learning Outcomes

Students will learn how to utilise various techniques for working with Lie algebras, and they will gain an understanding of parts of a major classification result.

Synopsis

Definition of Lie algebras, small-dimensional examples, some classical groups and their Lie algebras (treated informally). Ideals, subalgebras, homomorphisms, modules.

Nilpotent algebras, Engel's theorem; soluble algebras, Lie's theorem. Semisimple algebras and Killing form, Cartan's criteria for solubility and semisimplicity.

The root space decomposition of a Lie algebra; root systems, Cartan matrices and Dynkin diagrams. Classification of irreducible root systems. Description (with few proofs) of the classification of complex simple Lie algebras; examples.

Reading

1. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory,* Graduate Texts in Mathematics 9 (Springer–Verlag, 1972, reprinted 1997). Chapters 1–3 are relevant and part of the course will follow Chapter 3 closely.

2. B. Hall, *Lie Groups, Lie Algebras, and Representations. An Elementary Introduction,* Graduate Texts in Mathematics 222 (Springer–Verlag, 2003).

3. K. Erdmann, M. J. Wildon, *Introduction to Lie Algebras* (Springer–Verlag, 2006), ISBN: 1846280400.

Additional Reading

1. J.-P. Serre, *Complex Semisimple Lie Algebras* (Springer, 1987). Rather condensed, assumes the basic results. Very elegant proofs.

2. N. Bourbaki, *Lie Algebras and Lie Groups* (Masson, 1982). Chapters 1 and 4–6 are relevant; this text fills in some of the gaps in Serre's text.

C2.1b — Representation Theory of Symmetric Groups — Dr Erdmann — 16HT

Recommended Prerequisites

A thorough knowledge of linear algebra and the second year algebra courses; in particular familiarity with the symmetric groups, (symmetric) group actions, quotient vector spaces, isomorphism theorems and inner product spaces will be assumed. Some familiarity with basic representation theory from B2 (group algebras, simple modules, reducibility, Maschke's theorem, Wedderburn's theorem, characters) will be an advantage.

Overview

The representation theory of symmetric groups is a special case of the representation theory of finite groups. Whilst the theory over characteristic zero is well understood, this is not so over fields of prime characteristic. The course will be algebraic and combinatorial in flavour, and it will follow the approach taken by G. James. One main aim is to construct and parametrise the simple modules of the symmetric groups over an arbitrary field. Combinatorial highlights include combinatorial algorithms such as the Robinson-Schensted-Knuth correspondence. The final part of the course will discuss some finite-dimensional representations of the general linear group $GL_n(C)$, and connections with representations of symmetric groups. In particular we introduce tensor products, and symmetric and exterior powers.

Synopsis

Counting standard tableaux of fixed shape: Young diagrams and tableaux, standard-tableaux, Young-Frobenius formula, hook formula. Robinson-Schensted-Knuth algorithm and correspondence.

Construction of fundamental modules for symmetric groups: Action of symmetric groups on tableaux, tabloids and polytabloids; permutation modules on cosets of Young subgroups. Specht modules, and their standard bases. Examples and applications.

Simplicity of Specht modules in characteristic zero and classification of simple S_n-module over characteristic zero. Characters of symmetric groups, Murnaghan-Nakayama rule.

Submodule Theorem, construction of simple S_n-modules over a field of prime characteristic. Decomposition matrices. Examples and applications.

Some finite-dimensional $GL_n(C)$-modules, in particular the natural module, its tensor powers, and its symmetric and exterior powers. Connections with representations of S_n over C.

Reading

1. W. Fulton, *Young Tableaux,* London Mathematical Society Student Texts 35 (Cambridge University Press, 1997). From Part I and II.

2. D. Knuth, *The Art of Computer Programming, Volume 3* (Addison–Wesley, 1998). From Chapter 5.

3. B. E. Sagan, *The Symmetric Group: Representations, Combinatorial Algorithms, and Symmetric Functions,* Graduate Texts in Mathematics 203 (Springer–Verlag, 2000). Chapters 1 – 2.

Additional Reading

1. W. Fulton, J. Harris, *Representation Theory: A first course,* Graduate Texts in Mathematics, Readings in Mathematics 129 (Springer Verlag, 1991). From Part I.

2. G. James, *The Representation Theory of the Symmetric Groups,* Lecture Notes in Mathematics 682 (Springer Verlag, 1978).

3. G. James, A. Kerber, *The Representation Theory of the Symmetric Groups*, Encyclopaedia of Mathematics and its Applications 16, (Addison–Wesley, 1981). From Chapter 7.

4. R. Stanley, *Enumerative Combinatorics. Volume 2*, Cambridge Studies in Advanced Mathematics 62 (Cambridge University Press, 1999).

C2.2b: Representation Theory of Semisimple Lie Algebras — Dr Kremnizer — 16HT

Level: M-level   **Method of Assesment:** Written examination.

Weight: Half-unit. OSS paper code to follow.

Recommended Prerequisites

Course C2.1a. Some familiarity with Category Theory and basic homological algebra will be an advantage.

Overview

Representation theory of semisimple Lie algebras form a basic part of representation theory and has many important applications in mathematics and physics. The structures involved can be generalized to quantum groups and Kac-Moody Lie algebras. The aims of this course are to introduce techniques for understanding finite and infinite dimensional representations, and to understand the structure of the enveloping algebra using representation theory.

Synopsis

Review of Semisimple Lie Algebras, simple modules for sl_2, Highest weight modules, Verma modules and simple modules, finite dimensional modules, Harish-Chandra's theorem, Characters of finite dimensional modules. If time permits we will also discuss some topics in category O.

Reading List

1. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Graduate Texts in Mathematics 9 (Springer-Verlag, 1972, reprinted 1997).

2. J. E. Humphreys, *Representations of Semisimple Lie Algebras in the BGG Category O*, Graduate Studies in Mathematics 94 (American Mathematical Society, 2008).

3. R. W. Carter, *Lie algebras of finite and affine type* (Cambridge University Press, 2005).

Additional Reading

1. D. Gaitsgory, *Geometric representation theory*, lecture notes (Harvard University, 2005).

C3.1a: Topology and Groups — Dr Coward — 16MT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Half-unit, OSS paper code 2A63.

Only available to students who have not offered C3.1a Topology and Groups at Part B. From 2010/11 this course will be offered at Part B only.

Prerequisites

2nd year Groups in Action, 2nd year Topology.

Overview

This course introduces the important link between topology and group theory. On the one hand, associated to each space, there is a group, known as its fundamental group. This can be used to solve topological problems using algebraic methods. On the other hand, many results about groups are best proved and understood using topology. For example, presentations of groups, where the group is defined using generators and relations, have a topological interpretation. The endpoint of the course is the Nielsen-Shreier Theorem, an important, purely algebraic result, which is proved using topological techniques.

Synopsis

Homotopic mappings, homotopy equivalence. Simplicial complexes. Simplicial approximation theorem.

The fundamental group of a space. The fundamental group of a circle. Application: the fundamental theorem of algebra. The fundamental groups of spheres.

Free groups. Existence and uniqueness of reduced representatives of group elements. The fundamental group of a graph.

Groups defined by generators and relations (with examples). Tietze transformations.

The free product of two groups. Amalgamated free products.

The Seifert van Kampen Theorem.

Cell complexes. The fundamental group of a cell complex (with examples). The realization of any finitely presented group as the fundamental group of a finite cell complex.

Covering spaces. Liftings of paths and homotopies. A covering map induces an injection between fundamental groups. The use of covering spaces to determine fundamental groups: the circle again, and real projective n-space. The correspondence between covering spaces and subgroups of the fundamental group. Regular covering spaces and normal subgroups.

Cayley graphs of a group. The relationship between the universal cover of a cell complex, and the Cayley graph of its fundamental group. The Cayley 2-complex of a group.

The Nielsen-Schreier Theorem (every subgroup of a finitely generated free group is free) proved using covering spaces.

Reading

1. John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

Additional Reading

1. D. Cohen, *Combinatorial Group Theory: A Topological Approach,* Student Texts 14 (London Mathematical Society, 1989), Chs 1-7.

2. A. Hatcher, *Algebraic Topology* (CUP, 2001), Ch. 1.

3. M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chs. 1-7, 12, 17 .

4. D. L. Johnson, *Presentations of Groups,* Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chs. 1-5, 10,13.

5. W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chs. 1-4.

C3.2: Lie Groups and Differentiable Manifolds

Level: M-level.   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper codes to follow), or can be taken either as a half-unit in Lie Groups (OSS paper codes to follow) or a half-unit in Differentiable Manifolds (OSS paper codes to follow).

C3.2a: Lie Groups — Prof. Tillmann — 16MT

Recommended Prerequisites

2nd year Groups in Action, Topology, Multivariable Calculus.

Overview

The theory of Lie Groups is one of the most beautiful developments of pure mathematics in the twentieth century, with many applications to geometry, theoretical physics and mechanics, and links to both algebra and analysis. Lie groups are groups which are simultaneously manifolds, so that the notion of differentiability makes sense, and the group multiplication and inverse maps are differentiable. However this course introduces the theory in a more concrete way via groups of matrices, in order to minimise the prerequisites.

Learning Outcomes

Students will have learnt the basic theory of topological matrix groups and their representations. This will include a firm understanding of root systems and their role for representations.

Synopsis

The exponential map for matrices, Ad and ad, the Campbell–Baker–Hausdorff series.

Linear Groups, their Lie algebras and the Lie correspondence. Homomorphisms and coverings of linear groups. Examples including $SU(2)$, $SO(3)$ and $SL(2;\mathbb{R}) \cong SU(1,1)$.

The compact and complex classical Lie groups. Cartan subgroups, Weyl groups, weights, roots, reflections.

Informal discussion of Lie groups as manifolds with differentiable group structures; quotients of Lie groups by closed subgroups.

Bi-invariant integration on a compact group (statement of existence and basic properties only). Representations of compact Lie groups. Tensor products of representations. Complete reducibility, Schur's lemma. Characters, orthogonality relations.

Statements of Weyl's character formula, the theorem of the highest weight and the Borel–Weil theorem, with proofs for $SU(2)$ only.

Reading

1. W. Rossmann, *Lie Groups: An Introduction through Linear Groups*, (Oxford, 2002), Chapters 1–3 and 6.

2. A. Baker, *Matrix Groups: An Introduction to Lie Group Theory*, (Springer Undergraduate Mathematics Series).

Further Reading

1. J. F. Adams, *Lectures on Lie Groups* (University of Chicago Press, 1982).

2. R. Carter, G. Segal and I. MacDonald, *Lectures on Lie Groups and Lie Algebras* (LMS Student Texts, Cambridge, 1995).

3. J. F. Price, *Lie Groups and Compact Groups* (LMS Lecture Notes 25, Cambridge, 1977).

C3.2b: Differentiable Manifolds — Prof. Hitchin — 16HT

Recommended Prerequisites

2nd year core algebra, topology, multivariate calculus. Useful but not essential: groups in action, geometry of surfaces.

Overview

A manifold is a space such that small pieces of it look like small pieces of Euclidean space. Thus a smooth surface, the topic of the B3 course, is an example of a (2-dimensional) manifold.

Manifolds are the natural setting for parts of classical applied mathematics such as mechanics, as well as general relativity. They are also central to areas of pure mathematics such as topology and certain aspects of analysis.

In this course we introduce the tools needed to do analysis on manifolds. We prove a very general form of Stokes' Theorem which includes as special cases the classical theorems of Gauss, Green and Stokes. We also introduce the theory of de Rham cohomology, which is central to many arguments in topology.

Learning Outcomes

The candidate will be able to manipulate with ease the basic operations on tangent vectors, differential forms and tensors both in a local coordinate description and a global coordinate-free one; have a knowledge of the basic theorems of de Rham cohomology and some simple examples of their use; know what a Riemannian manifold is and what geodesics and harmonic forms are.

Synopsis

Smooth manifolds and smooth maps. Tangent vectors, the tangent bundle, induced maps. Vector fields and flows, the Lie bracket and Lie derivative.

Exterior algebra, differential forms, exterior derivative, Cartan formula in terms of Lie derivative. Orientability. Partitions of unity, integration on oriented manifolds.

Stokes' theorem. De Rham chomology and discussion of de Rham's theorem. Applications of de Rham theory including degree.

Riemannian metrics.

Reading

1. M. Spivak, *Calculus on Manifolds*, (W. A. Benjamin, 1965).

2. M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Vol. 1, (1970).

3. W. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, (Academic Press, 1986).

4. M. Berger and B. Gostiaux, *Differential Geometry: Manifolds, Curves and Surfaces.* Translated from the French by S. Levy, (Springer Graduate Texts in Mathematics, 115, Springer–Verlag (1988)) Chapters 0–3, 5–7.

5. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, (Springer Graduate Texts in Mathematics, 1994).

6. D. Barden and C. Thomas, *An Introduction to Differential Manifolds.* (Imperial College Press, London, 2003.)

C3.3 Abelian, Nilpotent and Solvable Groups and Groups, Trees, and Hyperbolic Spaces

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code to follow), or can be taken as either a half-unit in C3.2a (OSS paper code to follow) or a half-unit in C3.2b (OSS paper code to follow).

C3.3a Abelian, Nilpotent and Solvable Groups — Dr Drutu — 16MT

Recommended Prerequisites

Second year Groups in Action.

Overview

Abelian groups are some of the easiest to understand and most frequently met groups. They have as natural generalizations nilpotent, polycyclic and solvable groups, which generalizations are used in many different areas of algebra, geometry and topology. The course begins with the classification of (infinite) finitely generated Abelian groups. It continues with some of the main properties and results on nilpotent and solvable groups, illustrated with many examples, especially of linear groups. After introducing the growth function for infinite finitely generated groups and proving that nilpotent groups have polynomial growth, the course ends with the Milnor-Wolf Theorem. The latter theorem formulates a striking dichotomy in the class of solvable groups: such groups are either virtually nilpotent or of exponential growth.

