CALCULUS EXERCISES 1 — Curve Sketching

1. Sketch the graph of the curve
$$y = \frac{x^2 + 1}{(x-1)(x-2)}$$
carefully labelling any turning points and asymptotes.

2. The parabola $x = y^2 + ay + b$ crosses the parabola $y = x^2$ at $(1,1)$ making right angles.

Calculate the values of a and b.

On the same axes, sketch the two parabolas.

3. The curve C in the xy-plane has equation
$$x^2 + xy + y^2 = 1.$$
By solving $\mathrm{d}y/\mathrm{d}x = 0$, show that the maximum and minimum values taken by y are $\pm 2/\sqrt{3}$.

By changing to polar co-ordinates, $(x = r\cos\theta,\ y = r\sin\theta)$, sketch the curve C.

What is the greatest distance of a point on C from the origin?

4. Sketch the curve $y = x^3 + ax + b$ for a selection of values of a and b.

Suppose now that a is negative. Find the co-ordinates of the turning points of the graph and deduce that $y = 0$ has exactly two roots when
$$b = \pm\frac{2a}{3}\sqrt{\frac{-a}{3}}$$
For what values of b does the equation $y = 0$ have three distinct real roots?

5. On separate xu- and yu-axes sketch the curves $u = 8(x^3 - x)$ and $u = e^y/y$ labelling all turning points.

[Harder] Hence sketch the curve $e^y = 8y(x^3 - x)$.