CALCULUS EXERCISES 3 — Techniques of Integration

1. Evaluate

$$\int \frac{\ln x}{x}\ \mathrm{d}x, \qquad \int x \sec^2 x\ \mathrm{d}x, \qquad \int_3^\infty \frac{\mathrm{d}x}{(x-1)(x-2)}, \qquad \int_0^1 \tan^{-1} x\ \mathrm{d}x, \qquad \int_0^1 \frac{\mathrm{d}x}{e^x+1}.$$

2. Evaluate, using trigonometric and/or hyperbolic substitutions,

$$\int \frac{\mathrm{d}x}{x^2+1}, \qquad \int_1^2 \frac{\mathrm{d}x}{\sqrt{x^2-1}}, \qquad \int \frac{\mathrm{d}x}{\sqrt{4-x^2}}, \qquad \int_2^\infty \frac{\mathrm{d}x}{(x^2-1)^{3/2}}$$

3. By completing the square in the denominator, and using the substitution

$$x = \frac{\sqrt{2}}{3}\tan\theta - \frac{1}{3}$$

evaluate

$$\int \frac{\mathrm{d}x}{3x^2+2x+1}..$$

By similarly completing the square in the following denominators, and making appropriate trigonometric and/or hyperbolic substitutions, evaluate the following integrals

$$\int \frac{\mathrm{d}x}{\sqrt{x^2+2x+5}}, \qquad \int_0^\infty \frac{\mathrm{d}x}{4x^2+4x+5}.$$

4. Let $t = \tan\frac{1}{2}\theta$. Show that

$$\sin\theta = \frac{2t}{1+t^2}, \qquad \cos\theta = \frac{1-t^2}{1+t^2}, \qquad \tan\theta = \frac{2t}{1-t^2}$$

and that

$$\mathrm{d}\theta = \frac{2\ \mathrm{d}t}{1+t^2}.$$

Use the substitution $t = \tan\frac{1}{2}\theta$ to evaluate

$$\int_0^{\pi/2} \frac{\mathrm{d}\theta}{(1+\sin\theta)^2}.$$

5. Let

$$I_n = \int_0^{\pi/2} x^n \sin x\ \mathrm{d}x.$$

Evaluate I_0 and I_1.

Show, using integration by parts, that

$$I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n\,(n-1)\,I_{n-2}.$$

Hence, evaluate I_5 and I_6.