Learning Outcomes

Students will learn fundamentals about the various subclasses of solvable groups, about the algebraic and geometric features that they have in common, as well as about those that help to differentiate them.

Synopsis

Finitely generated groups, Cayley graphs.

Abelian groups and their classification.

Nilpotent groups, polycyclic groups, solvable groups; semidirect products, short exact sequences. Examples and properties of nilpotent and solvable groups.

Polynomial growth of nilpotent groups;

The Milnor-Wolf Theorem (solvable groups are either virtually nilpotent or have exponential growth).

Reading

1. M.I. Kargapolov, Ju. I. Merzljakov, *Fundamentals of the Theory of Groups* (Springer, New York, 1979). Chapters 6,7.

2. E. Vinberg, *A Course in Algebra* (American Mathematical Society, 2003). Chapters 10,11.

Additional Reading

1. G. Baumslag, *Lecture Notes on Nilpotent Groups* (American Mathematical Society, 1971).

2. G. Smith, O. Tabachnikova, *Topics in Group Theory* (Springer, 2002). Chapter 5.

3. Philip Hall, *Nilpotent Groups : notes of lectures given at the Canadian Mathematical Congress summer seminar, University of Alberta, 12-30 August, 1957* (Queen Mary College mathematics notes, 1969).

C3.3b: Groups, Trees and Hyperbolic Spaces — Dr Papazoglou — 16HT

Recommended Prerequisites

The Topology & Groups course C3.1a is a helpful prerequisite.

Overview

The aim of this course is to introduce the fundamental questions and techniques of combinatorial group theory and discuss their relationship to topology and geometry. One studies infinite groups via their action on 'nice' spaces. We will discuss in detail the Bass-Serre theory of group actions on trees and the basic notions of amalgams, HNN extensions and graphs of groups. As an application we will prove Kurosh's subgroup theorem and Stallings end theorem. Depending on time we plan to discuss also some of the following topics of geometric group theory: Quasi-sometries and Milnor-Svarc lemma. Hyperbolic groups. Isoperimetric inequalities and small cancellation groups.

Synopsis

Free groups. Group actions on Trees, Amalgams, HNN extensions. Classification of groups acting on trees, graphs of groups. Kurosh's theorem. Cayley graphs. Stallings ends theorem. If time allows: Quasi-isometries, Milnor-Svarc lemma Hyperbolic groups, Dehn functions, solution of the word and conjugacy problem for hyperbolic groups. Small cancellation groups.

Reading

1. Jean Pierre Serre, *Trees* (Springer, 2003).

2. Oleg Bogopolski, *Introduction to Group Theory* (EMS, 2008).

3. G. Baumslag, *Topics in Combinatorial Group Theory* (Birkhauser, 1993).

4. C.F. Miller, *Combinatorial Group Theory*, notes:
   http://www.ms.unimelb.edu.au/ cfm/notes/cgt-notes.pdf.

5. M. Bridson, A. Haefliger, *Metric Spaces of Non-positive Curvature, Part III* (Springer, 1999).

6. R. Lyndon, P. Schupp, *Combinatorial Group Theory* (Springer, 2001).

Additional Reading

1. W. Magnus, A. Karass, D. Solitar,*Combinatorial Group Theory: Presentations of Groups in Terms of Generators and Relations* (Dover Publications, 2004).

2. H. Short *et al.*, 'Notes on word hyperbolic groups', *Group Theory from a Geometrical Viewpoint, Proc. ICTP Trieste* (eds E. Ghys, A. Haefliger, A. Verjovsky, World Scientific 1990)

3. M. Kapovich, Lectures on Geometric Group Theory, notes: http://www.math.ucdavis.edu/ kapovich/eprints.html

C4.1: Functional Analysis, Banach and C*- algebras

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2464), or can be taken as either a half-unit in C4.1a (OSS paper code 2A64) or a half-unit in C4.1b (OSS paper code 2B64).

C4.1a: Functional Analysis — Prof. Haydon — 16MT

Recommended Prerequisites

Part A Topology, B4 Analysis

Overview

This course builds on B4, by extending the theory of Banach spaces and operators. As well as developing general methods that are useful in Operator Theory, we shall look in more detail at the structure and special properties of "classical" sequence-spaces and function-spaces.

Synopsis

Normed spaces and Banach spaces; dual spaces, subspaces, quotient spaces, direct sums and completions. Classical Banach spaces and their duals.

Compact sets and compact operators. Ascoli's theorem.

Hahn-Banach extension and separation theorems; the bidual space and reflexivity.

Weak and weak* topologies. The Banach-Alaoglu theorem and Goldstine's theorem. Weak compactness.

Baire's Category Theorem, Uniform Boundedness Principle, Open-Mapping Theorem, Closed-Graph Theorem.

Schauder bases; examples in classical spaces. Gliding-hump arguments.

Fredholm operators.

Reading

1. M. Fabian et al., *Functional Analysis and Infinite-Dimensional Geometry* (Canadian Math. Soc, Springer 2001), Chapters 1,2,3,6,7.

Alternative Reading

1. N. L. Carothers, *A Short Course on Banach Space Theory*, (LMS Student Text, CUP 2004).

C4.1b: Banach and C*- algebras — Dr Edwards — 16HT

Recommended Prerequisites

B4 Analysis.

Overview

The suggestion that the observables of quantum mechanics should be represented by linear operators on Hilbert spaces led Murray and von Neumann into a detailed study of algebras of bounded linear operators on Hilbert space. The abstract version of such algebras are known as C*-algebras and investigations into their structure continue today. Their importance now extends beyond functional analysis and physics into geometry and number theory.

Learning Outcome

The outcome of this course should be the appreciation and understanding of the main problems in Banach algebras and C*- algebras. Details are included in the synopsis below.

Synopsis

The course begins with an introduction to the general theory of Banach algebras and C*-algebras culminating in the Gelfand Representation Theorem for commutative C*-algebras. The second part of the course concentrates on the relationship that exists between the algebraic and geometric properties of non-commutative C*-algebras. The final part concerns itself with representations of C*-algebras on Hilbert spaces culminating with the Gelfand–Naimark Theorem.

Reading

1. G.I. Murphy, *C*-algebras and Operator Theory* (Academic, 1990), Chs. 2, 3 and parts of Ch. 5.

Alternative (more advanced) sources:

1. G.K. Pedersen, *C*-algebras and their Automorphism Groups* (Academic, 1979).

2. S. Sakai, *C*-algebras and W*-algebra* (Springer, 1971).

C5.1 Methods of Applied Functional Analysis

Level: M-level   **Method of assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2465), or C5.1a (OSS paper code 2A65) or C5.1b (OSS paper code 2B75) can be take as a half-unit (but see prerequisites for C5.1b).

C5.1a Methods of Functional Analysis for PDEs — Dr Kristensen — 16MT

Only available to students who have not offered C5.1a Methods of Functional Analysis for PDEs at Part B.

From 2010/11 this course will be offered at Part B only.

Recommended Prerequisites

Part A Integration. However, what is required is an ability to use integration techniques giving proper justification. There will be a 'Users' Guide to Integration' on the subject website so that anyone who has not done Part A Integration can read it up over the summer.

Overview

The course will introduce some of the modern techniques in partial differential equations that are central to the theoretical and numerical treatments of partial differential equations arising in science, geometry and other fields.

This course is a prerequisite for later Part C courses in Calculus of Variations and Fixed Point Methods for Nonlinear PDEs.

Learning Outcomes

Students will learn techniques and results, such as Sobolev spaces, weak convergence, weak solutions, the direct method of calculus of variations, embedding theorems, the Lax-Milgram Theorem, the Fredholm Alternative and the Hilbert-Schmidt Theorem and how to apply these to obtain existence and uniqueness results for elliptic partial differential equations.

Synopsis

Part I Function Spaces:

Why are function spaces important for partial differential equations?

Definition of Banach spaces, separability and dual spaces. Definition of Hilbert space. The spaces $L^p(\Omega)$, $1 \leq p \leq \infty$, where $\Omega \subset \mathbb{R}^n$ is open. Minkowski and Hölder inequalities. Statement that $L^p(\Omega)$ is a Banach space, and that the dual of L^p is $L^{p'}$, for $1 \leq p < \infty$. Statement that L^2 is a Hilbert space.

Weak and weak* convergence in L^p spaces. Examples. A bounded sequence in the dual of a separable Banach space has a weak* convergent subsequence.

Mollifiers and the density of smooth functions in L^p for $1 \leq p < \infty$.

Definition of weak derivatives and their uniqueness. Definition of Sobolev space $W^{m,p}(\Omega)$, $1 \leq p \leq \infty$. $H^m(\Omega) = W^{m,2}(\Omega)$. Definition of $W_0^{1,p}(\Omega)$, $1 \leq p < \infty$. Brief introduction to distributions.

Part II Elliptic Problems:

The direct method of calculus of variations. The Poincaré inequality. Proof of the existence and uniqueness of a weak solution to Poisson's equation $-\Delta u = f$, with zero Dirichlet boundary conditions and $f \in L^2(\Omega)$, with Ω bounded. Discussion of regularity of solutions.

The Lax-Milgram Theorem and Gårding's inequality. Existence and uniqueness of weak solutions to general linear uniformly elliptic equations.

Embedding theorems (proofs omitted except $W^{1,1}(a,b) \hookrightarrow C[a,b]$).

Compact operators and self adjoint operators. Fredholm Alternative and Hilbert-Schmidt Theorem. Examples including $-\Delta$ with zero Dirichlet boundary conditions.

Reading

1. Lawrence C. Evans, *Partial Differential Equations,* Graduate Studies in Mathematics (American Mathematical Society, 2004).

2. M. Renardy and R.C. Rogers, *An Introduction to Partial Differential Equations* (Springer-Verlag, New York, 2004).

Additional Reading

1. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989).

2. J. Rauch, *Partial Differential Equations* (Springer-Verlag, New York, 1992).

C5.1b Fixed Point Methods for Nonlinear PDEs — Prof. Niethammer — 16HT

Recommended Prerequisites

C5.1a: Methods of Functional Analysis for PDEs. Some knowledge of functional analysis, in particular Banach spaces (as in B4) and compactness (as in Part A Topology), is useful.

Overview

This course gives an introduction to the techniques of nonlinear functional analysis with emphasis on the major fixed point theorems and their applications to nonlinear differential equations and variational inequalities, which abound in applications such as fluid and solid mechanics, population dynamics and geometry.

Learning Outcomes

Besides becoming acquainted with the fixed point theorems of Banach, Brouwer and Schauder, students will see the abstract principles in a concrete context. Hereby they also reinforce techniques from elementary topology, functional analysis, Banach spaces, compactness methods, calculus of variations and Sobolev spaces.

Synopsis

Examples of nonlinear differential equations and variational inequalities. Banach's fixed point theorem and applications. Brouwer's fixed point theorem, proof via Calculus of Variations and Null-Lagrangians. Compact operators and Schauder's fixed point theorem. Applications of Schauder's fixed point theorem to nonlinear elliptic equations. Weak convergence in Lebesgue- and Sobolev spaces; Variational inequalities and monotone operators. Applications of monotone operator theory to nonlinear elliptic equations (p-Laplace, stationary Navier-Stokes)

Further Reading

1. E. Zeidler, *Nonlinear Functional Analysis I + II* (SpringerVerlag, 1986/89).

2. M. S. Berger, *Nonlinearity and Functional Analysis* (Academic Press, 1977).

3. K. Deimling, *Nonlinear Functional Analysis* (SpringerVerlag, 1985).

4. Lawrence C. Evans, *Partial Differential Equations,* Graduate Studies in Mathematics (American Mathematical Society, 2004).

5. L. Nirenberg, *Topics in Nonlinear Functional Analysis,* Courant Institute Lecture Notes (American Mathematical Society, 2001).

6. R.E. Showalter, *Monotone Operators in Banach Spaces and Nonlinear Partial Differential Equations,* Mathematical Surveys and Monographs, vol.49 (American Mathematical Society, 1997).

C5.2b: Calculus of Variations — Prof. Seregin — 16HT

Weight: Half-unit, OSS paper code to follow.

Recommended Prerequisites

C5.1a: Methods of Functional Analysis for PDEs. Some familiarity with the Lebesgue integral is essential, and some knowledge of elementary functional analysis (e.g. Banach spaces and their duals, weak convergence) an advantage.

Overview

The aim of the course is to give a modern treatment of the calculus of variations from a rigorous perspective, blending classical and modern approaches and applications.

Learning Outcomes

Students will learn rigorous results in the classical and modern one-dimensional calculus of variations and see possible behaviour and application of these results in examples. They will see some examples of multi-dimensional problems.

Synopsis

Classical and modern examples of variational problems (e.g. brachistochrone, models of phase transformations).

One-dimensional problems, function spaces and definitions of weak and strong relative minimizers. Necessary conditions; the Euler–Lagrange and Du Bois–Reymond equations, theory of the second variation, the Weierstrass condition. Sufficient conditions; field theory and sufficiency theorems for weak and strong relative minimizers. The direct method of the calculus of variations and Tonelli's existence theorem. Regularity of minimizers. Examples of singular minimizers and the Lavrentiev phenomenon. Problems whose infimum is not attained. Relaxation and generalized solutions. Isoperimetric problems and Lagrange multipliers.

Multi-dimensional problems, done via some examples.

