1. By writing \( \omega = a + ib \) (where \( a \) and \( b \) are real), solve the equation

\[
\omega^2 = -5 - 12i.
\]

Hence find the two roots of the quadratic equation

\[
z^2 - (4 + i)z + (5 + 5i) = 0.
\]

2. By substituting \( z = x + iy \) or \( z = re^{i\theta} \) into the following equations and inequalities, sketch the following regions of the complex plane on separate Argand diagrams:

- \( |z - 3 - 4i| < 5 \)
- \( \arg(z) = \pi/3 \)
- \( 0 \leq \text{Re}((iz + 3)/2) < 2 \)
- \( e^z = 1 \)
- \( \text{Im}(z^2) < 0 \)

3. Find the image of the point \( z = 2 + it \) under each of the following transformations.

- \( z \mapsto iz \)
- \( z \mapsto z^2 \)
- \( z \mapsto e^z \)
- \( z \mapsto 1/z \)

By letting \( t \) vary over all real values find the image of the line \( \text{Re} z = 2 \) under the same transformations.

4. (a) Given that \( e^{i\theta} = \cos \theta + i \sin \theta \), prove that

\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta.
\]

(b) Use De Moivre’s Theorem to show that

\[
\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta.
\]

5. (a) Let \( z = \cos \theta + i \sin \theta \) and let \( n \) be an integer. Show that

\[
2 \cos \theta = z + \frac{1}{z} \quad \text{and that} \quad 2i \sin \theta = z - \frac{1}{z}.
\]

Find expressions for \( \cos n\theta \) and \( \sin n\theta \) in terms of \( z \).

(b) Show that

\[
\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)
\]

and hence find \( \int_{0}^{\pi/2} \cos^5 \theta \, d\theta \).