Reading

1. G. Buttazzo, M. Giaquinta, S. Hildebrandt, *One-dimensional Variational Problems,* Oxford Lecture Series in Mathematics, Vol. 15 (OUP, 1998). Ch 1, Sections 1.1, 1.2 (treated differently in course), 1.3, Ch 2 (background), Ch 3, Sections 3.1, 3.2, Ch 4, Sections 4.1, 4.3.

Additional Reading

1. U. Brechtken-Manderscheid, *Introduction to the Calculus of Variations* (Chapman & Hall, 1991).

2. H. Sagan, *Introduction to the Calculus of Variations* (Dover, 1992).

3. J. Troutman, *Variational Calculus and Optimal Control* (Springer–Verlag, 1995).

C6.1a: Solid Mechanics — Dr Ortner — 16 MT

Level: M-level   **Method of Assessment:** Written examination.

Weight: Half-unit. OSS paper code 2466.

Recommended Prerequisites Second Year Mathematics core courses. It is not necessary to have taken any courses in fluid mechanics, though having done so provides some background in the use of similar concepts. Use is made of some elementary linear algebra in $\mathbb{R}^3$ (for example, eigenvalues, eigenvectors and diagonalization of symmetric matrices), and revision of this material, for example from the Mods Linear Algebra course, is useful preparation. The necessary material is summarized in the course.

The first module of 6 lectures is useful preparation for C6.2b Elasticity and Plasticity. Taken together the two courses will provide a broad overview of modern solid mechanics, with a variety of approaches.

Overview

Solid mechanics is a vital ingredient of materials science and engineering, and is playing an increasing role in biology. It has a rich mathematical structure. The aim of the course is to derive the basic equations of elasticity theory, the central model of solid mechanics, and give some interesting applications to the behaviour of materials.

Learning Outcomes

Students will learn basic techniques of modern continuum mechanics, such as kinematics of deformation, stress, constitutive equations and the relation between nonlinear and linearized models. They will also gain an insight into some recent developments in applications of mathematics to a variety of different materials.

Synopsis

(1) Nonlinear and linear elasticity (6 lectures)

Lagrangian and Eulerian descriptions of motion, analysis of strain. Balance laws of continuum mechanics. Frame-indifference. Cauchy and Piola-Kirchhoff stress. Constitutive equations for a nonlinear elastic material. Material symmetry, isotropy. Linear elasticity as a linearization of nonlinear elasticity.

(2) Exact solutions in elastostatics. (6 lectures)

Universal deformations for compressible materials. Incompressibility and models of rubber. Exact solutions for incompressible materials, including the Rivlin cube, simple shear, torsion of a cylinder, inflation of a balloon. Cavitation in polymers.

(3) Brittle fracture (4 lectures)

Griffith's theory of brittle fracture: Surface energy and energy release rate, singular solutions of linearized elasticity in a domain with a crack, stress intensity factors.

Variational theory of brittle fracture: formulation and examples.

Reading

1. R. J. Atkin & N. Fox, *An Introduction to the Theory of Elasticity* (Longman, 1980).

2. M.E. Gurtin, *An Introduction to Continuum Mechanics* (Academic Press, 1981).

Further Reading

1. Stuart S. Antman, *Nonlinear Problems of Elasticity,* Applied Mathematical Sciences v. 107 (Springer–Verlag, 1995).

2. Jerrold E. Marsden, Thomas J.R. Hughes, *Mathematical Foundations of Elasticity* (Prentice–Hall, 1983).

3. Philippe G. Ciarlet, *Mathematical Elasticity,* Studies in Mathematics and its Applications; v. 20, 27, 29 (North–Holland, 1988).

C6.2b: Elasticity and Plasticity — Dr Majumdar — 16HT

Level: M-level   **Method of Assessment:** Written examination.

Weight: Half-unit. OSS paper code 2467.

Recommended Prerequisites

Familiarity with classical and fluid mechanics (Part A) and simple perturbation theory (Part C course) will be useful. The complementary Part C course, Solid Mechanics, will be especially useful.

Overview

The course gives a rapid review of mathematical models for basic solid mechanics. Benchmark solutions are reviewed for static problems and wave propagation in linear-elastic materials. It is then shown how these results can be used as a basis for practically useful problems involving rods, plates, and shells. Also simple geometrically nonlinear models will be introduced to explain buckling, contact and fracture at the most basic level. Yield and plasticity will be discussed at a similar level, both microscopically and macroscopically and there will be a brief introduction to composite fields (composite materials, thermo- and visco-elasticity).

Synopsis

Review of tensors, conservation laws, Navier equations. Antiplane strain, plain strain, torsion. Elastic wave propagation, Rayleigh waves. Ad hoc approximations for thin materials; simple bifurcation theory and buckling; simple mixed boundary value problems, brittle fracture and smooth contact; simple ideas about homogenization, composite materials, thermo- and visco-elasticity.

Reading

1. R.M. Hill, *Mathematical Theory of Plasticity* (Oxford Clarendon Press, 1998).

2. A.E.H. Love, *Treatise on the Mathematical Theory of Elasticity* (Dover, 1944).

3. L.D. Landau and E.M. Lifshitz, *Theory of Elasticity* (Pergamon Press, 1986).

C6.3: Perturbation Methods and Applied Complex Variables

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2468), or can be taken as either a half-unit in C6.3a (OSS paper code 2A68) or a half-unit in C6.3b (OSS paper code 2B68).

C6.3a: Perturbation Methods — Dr Porter — 16MT

Recommended Prerequisites

Part A Differential Equations and Core Analysis (Complex Analysis). B5, B6 and B8 are helpful but not officially required.

Overview

Perturbation methods underlie numerous applications of physical applied mathematics: including boundary layers in viscous flow, celestial mechanics, optics, shock waves, reaction-diffusion equations, and nonlinear oscillations. The aims of the course are to give a clear and systematic account of modern perturbation theory and to show how it can be applied to differential equations.

Synopsis

Asymptotic expansions. Asymptotic evaluation of integrals (including Laplace's method, method of stationary phase, method of steepest descent). Regular and singular perturbation theory. Multiple-scale perturbation theory. WKB theory and semiclassics. Boundary layers and related topics. Applications to nonlinear oscillators. Applications to partial differential equations and nonlinear waves.

Reading

1. E.J. Hinch, *Perturbation Methods* (CUP, 1991), Chs. 1–3, 5–7.

2. C.M. Bender and S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers* (Springer, 1999), Chs. 6, 7, 9–11.

3. J. Kevorkian and J.D. Cole, *Perturbation Methods in Applied Mathematics* (Springer–Verlag, 1981), Chs. 1, 2.1–2.5, 3.1, 3.2, 3.6, 4.1, 5.2.

C6.3b: Applied Complex Variables — Prof. Chapman — 16HT

Recommended Prerequisites

The course requires second year core analysis (complex analysis). It continues the study of complex variables in the directions suggested by contour integration and conformal mapping. Part A Fluid Dynamics and Waves is desirable as it provides motivation for some of the topics studied.

Overview

The course begins where core second-year complex analysis leaves off, and is devoted to extensions and applications of that material. It is assumed that students will be familiar with inviscid two-dimensional hydrodynamics (Part A Fluid Dynamics and Waves) to the extent of the existence of a harmonic stream function and velocity potential in irrotational icompressible flow, and Bernoulli's equation.

Synopsis

1–2 Review of core real and complex analysis, especially contour integration, Fourier transforms.

2–4 Conformal mapping. Riemann mapping theorem (statement only). Schwarz–Christoffel formula. Solution of Laplace's equation by conformal mapping onto a canonical domain.

5–6 Applications to inviscid hydrodynamics: flow past an aerofoil and other obstacles by conformed mapping; free streamline flows, hodograph plane.

7–8 Flow with free boundaries in porous media. Construction of solutions using conformal mapping. The Schwarz function.

9–16 Transform methods, complex Fourier transform. Contour integral solutions of ODEs. Applications of Cauchy integrals and Plemelj formulae. Solution of mixed boundary value problems motivated by thin aerofoil theory and the theory of cracks in elastic solids. Riemann Hilbert problems. Wiener–Hopf method.

Reading

1. M. J. Ablowitz and A. S. Fokas, *Complex Variables: Introduction and Applications* (2nd edition, C.U.P., Cambridge, 2003). ISBN 0521534291.

2. J. Ockendon, Howison, Lacey and Movichan, *Applied Partial Differential Equations* (Oxford, 1999) Pages 195–212.

3. G. F. Carrier, M. Krook and C. E. Pearson, *Functions of a Complex Variable* (McGraw–Hill, New York, 1966). (Reprinted by Hod Books, 1983.) ISBN 0962197300 (Out of print).

C6.4a: Topics in Fluid Mechanics — Prof. Chapman & Dr Peppin — 16MT

Level: M-level   **Method of Assessment:** Written examination,

Weight: Half-unit. OSS paper code 2A74.

Recommended Prerequisites

B6 fluid mechanics.

Overview

The course will expand and illuminate the 'classical' fluid mechanics taught in the third year course B6, and illustrate its modern application in a number of different areas in industry and geoscience.

Synopsis

Convection: Earth's mantle and core; magma chambers. Stability, boundary layers, parameterised convection.

Rotating flows: atmosphere and oceans. Waves, geostrophy, quasi-geostrophy, baroclinic instability.

Two-phase flows: boilers, condensers, fluidised beds. Flow régimes. Homogeneous, drift-flux, two-fluid models. Ill-posedness, waves, density wave oscillations.

Thin film flows: coatings and foams. Lubrication theory: gravity flows, Marangoni effects. Droplet dynamics, contact lines, menisci. Drying and wetting.

Reading

1. J. S. Turner, *Buoyancy Effects in Fluids* (C. U. P., Cambridge, 1973).

2. A. E. Gill, *Atmosphere-Ocean Dynamics* (Academic Press, San Diego, 1982).

3. J. Pedlosky, *Geophysical Fluid Dynamics* (Springer–Verlag, Berlin, 1979).

4. D. A. Drew and S. L. Passman, *Theory of Multicomponent Fluids* (Springer–Verlag, Berlin, 1999).

5. P. B. Whalley, *Boiling, Condensation and Gas-Liquid Flow* (O. U. P., Oxford, 1987).

6. D. Weaire and S. Hutzler, *The Physics of Foams* (O. U. P., Oxford, 1999).

Further Reading

1. G. K. Batchelor, H. K. Moffatt and M. G. Worster (eds.), *Perspectives in Fluid Dynamics* (C. U. P., Cambridge, 2000).

C7.1b: Quantum Theory and Quantum Computers — Dr Hannabuss — 16HT

Level: M-level   **Method of Assessment:** Written examination.

Weight: Half-unit, OSS paper code 2A78.

Prerequisites

B7.1a Quantum Mechanics.

Overview

This course builds directly on the first course in quantum mechanics and covers a series of important topics, particularly features of systems containing several particles. The behaviour of identical particles in quantum theory is more subtle than in classical mechanics, and an understanding of these features allows one to understand the periodic table of elements and the rigidity of matter. It also introduces a new property of entanglement linking particles which can be quite widely dispersed.

There are rarely neat solutions to problems involving several particles, so usually one needs some approximation methods. In very complicated systems, such as the molecules of gas in a container, quantum mechanical uncertainty is compounded by ignorance about other details of the system and requires tools of quantum statistical mechanics.

Two state quantum systems enable one to encode binary information in a new way which permits superpositions. This leads to a quantum theory of information processing, and by exploiting entanglement to other ideas such as quantum teleportation.

Learning Outcomes

Students will be able to demonstrate knowledge and understanding of quantum mechanics of many particle systems, statistics, entanglement, and applications to quantum computing.

Synopsis

Identical particles, symmetric and anti-symmetric states, Fermi-Dirac and Bose-Einstein statistics and atomic structure.

Heisenberg representation, interaction representation, time dependent perturbation theory and Feynman-Dyson expansion. Approximation methods, Rayleigh-Schrödinger time-independent perturbation theory and variation principles. The Virial Theorem. Helium.

Mixed states, density operators. The example of spin systems. Purification.

Entanglement. The EPR paradox, Bell's inequalities, Aspect's experiment. GHZ states

Quantum information processing, qubits and quantum computing. The no-cloning theorem, quantum teleportation. Quantum logic gates. Quantum operations. The quantum Fourier transform. Grover's search algorithm.

Reading

K. Hannabuss, *Introduction to Quantum Mechanics* (OUP, 1997). Chapters 10–12 and 14, 16, supplemented by lecture notes on quantum computers on the web

Further Reading

A popular non-technical account of the subject:

A. Hey and P. Walters, *The New Quantum Universe* (Cambridge, 2003).

Also designed for an Oxford course, though only covering some material:

I. .P Grant, *Classical and Quantum Mechanics*, Mathematical Institute Notes (1991).

A concise account of quantum information theory:

S. Stenholm and K.-A. Suominen, *Quantum Approach to Informatics* (Wiley, 2005).

An encyclopaedic account of quantum computing:

M. A. Nielsen and I. L. Chuang, *Quantum Computation* (Cambridge University Press, 2000).

Even more paradoxes can be found in:

Y. Aharanov and D. Rohrlich, *Quantum Paradoxes* (Wiley–VCH, 2005).

Those who read German can find further material on entanglement in:

J. Audretsch, *Verschränkte Systeme* (Wiley–VCH, 2005).

Other accounts of the first part of the course:

L. I. Schiff, *Quantum Mechanics* (3rd edition, Mc Graw Hill, 1968).

B. J. Bransden and C. J. Joachain, *Introduction to Quantum Mechanics* (Longman, 1995).

A. I. M. Rae, *Quantum Mechanics* (4th edition, Institute of Physics, 1993).

John Preskill's on-line lecture notes (http://www.theory.caltech.edu/ preskill/ph219/index.html).

C7.2b: General Relativity I — Prof. Chruściel — 16HT

Level: M-level   **Method of Assessment:** Written examination.

Weight: Half-unit. OSS paper code 2B78.

Recommended Prerequisites

B7.2a Relativity and Electromagnetism.

Overview

The course is intended as an elementary introduction to general relativity, the basic physical concepts of its observational implications, and the new insights that it provides into the nature of space time, and the structure of the universe. Familiarity with special relativity and electromagnetism as covered in the B7 course will be assumed. The lectures will review Newtonian gravitation, tensor calculus and continuum physics in special relativity, physics in curved space time and the Einstein field equations. This will suffice for an account of simple applications to planetary motion, the bending of light and the existence of black holes.

Learning Outcomes

This course starts by asking how the theory of gravitation can be made consistent with the special-relativistic framework. Physical considerations (the principle of equivalence, general covariance) are used to motivate and illustrate the mathematical machinery of tensor calculus. The technical development is kept as elementary as possible, emphasising the use of local inertial frames. A similar elementary motivation is given for Einstein's equations and the Schwarzschild solution. Orbits in the Schwarzschild solution are given a unified treatment which allows a simple account of the three classical tests of Einstein's theory. Finally, the analysis of extensions of the Schwarzschild solution show how the theory of black holes emerges and exposes the radical consequences of Einstein's theory for space-time structure. Cosmological solutions are not discussed.

The learning outcomes are an understanding and appreciation of the ideas and concepts described above.

Synopsis

Review of Newtonian gravitation theory and problems of constructing a relativistic generalisation. Review of Special Relativity. The equivalence principle. Tensor formulation of special relativity (including general particle motion, tensor form of Maxwell's equations and the energy momentum-tensor of dust). Curved space time. Local inertial coordinates. General coordinate transformations, elements of Riemannian geometry (including connections, curvature and geodesic deviation). Mathematical formulation of General Relativity, Einstein's equations (properties of the energy-momentum tensor will be needed in the case of dust only). The Schwarzschild solution; planetary motion, the bending of light, and black holes.

Reading

1. L.P. Hughston and K.P. Tod, *An Introduction to General Relativity,* LMS Student Text 5 (London Mathematical Society, CUP, 1990), Chs 1–18.

2. N.M.J. Woodhouse, *Notes on Special Relativity*, Mathematical Institute Notes. Revised edition; published in a revised form as *Special Relativity, Lecture notes in Physics m6* (Springer–Verlag, 1992), Chs 1–7

Further Reading

1. B. Schutz, *A First Course in General Relativity* (CUP, 1990).

2. R.M. Wald, *General Relativity* (Chicago, 1984).

3. W. Rindler, *Essential Relativity* (Springer–Verlag, 2nd edition, 1990).

C7.4 — Theoretical Physics

Note: This unit is offered by the Physics Department.

This course will be available to Part C Maths students for the last time in 2009/10.

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit only. OSS paper code 2756.

Recommended Prerequisites

Part A Electromagnetism, Part A Classical Mechanics, B7.1a Quantum Mechanics, C7.1b Quantum Theory and Quantum Computers, B7.2a Special Relativity and Electromagnetism.

Overview

This course is intended to give an introduction to some aspects of field theory and related ideas. These are important in particular for treating systems with an infinite number of degrees of freedom. An aim is to present some core ideas and important applications in a unified way. These applications include the classical mechanics of continuum systems, the quantum mechanics and statistical mechanics of many-particle systems, and some basic aspects of relativistic quantum field theory.

C7.4a Theoretical Physics I — Prof. Chalker and Dr Lukas — 24MT

Synopsis

1. The mathematical description of systems with an infinite number of degrees of freedom: functionals, functional differentiation, and functional integrals. Multi-dimensional Gaussian integrals. Random fields: properties of a Gaussian field. Perturbation theory for non-Gaussian functional integrals. Path integrals and quantum mechanics. Treatment of free particle and of harmonic oscillator. [5 lectures]

2. Classical field theory: fields, Lagrangians and Hamiltonians. The least action principle and field equations. Space-time and internal symmetries: $U(1)$ example, Noether current. The idea of an irreducible representation of a group. Irreducible representations of $SU(2)$ and application to global internal symmetry. Simple representations of the Lorentz group via $SU(2) \times SU(2)$ without proof. $U(1)$ gauge symmetry, action of scalar QED and derivation of Maxwell's equations in covariant form. [5 lectures]

3. Landau theory and phase transitions: phase diagrams, first-order and continuous phase transitions. Landau–Ginsburg–Wilson free energy functionals. Examples including liquid crystals. Critical phenomena and scaling theory. [5 lectures]

4. The link between quantum mechanics and the statistical mechanics of one-dimensional systems via Wick rotation. Transfer matrices for one-dimensional systems in statistical mechanics. [4 lectures]

5. Stochastic processes and path integrals: the Langevin and Fokker–Planck equation. Brownian motion of single particle. Rouse model of polymer dynamics. [4 lectures]

C7.4b — Theoretical Physics II — Prof. Chalker and Dr Lukas — 16HT

Synopsis

1. Canonical quantisation and connection to many body theory: quantised elastic waves; quantisation of free scalar field theory; many-particle quantum systems. [4 lectures]

2. Path integrals and quantum field theory: generating functional and free particle propagator for scalar and $U(1)$ gauge fields (in Lorentz gauge). [5 lectures]

3. Perturbation theory at tree level for decay and scattering processes. Examples from pure scalar theories and scalar QED. Goldstone theorem. [4 lectures]

4. Canonical transformations in quantum field theory: Bogoliubov transformations applied to bose condensates, magnons in antiferromagnets, and to BCS theory. [4 lectures]

[Total: 40 lectures]

Reading

The lecturers are aware of no book that presents all parts of this course in a unified way and at an appropriate level. For this reason, detailed lecture notes will be made available.

Some books that cover parts of the course are:

1. D. Bailin and A. Love, *Introduction to Gauge Field Theory*, mainly chapters 1 – 6 for sections 1, 2, 6, 7 and 8.

2. R. P. Feynman, *Statistical Mechanics*, mainly chapters 3, 4 and 6 for sections 1, 6 and 9.

3. F. Reif, *Statistical and Thermal Physics*, chapter 15 for section 5.

4. J. M. Yeomans, *Statistical Mechanics of Phase Transitions*, chapters 1 – 5 for sections 3 and 4.

C8.1: Mathematics and the Environment and Mathematical Physiology

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2481), or can be taken as either a half-unit in C8.1a (OSS paper code 2A81) or a half-unit in C8.1b (OSS paper code 2B81).

C8.1a: Mathematics and the Environment — Dr Sander & T.B.C. — 16MT

Recommended Prerequisites

B6 highly recommended.

Overview

The aim of the course is to illustrate the techniques of mathematical modelling in their particular application to environmental problems. The mathematical techniques used are drawn from the theory of ordinary differential equations and partial differential equations. However, the course does require the willingness to become familiar with a range of different scientific disciplines. In particular, familiarity with the concepts of fluid mechanics will be useful.

Synopsis

Applications of mathematics to environmental or geophysical problems involving the use of models with ordinary and partial differential equations. Examples to be considered are: Climate dynamics. River flow and sediment transport. Glacier dynamics.

Reading

1. A. C. Fowler, *Mathematics and the Environment*, Mathematical Institute Notes (Revised edition, September 2004.)

2. K. Richards, *Rivers* (Methuen, 1982).

3. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, New York, 1974).

4. W. S. B. Paterson, *The Physics of Glaciers* (3rd edition, Pergamon Press, 1994).

5. J. T. Houghton, *The Physics of Atmospheres* (3rd ed., C.U.P., Cambridge, 2002).

C8.1b: Mathematical Physiology — Prof. Bressloff — 16HT

Recommended Prerequisites

B8a highly recommended.

Overview

The course aims to provide an introduction which can bring students within reach of current research topics in physiology, by synthesising a coherent description of the physiological background with realistic mathematical models and their analysis. The concepts and treatment of oscillations, waves and stability are central to the course, which develops ideas introduced in the more elementary B8a course. In addition, the lecture sequence aims to build understanding of the workings of the human body by treating in sequence problems at the intracellular, intercellular, whole organ and systemic levels.

Synopsis

Review of enzyme reactions and Michaelis–Menten theory. Trans-membrane ion transport: Hodgkin–Huxley and Fitzhugh–Nagumo models.

Excitable media; wave propagation in neurons.

Calcium dynamics: calcium-induced calcium release. Intracellular oscillations and wave propagation.

The electrochemical action of the heart. Spiral waves, tachycardia and fibrillation. The heart as a pump. Regulation of blood flow.

Respiration and CO_2 control. Mackey and Grodins models.

Regulation of stem cell and blood cell production. Dynamical diseases.

Reading

The principal text is:

1. J. Keener and J. Sneyd, *Mathematical Physiology* (Springer–Verlag, 1998). Chs. 1, 4, 5, 9–12, 14–17. [Or: Second edition Vol I: Chs. 1, 2, 4, 5, 6, 7. Vol II: Chs. 11, 13, 14. (Springer–Verlag, 2009)]

Subsidiary mathematical texts are:

1. J. D. Murray, *Mathematical Biology* (Springer–Verlag, 2nd ed., 1993). [Third edition, Vols I and II, (Springer–Verlag, 2003).]

2. L. A. Segel, *Modeling Dynamic Phenomena in Molecular and Cellular Biology* (CUP, 1984).

3. L. Glass and M. C. Mackey, *From Clocks to Chaos* (Princeton University Press, 1988).

4. P. Grindrod, *Patterns and Waves* (Oxford University Press, 1991).

General physiology texts are:

1. R. M. Berne and M. N. Levy, *Principles of Physiology* (2nd ed., Mosby, St. Louis, 1996).

2. J. R. Levick, *An Introduction to Cardiovascular Physiology* (3rd ed. Butterworth–Heinemann, Oxford, 2000).

3. A. C. Guyton and J. E. Hall, *Textbook of Medical Physiology* (10th ed., W. B. Saunders Co., Philadelphia, 2000).

C9.1: Analytic Number Theory and Elliptic Curves

Level: M-Level.   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2482), or can be taken as either a half-unit in C9.1a (OSS paper code 2A82) or a half-unit in C9.1b (OSS paper code 2B82).

C9.1a: Analytic Number Theory — Dr Bui — 16MT

Prerequisites

Complex analysis (holomorphic and meromorphic functions, Cauchy's Residue Theorem, Evaluation of integrals by contour integration, Uniformly convergent sums of holomorphic functions). Elementary number theory (Unique Factorization Theorem).

Overview

The course aims to introduce students to the theory of prime numbers, showing how the irregularities in this elusive sequence can be tamed by the power of complex analysis. The course builds up to the Prime Number Theorem which is the corner-stone of prime number theory, and culminates in a description of the Riemann Hypothesis, which is arguably the most important unsolved problem in modern mathematics.

Learning Outcomes

Students will learn to handle multiplicative functions, to deal with Dirichlet series as functions of a complex variable, and to prove the Prime Number Theorem and simple variants.

Synopsis

Introductory material on primes.

Arithmetic functions — Möbius function, Euler function, Divisor function, Sigma function — multiplicativity.

Dirichlet series — Euler products — von Mangoldt function.

Riemann Zeta-function — analytic continuation to $Re(s) > 0$.

Non-vanishing of $\zeta(s)$ on $Re(s) = 1$.

Proof of the prime number theorem.

The Riemann hypothesis and its significance.

The Gamma function, the functional equation for $\zeta(s)$, the value of $\zeta(s)$ at negative integers.

Reading

1. T.M. Apostol, *Introduction to Analytic Number Theory,* Undergraduate Texts in Mathematics (Springer–Verlag, 1976). Chapters 2,3,11,12 and 13.

2. M. Ram Murty, *Problems in Analytic Number Theory* (Springer, 2001). Chapters 1 – 5.

3. G.H. Hardy and E.M. Wright, *An Introduction to the Theory of Numbers* (Fifth edition, OUP, 1979). Chapters 16 ,17 and 18.

4. G.J.O. Jameson, *The Prime Number Theorem,* LMS Student Texts 53 (CUP, 2003).

C9.1b Elliptic Curves — Prof. Heath-Brown — 16 lectures HT

Recommended Prerequisites

It is helpful, but not essential, if students have already taken a standard introduction to algebraic curves and algebraic number theory. For those students who may have gaps in their background, I have placed the file "Preliminary Reading" permanently on the Elliptic Curves webpage, which gives in detail (about 30 pages) the main prerequisite knowledge for the course. Go first to: http://www.maths.ox.ac.uk/courses/material then click on "C9.1b Elliptic Curves" and then click on the pdf file "Preliminary Reading".

Overview

Elliptic curves give the simplest examples of many of the most interesting phenomena which can occur in algebraic curves; they have an incredibly rich structure and have been the testing ground for many developments in algebraic geometry whilst the theory is still full of deep unsolved conjectures, some of which are amongst the oldest unsolved problems in Mathematics. The course will concentrate on arithmetic aspects of elliptic curves defined

over the rationals, with the study of the group of rational points, and explicit determination of the rank, being the primary focus. Using elliptic curves over the rationals as an example, we will be able to introduce many of the basic tools for studying arithmetic properties of algebraic varieties.

Learning Outcomes

On completing the course, students should be able to understand and use properties of elliptic curves, such as the group law, the torsion group of rational points, and 2-isogenies between elliptic curves. They should be able to understand and apply the theory of fields with valuations, emphasising the p-adic numbers, and be able to prove and apply Hensel's Lemma in problem solving. They should be able to understand the proof of the Mordell–Weil Theorem for the case when an elliptic curve has a rational point of order 2, and compute ranks in such cases, for examples where all homogeneous spaces for descent-via-2-isogeny satisfy the Hasse principle. They should also be able to apply the elliptic curve method for the factorisation of integers.

Synopsis

Non-singular cubics and the group law; Weierstrass equations.
Elliptic curves over finite fields; Hasse estimate (stated without proof).
p-adic fields (basic definitions and properties).
1-dimensional formal groups (basic definitions and properties).
Curves over p-adic fields and reduction mod p.
Computation of torsion groups over $\mathbb{Q}$; the Nagell–Lutz theorem.
2-isogenies on elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Weak Mordell–Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Height functions on Abelian groups and basic properties.
Heights of points on elliptic curves defined over $\mathbb{Q}$; statement (without proof) that this gives a height function on the Mordell–Weil group.
Mordell–Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Explicit computation of rank using descent via 2-isogeny.
Public keys in cryptography; Pollard's $(p-1)$ method and the elliptic curve method of factorisation.

Reading

1. J.W.S. Cassels, *Lectures on Elliptic Curves,* LMS Student Texts 24 (Cambridge University Press, 1991).

2. N. Koblitz, *A Course in Number Theory and Cryptography,* Graduate Texts in Mathematics 114 (Springer, 1987).

3. J.H. Silverman and J. Tate, *Rational Points on Elliptic Curves,* Undergraduate Texts in Mathematics (Springer, 1992).

4. J.H. Silverman, *The Arithmetic of Elliptic Curves,* Graduate Texts in Mathematics 106 (Springer, 1986).

Further Reading

1. A. Knapp, *Elliptic Curves, Mathematical Notes 40* (Princeton University Press, 1992).

2. G, Cornell, J.H. Silverman and G. Stevans (editors), *Modular Forms and Fermat's Last Theorem* (Springer, 1997).

3. J.H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves,* Graduate Texts in Mathematics 151 (Springer, 1994).

C10.1 Stochastic Differential Equations and Brownian Motion in Complex Analysis

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2483), or can be taken as either a Half-unit in C10.1a (OSS paper code 2A83) or a Half-unit in C10.1b (OSS paper code 2B83).

C10.1a: Stochastic Differential Equations —Prof. Lyons—16MT

Recommended Prerequisites

A course on integration (in future years Part A integration will be required) and an introduction to discrete martingales, B10a Martingales Through Measure Theory, is expected.

Overview

Stochastic differential equations have been used extensively in many areas of application, including finance and social science as well as Chemistry. This course develops the basic theory of Itô's calculus and stochastic differential equations, and gives a few applications.

Learning Outcomes

The student will have developed an appreciation of stochastic calculus as a tool that can be used for defining and understanding diffusive systems.

Synopsis

Itô's calculus: stochastic integrals with respect to martingales, Itô's lemma, Levy's theorem on characteristic of Brownian motion, exponential martingales, exponential inequality,

Girsanov's theorem, The Martingale Representation Theorem. Stochastic differential equations: strong solutions, questions of existence and uniqueness, diffusion processes, Cameron–Martin formula, weak solution and martingale problem. Some selected applications chosen from option pricing, stochastic filtering etc.

Reading — Main Texts

1. Dr Qian's online notes:

   www.maths.ox.ac.uk/courses/2007/part-c/c101a-stochastic-differential-equations/material

2. B. Oksendal, *Stochastic Differential Equations: An introduction with applications* (Universitext, Springer, 6th edition). Chapters II, III, IV, V, part of VI, Chapter VIII (F).

3. F. C. Klebaner, *Introduction to Stochastic Calculus with Applications* (Imperial College Press, 1998, second edition 2005). Sections 3.1 – 3.5, 3.9, 3.12. Chapters 4, 5, 11.

Alternative Reading

1. H. P. McKean, *Stochastic Integrals* (Academic Press, New York and London, 1969).

Further Reading

1. N. Ikeda & S. Watanabe, *Stochastic Differential Equations and Diffusion Processes* (North–Holland Publishing Company, 1989).

2. I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus,* Graduate Texts in Mathematics 113 (Springer–Verlag, 1988).

3. L. C. G. Rogers & D. Williams, *Diffusions, Markov Processes and Martingales Vol1 (Foundations) and Vol 2 (Ito Calculus)* (Cambridge University Press, 1987 and 1994).

C10.1b: Brownian Motion in Complex Analysis — Dr Cass — 16HT

Recommended Prerequisites

Part A Analysis (and Part A Integration for future years), (B10a) Martingales through Measure and (C10.1) Stochastic Differential Equations.

Overview

Randomness plays a key role in determining the behaviour of many high dimensional systems and so is intimately connected with applications. However, it also plays a key role in our understanding of many aspects of pure mathematics. This course will look at the deep interaction between 2 dimensional Brownian motion and complex analysis. At the core of these interactions is the conformal invariance of Brownian motion observed by Levy and the relationship with Harmonic Functions (based on martingales) first observed by Kakutani and Doob.

Since that time there have been many developments and connections. The Hardy spaces of Fefferman and Stein, Value Distribution Theory, and most recently the stochastic Lowener equation (a topic of current and very exciting research).

We will use the conformal properties of Brownian motion to examine and prove some deep theorems about value distributions for complex functions.

Learning Outcomes

The student will have developed an appreciation of the role that Brownian motion and martingales can play in pure mathematics.

Synopsis

Brownian motion. Continuous martingales and Levy's characterisation in terms of Brownian motion. Conformal invariance of Brownian motion. Brownian motion tangles about two points and a proof of Picard's theorems. Harmonic functions on the disk and the solution of the Dirichlet problem. Burkholder's Inequalities in Hardy spaces. Harmonic and superharmonic functions — via probability. [Nevanlinna's Theorems].

Reading

There isn't a perfect book for this course and we will refer to research papers to a limited extent.

1. The Notes of Prof. Lyons

2. D. Burkholder, *Distribution Function Inequalities for Martingales*, Ann. Probability, 1 (1973) 19–42.

Further Reading

1. McKean, *Stochastic Integrals* (1969). Hard, short, with much relevant material and some mistakes! Excellent for the able!

2. K. E. Petersen, *Brownian Motion, Hardy Spaces and Bounded Mean Oscillation,* LMS Lecture Note Series, 28 (Cambridge University Press, 1977).

3. T. K. Carne, 'Brownian Motion and Nevanlinna Theory'. *Proc. London. Math. Soc.* (52) (1986), 349–68.

4. Richard F. Bass, *Probabilistic Techniques in Analysis* (Springer–Verlag, New York Inc, 1995) ISBN: 0387943870.

5. Lars Ahlfors, *Complex Analysis* (McGraw–Hill, 1979) ISBN: 0070006571.

6. Jean-Claude Gruet, 'Nevanlinna Theory, Fuchsian Functions and Brownian Motion Windings', Source: *Rev. Mat. Iberoamericana* 18 (2002), no. 2, 301–324.

C11.1: Graph Theory and Probabilistic Combinatorics

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2485), or C11.1a may be taken as a half-unit (OSS paper code 2A85), C11.1b may not.

C11.1a: Graph Theory — Prof. Scott — 16MT

Recommended Prerequisites

None beyond elementary probability theory.

Overview

Graphs are among the simplest mathematical structures, but nevertheless have a very rich and well-developed structural theory. Graph Theory is an important area of mathematics, and also has many applications in other fields such as computer science.

The main aim of the course are to introduce the analysis of discrete structures, and particularly the use of extremal methods.

Learning Outcomes

The student will have developed an appreciation of extremal methods in the analysis and understanding of graphical structures.

Synopsis

Introduction. Trees. Euler circuits. Planar graphs.
Matchings and Hall's Theorem. Connectivity and Menger's Theorem.
Extremal problems. Long paths and cycles. Turán's Theorem. Erdős–Stone Theorem.
Graph colouring. The Theorem of Brooks. The chromatic polynomial.
Ramsey's Theorem.
Szemerédi's Regularity Lemma.

Reading

1. B. Bollobás, *Modern Graph Theory,* Graduate Texts in Mathematics 184 (Springer–Verlag, 1998)

Further Reading

1. J. A. Bondy and U. S. R. Murty, *Graph Theory: An Advanced Course*, Graduate Texts in Mathematics 244 (Springer-Verlag, 2007).

2. R. Diestel, *Graph Theory,* Graduate Texts in Mathematics 173 (third edition, Springer–Verlag, 2005).

3. D. West, *Introduction to Graph Theory* (second edition, Prentice–Hall, 2001).

C11.1b: Probabilistic Combinatorics — Prof. Riordan — 16HT

Recommended Prerequisites

C11.1a Graph Theory and Part A Probability.

Overview

Probabilistic combinatorics is a very active field of mathematics, with connections to other areas such as computer science and statistical physics. Probabilistic methods are essential for the study of random discrete structures and for the analysis of algorithms, but they can also provide a powerful and beautiful approach for answering deterministic questions. The aim of this course is to introduce some fundamental probabilistic tools and present a few applications.

Learning Outcomes

The student will have developed an appreciation of probabilistic methods in discrete mathematics.

Synopsis

Spaces of random graphs. Threshold functions.
First and second moment methods. Chernoff bounds. Applications to Ramsey numbers and random graphs.
Lovasz Local Lemma. Two-colourings of hypergraphs (property B).
Poisson approximation, and application to the distribution of small subgraphs. Janson's inequality.
Concentration of measure. Martingales and the Azuma–Hoeffding inequality.
Chromatic number of random graphs.

Talagrand's inequality.

Reading

1. N. Alon and J.H. Spencer, *The Probabilistic Method* (second edition, Wiley, 2000).

Further Reading

1. B. Bollobás, *Random Graphs* (second edition, CUP, 2001).

2. M. Habib, C. McDiarmid, J. Ramirez-Alfonsin, B. Reed, ed., *Probabilistic Methods for Algorithmic Discrete Mathematics* (Springer, 1998).

3. S.Janson, T. Luczak and A.Rucinski, *Random Graphs* (John Wiley and Sons, 2000).

4. M. Mitzenmacher and E. Upfal, *Probability and Computing : Randomized Algorithms and Probabilistic Analysis* (Cambridge University Press, New York (NY), 2005).

5. M. Molloy and B. Reed, *Graph Colouring and the Probabilistic Method* (Springer, 2002).

6. R. Motwani and P. Raghavan, *Randomized Algorithms* (CUP, 1995).

Numerical Linear Algebra and Analysis

C12.1 Numerical Linear Algebra and Continuous Optimization

Level: M-level   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2488), or can be taken a either a half-unit in C12.1a (OSS paper code 2A88) or C12.1b (OSS paper code 2B88).

C12.1a Numerical Linear Algebra — Prof. Trefethen — 16MT

Recommended Prerequisites

Only elementary linear algebra is assumed in this course. The part A Numerical Analysis course would be helpful, indeed some swift review and extensions of some of the material of that course is included here.

Overview

Linear Algebra is a central and widely applicable part of Mathematics. It is estimated that many (if not most) computers in the world are computing with matrix algorithms at any moment in time whether these be embedded in visualization software in a computer game or calculating prices for some financial option. This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices.

Numerical Methods for solving linear systems of equations, computing eigenvalues and singular values and various related problems involving matrices are the main focus of this course.

Synopsis

Common problems in linear algebra. Matrix structure, singular value decomposition. QR factorization, the QR algorithm for eigenvalues. Direct solution methods for linear systems, Gaussian elimination and its variants. Iterative solution methods for linear systems.

Chebyshev polynomials and Chebyshev semi-iterative methods, conjugate gradients, convergence analysis, preconditioning.

Reading List

L. N. Trefethen and D. Bau III, *Numerical Linear Algebra* (SIAM, 1997).

J. W. Demmel, *Applied Numerical Linear Algebra* (SIAM, 1997).

A. Greenbaum, *Iterative Methods for Solving Linear Systems* (SIAM, 1997).

G. H. Golub and C. F. van Loan, *Matrix Computations* (John Hopkins University Press, 3rd edition, 1996).

H. C. Elman, D. J. Silvester and A. J. Wathen, *Finite Elements and Fast Iterative Solvers* (OUP, 1995), only chapter 2.

C12.1b Continuous Optimization — Dr Robinson — 16HT

Overview

Optimization deals with the problem of minimising or maximising a mathematical model of an objective function such as cost, fuel consumption etc. under a set of side constraints on the domain of definition of this function. Optimization theory is the study of the mathematical properties of optimization problems and the analysis of algorithms for their solution. The aim of this course is to provide an introduction to nonlinear continuous optimization specifically tailored to the background of mathematics students.

Synopsis

Part 1: Unconstrained Optimization
Optimality conditions, Newton's method for nonlinear systems, Convergence rates, Steepest descent method, General line search methods (alternative search directions, e.g. Newton, CG, BFG, ...), Trust region methods, Inexact evaluation of linear systems, iterative methods and the role of preconditioners.

Part 2: Constrained Optimization
Optimality/KKT conditions, Lagrange Multipliers, Penalty methods and SQP for equality constrained optimization, Interior penalty / barrier methods for inequality constrained optimization.

Reading List

Lecture notes will be made available for downloading from the course webpage.
To complement the notes, reading assignments will be given from the book of J.Nocedal and S.J.Wright, *Numerical Optimisation*, (Springer, 1999).

C12.2 Approximation Theory and Finite Element Methods

Level: M-Level.   **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2489), or can be taken as either a half-unit in C12.2a (OSS paper code 2A89) or a half-unit in C12.2b (OSS paper code 2B89).

C12.2a Approximation of Functions — Dr Sobey — 16MT

Recommended Prerequisites

None

Overview

The central idea in approximation of functions can be illustrated by the question: Given a set of functions A and an element $u \in A$, if we select a subset $B \subset A$, can we choose an element $U \in B$ so that U approximates u in some way? The course focuses on this question in the context of functions when the way we measure 'goodness' of approximation is either with an integral least square norm or with an infinity norm of the difference $u - U$. The choice of measure leads to further questions: is there a best approximation; if a best approximation exists, is it unique, how accurate is a best approximation and can we develop algorithms to generate good approximations? This course aims to give a grounding in the advanced theory of such ideas, the analytic methods used and important theorems for real functions. As well as being a beautiful subject in its own right, approximation theory is

the foundation for many of the algorithms of computational mathematics and numerical analysis.

Synopsis

Introduction to approximation. Approximation in L^2. Approximation in L^∞: Oscillation Theorem, Exchange Algorithm. Approximation with splines. Rational approximation. Approximation of periodic functions.

Reading

1. M. J. D. Powell, *Approximation Theory and Methods* (CUP).

2. P. J. Davis, *Interpolation & Approximation* (Dover).

C12.2b Finite Element Methods for Partial Differential Equations — Dr Wathen — 16HT

Recommended Prerequisites

No formal prerequisites are assumed. The course builds on elementary calculus, analysis and linear algebra and, of course, requires some acquaintance with partial differential equations such as the material covered in the Maths Mods Waves and Diffusion course, in particular the Divergence Theorem. Part A Numerical Analysis would be helpful but is certainly not essential. Function Space material will be introduced in the course as needed.

Overview

Computational algorithms are now widely used to predict and describe physical and other systems. Underlying such applications as Weather Forecasting, Civil Engineering (design of structures) and Medical Scanning are numerical methods which approximately solve partial differential equation problems. This course gives a mathematical introduction to one of the more widely used methods: the finite element method.

Synopsis

Finite element methods represent a powerful and general class of techniques for the approximate solution of partial differential equations. The aim of this course is to introduce these methods for boundary value problems for the Poisson and related elliptic partial differential equations. Attention will be paid to formulation, analysis, implementation and applicability of these methods.

Reading List

The main text will be

Howard Elman, David Silvester & Andy Wathen, *Finite Elements and Fast Iterative Solvers* (Oxford University Press, 2005) [mainly Chapters 1 and 5]

and some of the introductory material is usefully covered in

Endre Suli and David Mayers, *An Introduction to Numerical Analysis* (Cambridge University Press, 2003) [Chapter 11 and in particular Chapter 14]

and

David Silvester, *A Finite Element Primer*, notes which can be found at
http://www.maths.manchester.ac.uk/ djs/primer.pdf

Another book on finite elements which may be useful for different parts of the course is

Claes Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Method* (Cambridge University Press, 1990). (Which is unfortunately out of print) [Chapter 1-4]

and the Computing Lab lecture notes

Endre Suli, *Finite Element Methods for Partial Differential Equations*, Oxford University Computing Laboratory, 2001.

are also useful.

CD : Dissertations on a Mathematical Topic

Level : M-level

Weight : Half unit or whole unit.

You may offer either a whole-unit or a half-unit Dissertation in a Mathematical topic.

The *Guidance Notes on Extended Essays and Dissertations in Mathematics* by the Projects Committee and application form are available at:

http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

A CD Dissertation at M-level would be 10,000 words in length for a whole-unit or 5,000 words in length for a half-unit. A whole-unit is equivalent to a 32-hour lecture course and a half-unit is equivalent to a 16-hour lecture course. Each project is double blind marked. The marks are reconciled following discussion between the two assessors and overseen by examiners.

Application You must apply to the Mathematics Project Committee in advance for approval. Proposals should be addressed to The Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room DH61, Dartington House, and must be received before 12 noon on Friday of Week 3 of Michaelmas Full Term. Note that for approval of the topic of a CD dissertation the subject matter must have a substantial mathematical content. Once your

title has been approved, it may only be changed by approval of Chairman of the Project's Committee.

Assessment

Dissertations will be assigned USMs according to the same principles as Mathematics papers. In arriving at these marks, the relative weights given to content, mathematics, and presentation will be 25%, 50% and 25%, respectively. Here is a brief explanation of these terms and how they may be interpreted by assessors and examiners :

Content: appropriate material selected. The examiners are looking for some of your own thoughts and contributions: you must do more than rehash text books and lecture notes; you should use original sources; you must not plagiarise.

Mathematics: proofs and assertions should be correct, and the mathematics should be appropriate for the level of study. In applied topics, the derivation of the model should be properly justified.

Presentation: the examiners are looking at how clear the mathematics is and if it is well laid out; the quality of exposition (clear, well-thought out, well-organised); English style (spelling, grammar, readability). Sources should be properly acknowledged, references should be properly cited. Give some thought to notation, choice of typeface, and numbering of equations and sections. Do not fail to number the pages. Be sure to supply complete and accurate references for all the sources used in completing the project, and be sure to cite them properly in the text.

Excellent brief advice on mathematical writing is to be found on the London Mathematical Society website at http://www.lms.ac.uk/publications/documents/writing.pdf.

Students are invited to give an informal presentation of their work to their supervisor before the end of Hilary Term. Although not formally assessed this provides useful feedback to the student and is part of the learning process.

Generally, students would have approximately 8 hours of supervision for a whole-unit dissertation distributed over the two terms. Similarly, for the half-unit approximately 4 hours of supervision. There are special lectures given on projects, in TT and MT on projects in general, also lectures on Latex. See the lecture List for details.

Submission of Dissertation FOUR copies of your dissertation, identified only by your candidate number, should be sent to the Chairman of Examiners, FHS of Mathematics Part C, Examination Schools, Oxford, to arrive no later than **12 noon on Friday of week 9, Hilary Term 2010.**

3 Other Units

MS: Statistics Half-units

MS1a: Graphical Models and Inference — Prof. Lauritzen — 16MT

Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.

Method of Asessment: Written examination.

Weight: Half-unit. OSS paper code 2A55. Whole unit with MS1b (OSS paper code 2655).

Recommended Prerequisites

OBS1 Applied statistics and OBS2a Foundations of Statistical Inference would be helpful but not essential.

Overview

Graphical models have become increasingly important in many areas where statistics play a role. They enable the description and analysis of complex stochastic systems via their natural modularity, expressed in terms of (mathematical) graphs which encode conditional independence structure. The modules correspond typically to well-understood, classical models. This course builds upon develops the specific theory and computational tools needed in the analysis of graphical models for categorical and multivariate Gaussian data as well as Bayesian graphical models for complex stochastic systems.

Synopsis

Topics include:

1. Conditional independence and Markov properties.

2. Log-linear graphical models for categorical data.

3. Gaussian graphical models.

4. Graphical models for complex stochastic systems

Reading

1. D. Edwards, *Introduction to Graphical Models* (2nd ed., Springer–Verlag, New York, 2002).

2. S. L. Lauritzen, *Graphical Models* (Oxford University Press, Oxford, 1996).

3. P. J. Green, N. L. Hjort and S. Richardson, eds. *Highly Structured Stochastic Systems*, (Oxford University Press, Oxford, 2003).

MS1b: Statistical Data Mining — Dr Meinshausen — 12HT and 4 labs

Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.
This course will be delivered using a combination of 12 lectures and 4 one-hour computer laboratory sessions.

Method of Assessment: Mini-project. The mini-projects are distributed to candidates from noon on Monday of week 8 in the term in which the course is being taught. Two copies should be submitted to the Exam Schools by noon Monday week 1 of the following term.

Weight: Half-unit. OSS code to follow.

Recommended Prerequisites

Part A Probability and Statistics. OBS1 Applied Statistics would be an advantage.

Overview

'Data mining' is now widely used to find interesting patterns in large databases, for example in insurance, in marketing and in many scientific fields. With large amounts of data we can search for quite subtle patterns. This course concentrates on the statistical tools used to identify patterns, and then to identify those which are interesting not just the result of chance associations.

Synopsis

Fundamentals of pattern recognition, machine learning and data mining. Exploratory methods: principal components analysis, biplots, independent component analysis, multidimensional scaling.

Cluster Analysis: K-means, hierarchical methods, vector quantisation, self-organising maps.

Linear discriminant analysis, logistic discrimination, linear separation.

Feed-forward neural networks, Classification trees, ensemble methods, V-fold cross-validation.

Reading

1. C. Bishop, *Neural Networks for Pattern Recognition* (Oxford University Press, 1995).

2. D. Hand, H. Mannila, P. Smyth, *Principles of Data Mining* (MIT Press, 2001).

3. I. H. Witten and E. Franke, *Data Mining. Practical Machine Learning Tools and Techniques with Java Implementations* (Morgan Kaufmann, 2000).

Further Reading

1. B. D. Ripley, *Pattern Recognition and Neural Networks* (Cambridge University Press, 1996).

MS2a: Bioinformatics and Computational Biology — Prof. Hein — 16MT

Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.

Method of Assessment: Mini-project. The mini-projects are distributed to candidates from noon on Monday of week 8 in the term in which the course is being taught. Two copies should be submitted to the Exam Schools by noon Monday week 1 of the following term.

Weight: Half-unit (OSS paper code 2A56). Whole-unit if taken with MS2b (OSS paper code 2656).

Recommended Prerequisites

None. In particular, no previous knowledge of Genetics will be necessary.

Overview

Modern molecular biology generates large amounts of data, such as sequences, structures and expression data, that needs different forms of statistical analysis and modelling to be properly interpreted. The fields of Bioinformatics and Computational Biology have this as their subject matter and there is no sharp boundary between them. Bioinformatics has an applied flavour while Computational Biology is viewed as the study of the models, statistical methodology and algorithms needed to do bioinformatics analysis. This course aims to present core topics of these fields with and emphasis on modelling and computation.

Synopsis

Fundamental Data Structures in Biology: Sequences, Genes and RNA secondary structure.

Stochastic Models of Sequence and Genome Evolution including models of single nucleotide/amino acid/codon evolution.

Phylogenies: enumerating phylogenies, the probability of sequences related by a specified phylogeny, the minimal number of events needed to explain a data set (Parsimony).

Likelihood and algorithms (Markov Chain Monte Carlo) for inference based on the likelihood. Software packages for sample-based inference.

Alignment Algorithms. Comparing 2 strings, an arbitrary number of strings, find segments of high similarity in 2 strings.

Common Patterns in a set of Sequences.

Reading

1. C. Semple and M. Steel, *Phylogenetics* (Oxford University Press, 2003).

2. Durbin et al., *Biological Sequence Analysis* (Cambridge University Press, 1998).

3. T. Jiang et al., (editors) *Current Topics in Computational Biology* (MIT Press, 2003).

4. M. S. Waterman et al., *Computational Genome Analysis: An Introduction* (Springer, 2004).

MS2b: Stochastic Models in Mathematical Genetics — Prof. Griffiths & Dr Myers — 16HT

Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.

Method of Assessment: Written examination.

Weight: Half-unit. OSS paper code 2B56.

Overview

The aim of the lectures is to introduce modern Stochastic models in Mathematical Population Genetics and give examples of real world applications of these models. Stochastic and Graph theoretic properties of coalescent and gene trees are studied in the first eight lectures. Extensions to model additional key biological phenomena, and applications, are studied in the second eight lectures.

Synopsis

Evolutionary models in Mathematical Genetics: The Wright-Fisher model. The Genealogical Markov chain describing the number ancestors back in time of a collection of genes.

The Coalescent process describing the stochastic behaviour of the ancestral tree of a collection of genes. Mutations on ancestral lineages in a coalescent tree. Inferring the time to the most recent common ancestor in a sample of genes from the number of mutations occurring to the genes. Models with a variable population size.

The frequency spectrum and age of a mutation. Ewens' sampling formula for the probability distribution of the allele configuration of genes in a sample in the infinitely-many-alleles model. Hoppe's urn model for the infinitely-many-alleles model.

The infinitely-many-sites model of mutations on DNA sequences. Gene trees as perfect phylogenies describing the mutation history of a sample of DNA sequences. Graph theoretic constructions and characterizations of gene trees from DNA sequence variation. Gusfield's construction algorithm of a tree from DNA sequences. Examples of gene trees from data. The probability distribution of a gene tree.

Modelling biological forces in Population Genetics:

Recombination. The effect of recombination on genealogies. Detecting recombination events under the infinitely-many-sites model. Hudson's algorithm. Haplotype bounds on recombination events.

Modelling recombination in the Wright-Fisher model. The coalescent process with recombination: the ancestral recombination graph. Properties of the ancestral recombination graph. Applications of coalescent-based methods to the estimation of historical recombination rate.

Introduction to diffusion theory. Tracking mutations forward in time in the Wright-Fisher model. Modelling the frequency of a neutral mutation in the population via a diffusion process limit. The generator of a diffusion process with two allelic types. The probability of fixation and expected time to loss or fixation of a mutation. The frequency spectrum of a mutation.

Genic selection. Extension of results from neutral to selection case. Behaviour of selected mutations. Brief discussion of modern approaches to detecting selection from variation data.

Reading

1. R. Durrett, *Probability Models for DNA Sequence Evolution*, (Springer, 2002).

2. W. J. Ewens, *Mathematical Population Genetics* (2nd ed, Springer, 2004).

3. J. R. Norris, *Markov Chains* (Cambridge University Press, 1999).

4. M. Slatkin and M. Veuille, *Modern Developments in Theoretical Population Genetics* (Oxford Biology, 2002).

5. S. Tavaré and O. Zeitouni, *Lectures on Probability Theory and Statistics, Ecole d'Eté de Probabilités de Saint-Flour XXXI — 2001*, Lecture Notes in Mathematics 1837. (Springer, 2004).

MS3b: Lévy Processes and Finance — Dr Winkel — 16HT

Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.

Method of Assessment: Written examination.

Weight: Half-unit. OSS paper code 2B57.

Recommended Prerequisites

Part A Probability is a prerequisite. BS3a/B12a Applied Probability or B10a Martingales and Financial Mathematics would be useful, but are by no means essential; some material from these courses will be reviewed without proof.

Overview

Lévy processes form a central class of stochastic processes, contain both Brownian motion and the Poisson process, and are prototypes of Markov processes and semimartingales. Like Brownian motion, they are used in a multitude of applications ranging from biology and physics to insurance and finance. Like the Poisson process, they allow to model abrupt moves by jumps, which is an important feature for many applications. In the last ten years Lévy processes have seen a hugely increased attention as is reflected on the academic side by a number of excellent graduate texts and on the industrial side realising that they provide versatile stochastic models of financial markets. This continues to stimulate further research in both theoretical and applied directions. This course will give a solid introduction to some of the theory of Lévy processes as needed for financial and other applications.

Synopsis

Review of (compound) Poisson processes, Brownian motion (informal), Markov property. Connection with random walks, [Donsker's theorem], Poisson limit theorem. Spatial Poisson processes, construction of Lévy processes.

Special cases of increasing Lévy processes (subordinators) and processes with only positive jumps. Subordination. Examples and applications. Financial models driven by Lévy processes. Stochastic volatility. Level passage problems. Applications: option pricing, insurance ruin, dams.

Simulation: via increments, via simulation of jumps, via subordination. Applications: option pricing, branching processes.

Reading

1. J.F.C. Kingman, *Poisson Processes* (Oxford University Press, 1993), Ch.1–5, 8.

2. A.E. Kyprianou, *Introductory Lectures on Fluctuations of Lévy Processes with Applications* (Springer, 2006), Ch. 1–3, 8–9.

3. W. Schoutens, *Lévy Processes in Finance: Pricing Financial Derivatives* (Wiley, 2003).

Further Reading

1. J. Bertoin, *Lévy Processes* (Cambridge University Press, 1996), Sect. 0.1–0.6, I.1, III.1–2, VII.1.

2. K. Sato, *Lévy Processes and Infinite Divisibility* (Cambridge University Press, 1999), Ch. 1–2, 4, 6, 9.

MS5b: High-throughput Data Analysis — Dr Clifford — 16HT

Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.

Method of Assessment: Written examination.

Weight: Half-unit. OSS paper code to follow.

Recommended Prerequisites

Part A (second-year) courses in Statistics and Probability.

Learning Outcomes

Understanding the problems of working with high-throughput, high-frequency data-streams in applications such as finance, engineering and environmental monitoring. Understanding the limitations of established statistical techniques in dealing with such data. Development of working knowledge and experience with conventional and simulation-based methods for online statistical inference.

Understanding how to construct informative 'data sketches' from transiently observed data-streams. Experience in the use of such sketches.

Synopsis

Illustrations of the problems of high-throughput, high-frequency data analysis in finance, internet monitoring and object-tracking. Defining the objective, namely the construction of efficient statistical algorithms that operate with limited storage and process with one pass over the data. Online data analysis. Introduction to dynamic statistical models. Gaussian

and non-Gaussian models; Kalman filter, extended Kalman filter. Estimating volatility and cross-correlation in high-frequency financial time series. Problems of asynchronicity and scale.

Simulation based online filtering: concept of particle filters, design, optimisation, performance characteristics, connections with the Kalman filter. Applications to object tracking and parameter estimation in mathematical models of finance. Data-sketching. Probabilistic counting. Estimation of frequency moments by stable-law projection. Comparison of data streams via data sketches. Estimation of entropy and cross-entropy. Application to MCMC convergence diagnostics and monitoring of internet traffic.

Reading

1. A. Doucet, N. De Freitas and N.J. Gordon, *Sequential Monte Carlo Methods in Practice* (Springer, 2001).

2. A. Doucet and A.M. Johansen, *A tutorial on particle filtering and smoothing: fifteen years later*, Technical report, Department of Statistics, University of British Columbia,http://www.cs.ubc.ca/ arnaud/doucet_johansen_tutorialPF.pdf, 2008.

3. J.P. Nolan, *Information on Stable Distributions*, http://academic2.american.edu/j̃pnolan/stable/stable.html

4. P. Indyk, Selected readings of his papers on data sketching: as indicated in the lectures.

Background Reading: Financial Applications

1. P. Lequeux (Editor), *Financial Markets Tick by Tick* (Wiley, 1999).

MS6a: The Analysis of Biological Networks — Prof. Hein, with Dr Lyngsø & Dr Thatte — 16HT

Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.

Method of Assessment: Mini-project. The mini-projects are distributed to candidates from noon on Monday of week 8 in the term in which the course is being taught. Two copies should be submitted to the Exam Schools by noon Monday week 1 of the following term.

Weight: Half-unit. OSS paper code to follow.

Recommended Prerequisites

None. In particular, no previous knowledge of Genetics will be necessary.

Learning Outcomes

Modern molecular biology generates large amounts of data, that allows the characterisation of biological systems. In the describing the dynamics and relationships of biological systems, network is the most central concept. At the cellular level there are 4 main classes: metabolic, regulatory, signalling and protein interaction networks. Such networks pose a set of common problems, such as inference, evolution, enumeration, comparison and dynamics. This course aims to present core techniques and problems of these topics with and emphasis on modelling and computation.

Synopsis

Dynamic High Throughput Data in Biology
Statistical analysis of Dynamic High Throughput Data
Biological Networks and their Temporal Dynamics
Evolution of Networks
Network Algorithms
Network Inference
Combinatorial Methods of Network Comparison
Probability Theory of Networks
Reconstruction Problems in Networks
Sampling Networks
Network Robustness

Reading

1. U. Alon, *An Introduction to Systems Biology: Design Principles of Biological Circuits* (CRC Press, 2006).

2. Davies, Rafnar, Hellenthal and Hein, *Integrative Genomics and Functional Explanation* (2009)

3. Gillespie, Stochastic Simulation of Chemical Kinetics, *Ann. Rev. Phys. Chem.* 58:35-55 (2007).

Computer Science: Half Units

Please see the Computing Laboratory website for full, up-to-date course information.

Please note that these three courses will be examined by mini-project (as for MSc students). Mini-projects will be handed out to candidates on the last Friday of the term in which the subject is being taught, and you will have to hand it in to the Exam Schools by noon on Monday of Week 1 of the following term. The mini-project will be designed to be completed

in about three days. It will include some questions that are more open-ended than those on a standard sit-down exam. The work you submit should be your own work, and include suitable references.

Please note that the Computer Science courses in Part C are 50% bigger than those in earlier years, i.e. for each Computer Science course in the 3rd year undergraduates are expected to undertake about 10 hours of study per week, but 4th year courses will each require about 15 hours a week of study. Lecturers are providing this extra work in a variety of ways, e.g. some will give 16 lectures with extra reading, classes and/or practicals, whereas others will be giving 24 lectures, and others still will be doing something in between. Students will need to look at each synopsis for details on this.

CCS1a: Categories, Proofs and Processes — Lecturer T.B.C. — 20 lectures (plus extra reading) HT

Level: M-level.   **Method of Assessment:** Mini-project.

Weight: Half-unit. OSS code 292.

Recommended Prerequisites: Some familiarity with basic discrete mathematics: sets, functions, relations, mathematical induction. Basic familiarity with logic: propositional and predicate calculus. Some first acquaintance with abstract algebra: vector spaces and linear maps, and/or groups and group homomorphisms. Some familiarity with programming, particularly functional programming, would be useful but is not essential.

Overview

Category Theory is a powerful mathematical formalism which has become an important tool in modern mathematics, logic and computer science. One main idea of Category Theory is to study mathematical 'universes', collections of mathematical structures and their structure-preserving transformations as mathematical structures in their own right, i.e. categories — which have their own structure-preserving transformations (functors). This is a very powerful perspective, which allows many important structural concepts of mathematics to be studied at the appropriate level of generality, and brings many common underlying structures to light, yielding new connections between apparently different situations.

Another important aspect is that set-theoretic reasoning with elements is replaced by reasoning in terms of arrows. This is more general, more robust, and reveals more about the intrinsic structure underlying particular set-theoretic representations.

Category theory has many important connections to logic. We shall in particular show how it illuminates the study of formal proofs as mathematical objects in their own right. This will involve looking at the Curry–Howard isomorphism between proofs and programs, and at Linear Logic, a resource-sensitive logic. Both of these topics have many important applications in Computer Science.

Category theory has also deeply influenced the design of modern (especially functional) programming languages, and the study of program transformations. One exciting recent

development we will look at will be the development of the idea of coalgebra, which allows the formulation of a notion of coinduction, dual to that of mathematical induction, which provides powerful principles for defining and reasoning about infinite objects.

This course will develop the basic ideas of Category Theory, and explore its applications to the study of proofs in logic, and to the algebraic structure of programs and programming languages.

Learning Outcomes

To master the basic concepts and methods of categories.

To understand how category-theory can be used to structure mathematical ideas, with the concepts of functoriality, naturality and universality; and how reasoning with objects and arrows can replace reasoning with sets and elements. To learn the basic ideas of using commutative diagrams and unique existence properties.

To understand the connections between categories and logic, focussing on structural proof theory and the Curry–Howard isomorphism.

To understand how some basic forms of computational processes can be modelled with categories.

Synopsis

- Introduction to category theory. Categories, functors, natural transformations. Isomorphisms. monics and epics. Products and coproducts. Universal constructions. Cartesian closed categories. Symmetric monoidal closed categories. The ideas will be illustrated with many examples, from both mathematics and Computer Science.

- Introduction to structural proof theory. Natural deduction, simply typed lambda calculus, the Curry–Howard correspondence. Introduction to Linear Logic. The connection between logic and categories.

- Further topics in category theory. Algebras, coalgebras and monads. Connections to programming (structural recursion and corecursion), and to programming languages (monads as types for computational effects).

- Categories: 10 lectures. 1 each for the following topics:

  - Categories

  - Monics, epics, isomorphisms

  - Products and coproducts

  - Limits and colimits

- Functors and natural transformations

- Universal arrows

- Adjunctions

- Monads

- Cartesian closed categories

- Symmetric monoidal closed categories

- Connection to Logic: 4 lectures

- Algebras and coalgebras: 4 lectures.

Reading List

Lecture notes will be provided.

The following books provide useful background reading.

1. Pierce, *Basic Category Theory for Computer Science.*

2. Lawvere and Schanuel, *Conceptual Mathematics.*

3. Saunders Mac Lane, *Categories for the Working Mathematician.*

4. R. Crole, *Categories and Types.*

5. Girard, Lafont and Taylor, *Proofs and Types.*

6. Bird and de Moor, *Algebra of Programming.*

Of these, the book by Pierce provides a very accessible and user-friendly first introduction to the subject.

CCS3b: Quantum Computer Science — Lecturer T.B.C. — 20 lectures (plus extra reading) HT

Level: M-level.   **Method of Assessment:** Mini-project.

Weight: Half-unit. OSS paper code 294.

Recommended Prerequisites:

We do not assume any prior knowledge of quantum mechanics. Some knowledge of basic linear algebraic notions such as vector spaces and matrices is however a prerequisite. The course notes do comprise an overview of this material so we advise students with a limited background in linear algebra to consult the course notes before the course starts.

Overview

Both physics and computer science have been very dominant scientific and technological disciplines in the previous century. Quantum Computer Science aims at combining both and hence promises to play an important similar role in this century. Combining the existing expertise in both fields proves to be a non-trivial but very exciting interdisciplinary journey. Besides the actual issue of building a quantum computer or realizing quantum protocols it involves a fascinating encounter of concepts and formal tools which arose in distinct disciplines.

This course provides an interdisciplinary introduction to the emerging field of quantum computer science, explaining (very) basic quantum mechanics (including finite dimensional Hilbert spaces and the tensor product thereof), quantum entanglement, its structure and its physical consequences (e.g. non-locality, no-cloning principle), and introduces qubits. We give detailed discussions of some key algorithms and protocols such as Shor's factorization algorithm, quantum teleportation and quantum key exchange, and analyze the challenges these pose for computer science, mathematics etc. We also provide a more conceptual semantic analysis of some of the above. Other important issues such as quantum information theory (including mixed states) will also be covered (although not in great detail). We also discuss alternative computational paradigms and models for the circuit model, we argue the need for high-level methods, provide some recent results concerning graphical language and categorical semantics for quantum informatics and delineate the remaining scientific challenges for the future.

Learning Outcomes

The student will know by the end of the course what quantum computing and quantum protocols are about, why they matter, and what the scientific prospects concerning are. This includes a structural understanding of some basic quantum mechanics, knowledge of important algorithms such as Shor's algorithm and important protocols such as quantum teleportation. He/she will also know where to find more details and will be able to access these. Hence this course also offers computer science and mathematics students a first stepping-stone for research in the field, with a particular focus on the newly developing field

of quantum computer science semantics, to which Oxford University Computing Laboratory has provided pioneering contributions.

Synopsis

1. Historical and physical context
1.1 The birth of quantum mechanics
1.2 The status of quantum mechanics
1.3 The birth of quantum informatics
1.4 The status of quantum informatics

2 Qubits vs. bits
2.1 Acting on qubits
2.2 Describing a qubit with complex numbers
2.3 Describing two qubits

3 von Neumanns pure state formalism
3.1 Hilbert space
3.2 Matrices
3.3 Tensor structure
3.4 Dirac notation

4 Protocols from entanglement
4.1 Bell-base and Bell-matrices
4.2 Teleportation and entanglement swapping

5 The structure of entanglement and graphical calculus
5.1 Graphical calculus for quantum theory
5.2 Map-state duality and compositionality
5.3 The logic of bipartite entanglement
5.4 Quantifying entanglement
5.5 Trace

6 Algorithms and gates
6.1 Special gates
6.2 The Deutch–Jozsa algorithm
6.3 Grover's algorithm
6.4 Shor's factoring algorithm
6.4.1 Period finding
6.4.2 Factoring and code-breaking
6.5 Quantum key distribution

7 Mixed states

8 Quantum logic and Gleason's theorem

9 Mixed operations

10 Semantics for quantum computing
10.1 Quantum computing with relations
10.2 Quantum computing with categories
10.3 Further reading

Reading List

Lecture notes which cover the whole course and which provide detailed pointers to additional reading will be made available. Standard books on the subject that might be of use are:

1. J. Gruska, *Quantum Computing* (McGraw–Hill, 1999).

2. M. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).

3. A. Yu. Kitaev, A. H. Shen and M. N. Vyalyi,*Classical and Quantum Computing*, Graduate Studies in Mathematics 47 (American Mathematical Society, 2001).

On-line available courses elsewhere which can be consulted are:

1. By Braunstein: http://www.weizmann.ac.il/chemphys/schmuel/comp/comp.html

2. By Bub:http://arxiv.org/abs/quant-ph/0512125

3. By Preskill http://www.theory.caltech.edu/people/preskill/ph229/

A different angle which is also very much reflected in the course is available at:

1. http://arxiv.org/abs/quant-ph/0510032

2. http://arxiv.org/abs/quant-ph/0506132

CCS4b — Automata, Logics and Games — Prof. Ong — 16 lectures (plus extra reading) HT

Level: M-level.   **Method of Assessment:** Mini-project.

Weight: Half-unit.

Recommended Prerequisites: Knowledge of first-order predicate calculus will be assumed. Familiarity with the basics of Finite Automata Theory and Computability (for

example, as covered by the course, Models of Computation) and Complexity Theory would be very helpful. Candidates who do not have the required background are expected to have taken the course, Introduction to the Theory of Computation.

Overview

To introduce the mathematical theory underpinning the Computer-Aided Verification of computing systems. The main ingredients are:

- Automata (on infinite words and trees) as a computational model of state-based systems.

- Logical systems (such as temporal and modal logics) for specifying operational behaviour.

- Two-person games as a conceptual basis for understanding interactions between a system and its environment.

Learning Outcomes

At the end of the course students will be able to:

1. Describe in detail what is meant by a Buchi automaton, and the languages recognised by simple examples of Buchi automata.

2. Use linear-time temporal logic to describe behavioural properties such as recurrence and periodicity, and translate LTL formulas to Buchi automata.

3. Use S1S to define omega-regular languages, and give an account of the equivalence between S1S definability and Buchi recognisability.

4. Explain the intuitive meaning of simple modal mu-calculus formulas, and describe the correspondence between property-checking games and modal mu-calculus model checking.

Synopsis

- Automata on infinite words. Buchi automata: Closure properties. Determinization and Rabin automata.

- Nonemptiness and Nonuniversality problems for Buchi automata.

- Linear temporal logic and alternating Buchi automata.

- Modal mu-calculus: Fundamental Theorem, decidability and finite model property. Parity Games and the Model-Checking Problem: memoryless determinacy, algorithmic issues.

- Monadic Second-order Logic and its relationship with the modal mu-calculus.

Reading List

Selected parts from:

1. J. Bradeld and C. P. Stirling, *Modal logics and $^1-calculi$* (in J. Bergstra, A. Ponse, and S. Smolka, editors), Handbook of Process Algebra, pages 293–332 (Elsevier, North–Holland, 2001).

2. B. Khoussainov and A. Nerode, *Automata Theory and its Applications,* Progress in Computer Science and Applied Logic (Volume 21. Birkhauser, 2001).

3. C. P. Stirling, *Modal and Temporal Properties of Processes,* Texts in Computer Science (Springer–Verlag, 2001).

4. W. Thomas, *Languages, Automata and Logic* (in G. Rozenberg and A. Salomaa, editors), Handbook of Formal Languages, volume 3 (Springer–Verlag, 1997).

5. M. Y. Vardi, *An Automata — Theoretic Approach to Linear Temporal Logic*, in Logics for Concurrency: Structure versus Automata, ed. F. Moller and G. Birtwistle, LNCS vol. 1043, pp. 238–266, (Springer–Verlag, 1996).
   The only copy of the above book is in the RSL on open shelves. However, the article in question is available in pdf form online at this address:
   http://folli.loria.fr/cds/1998/pdf/degiacomo-nardi/vardi.pdf

Philosophy

Rise of Modern Logic — Dr Isaacson — HT

Please note this is the draft synopsis. For further details on the Philosophy courses (including details of method of assessment, etc.) please refer to the Philosophy Lectures Prospectus which is published at

http://www.philosophy.ox.ac.uk/ prior to the start of each term (usually in 0th Week).

Lectures on the Rise of Modern Logic have been provisionally arranged for 2009/10 and are likely to run in Hilary Term.

Method of Assessment: A three hour examination paper in Trinity Term and a 5000 word essay to be submitted by Week 0 in Trinity Term.

Weight: Whole-unit.

Essential Prerequisites

Philosophy of Mathematics, B1 Logic and Set Theory.

Synopsis

The eight lectures shall concentrate on Frege's and Russell's logicism, the origins of set theory (Cantor and Zermelo), Hilbert's programme and the impact of Gödel's theorems on it. Further topics will include the rise of first-order logic and intuitionism.

Further details can be found in the Mathematics & Philosophy Handbook.

Reading

Much of the relevant material is contained in

Jean van Heijenoort (ed.), *From Frege to Gödel. A Source Book in Mathematical Logic, 1879–1931* (Harvard University Press).

OD : Dissertations on a Mathematically related Topic

Level : M-level

Weight : Half unit or whole unit.

You may offer either a whole-unit or a half-unit Dissertation in a Mathematically related topic. For example, applications of mathematics to another field (eg Maths in Music), historical topics, topics concentrating on the analysis of statistical data, or topics concentrating on the production of computer-generated data will be acceptable as an Other mathematical option. (Topics in Mathematical education are not allowed).

The *Guidance Notes on Extended Essays and Dissertations in Mathematics* by the Projects Committee and application form are available at:

http://www.maths.ox.ac.uk/current-students/undergraduates/projects/. An OD Dissertation would be 10,000 words in length for a whole-unit or 5,000 words in length for a half-unit. A whole-unit is equivalent to a 32-hour lecture course and a half-unit is equivalent to a 16-hour lecture course. Each project is double blind marked. The marks are reconciled following discussion between the two assessors and overseen by examiners.

Application You must apply to the Mathematics Project Committee in advance for approval. Proposals should be addressed to The Chairman of the Projects Committee, c/o Mr Yan-Chee Yu, Room DH61, Dartington House, and must be received before 12 noon on Friday of Week 3 of Michaelmas Full Term. Note that a CD dissertation must have a substantial mathematical content. Once your title has been approved, it may only be changed by approval of Chairman of the Project's Committee.

Assessment

Dissertations will be assigned USMs according to the same principles as Mathematics papers. In arriving at these marks, the relative weights given to content and presentation will be 75%, and 25%, respectively. The topic should have an appropriate intellectual content which could be broader than that of a CD Dissertation, but it should have close connections with mathematics. Here is a brief explanation of these terms and how they may be interpreted by assessors:

Content: appropriate range and suitability of material selected. The examiners are looking for some of your own thoughts and contributions: you must do more than rehash text books and lecture notes; use of original sources (such as research publications) or some effort to unify diverse treatments of a subject; you must not plagiarise. Connections with Mathematics : the Mathematics, assertions and proofs should be correct, and the mathematics should be appropriate for the level of study.

Presentation: the examiners are looking at how clear the mathematics is and if it is well laid out; the quality of exposition (clear, well-thought out, well-organised); English style (spelling, grammar, readability). Sources should be properly acknowledged, references should be properly cited. Give some thought to notation, choice of typeface, and numbering of equations and sections. Do not fail to number the pages. Be sure to supply complete and accurate references for all the sources used in completing the project, and be sure to cite them properly in the text.

Excellent brief advice on mathematical writing is to be found on the London Mathematical Society website at http://www.lms.ac.uk/publications/documents/writing.pdf.

Students are invited to give an informal presentation of their work to their supervisor before the end of Hilary Term. Although not formally assessed this provides useful feedback to the student and is part of the learning process.

Generally, students would have approximately 8 hours of supervision for a whole-unit dissertation distributed over the two terms. Similarly, for the half-unit approximately 4 hours of supervision.

Submission of Dissertation FOUR copies of your dissertation, identified only by your candidate number, should be sent to the Chairman of Examiners, FHS of Mathematics Part C, Examination Schools, Oxford, to arrive no later than **12 noon on Friday of week 9, Hilary Term 2010.**

4 Language Classes: French and German

Language courses in French and German offered by the University Language Centre.

Students in the FHS Mathematics may apply to take language classes. In 2009-2010, French and German language classes will be run in MT and HT. We have a limited number of places but if we have spare places we will offer these to joint school students , Mathematics and Computer Science, Mathematics and Philosophy and Mathematics and Statistics.

Two levels of French courses are offered, a lower level for those with a good pass at GCSE, and a higher level course for those with A/S or A level. Acceptance on either course will depend on satisfactory performance in the Preliminary Qualifying Test held in Week 1 of Michaelmas Term (Monday, 17.00-19.00 at the Language Centre). Classes at both levels will take place on Mondays, 17.00-19.00. A single class in German at a lower or higher level will be offered on the basis of the performances in the Preliminary Qualifying Test, held at the same time as the French test. Classes will also be held on Mondays, 17-00-19.00.

Performance on the course will not contribute to the class of degree awarded. However, upon successful completion of the course, students will be issued with a certificate of achievement which will be sent to their college.

Places at these classes are limited, so students are advised that they must indicate their interest at this stage. If you are interested but are unable to attend this presentation for some reason please contact the Academic Administrator in the Mathematical Institute (academic.administrator@maths.ox.ac.uk; (6)15203) as soon as possible.

Aims and rationale

The general aim of the language courses is to develop the student's ability to communicate (in both speech and writing) in French or German to the point where he or she can function in an academic or working environment in a French-speaking or German-speaking country.

The courses have been designed with general linguistic competence in the four skills of reading, writing, speaking and listening in mind. There will be opportunities for participants to develop their own particular interests through presentations and assignments.

Form and Content

Each course will consist of a thirty-two contact hour course, together with associated work. Classes will be held in the Language Centre at two hours per week in Michaelmas and Hilary Terms.

The content of the courses is based on coursebooks together with a substantial amount of supplementary material prepared by the language tutors. Participants should expect to spend an additional two hours per week on preparation and follow-up work.

Each course aims to cover similar ground in terms of grammar, spoken and written language and topics. Areas covered will include:

Grammar:

- all major tenses will be presented and/or revised, including the subjunctive

- passive voice

- pronouns

- formation of adjectives, adverbs, comparatives

- use of prepositions

- time expressions

Speaking

- guided spoken expression for academic, work and leisure contact (e.g. giving presentations, informal interviews, applying for jobs)

- expressing opinions, tastes and preferences

- expressing cause, consequence and purpose

Writing

- Guided letter writing for academic and work contact

- Summaries and short essays

Listening

- Listening practice of recorded materials (e.g. news broadcasts, telephone messages, interviews)

- developing listening comprehension strategies

Topics (with related readings and vocabulary, from among the following)

- life, work and culture

- the media in each country

- social and political systems

- film, theatre and music

- research and innovation

- sports and related topics

- student-selected topics

Teaching staff

The courses are taught by Language Centre tutors or Modern Languages Faculty instructors.

Teaching and learning approaches

Each course uses a communicative methodology which demands active participation from students. In addition, there will be some formal grammar instruction. Students will be expected to prepare work for classes and homework will be set. Students will also be given training in strategies for independent language study, including computer-based language learning, which will enable them to maintain and develop their language skills after the course.

Entry

Two classes in French and one in German at (probably at Basic and Threshold levels) will be formed according to level of French/German at entry. The minimum entry standard is a good GCSE Grade or equivalent. Registration for each option will take place at the start of Michaelmas Term. A preliminary qualifying test is then taken which candidates must pass in order to be allowed to join the course.

Learning Outcomes

The learning outcomes will be based on internationally agreed criteria for specific levels (known as the ALTE levels), which are as follows:

Basic Level (corresponds to ALTE Level 2 "Can-do" statements)

- Can express opinions on professional, cultural or abstract matters in a limited way, offer advice and understand instructions.

- Can give a short presentation on a limited range of topics.

- Can understand routine information and articles, including factual articles in newspapers, routine letters from hotels and letters expressing personal opinions.

- Can write letters or make notes on familiar or predictable matters.

Threshold Level (corresponds to ALTE Level 3 "Can-do" statements)

- Can follow or give a talk on a familiar topic or keep up a conversation on a fairly wide range of topics, such as personal and professional experiences, events currently in the news.

- Can give a clear presentation on a familiar topic, and answer predictable or factual questions.

- Can read texts for relevant information, and understand detailed instructions or advice.

- Can make notes that will be of reasonable use for essay or revision purposes.

- Can make notes while someone is talking or write a letter including non- standard requests.

Assessment

There will a preliminary qualifying test in Michaelmas Term. There are three parts to this test: Reading Comprehension, Listening Comprehension, and Grammar. Candidates who have not studied or had contact with French or German for some time are advised to revise thoroughly, making use of the Language Centre's French or German resources.

Students' achievement will be assessed through a variety of means, including continuous assessment of oral performance, a written final examination, and a project or assignment chosen by individual students in consultation with their language tutor.

Reading comprehension, writing, listening comprehension and speaking are all examined. The oral component may consist of an interview, a presentation or a candidate's performance in a formal debate or discussion.