# Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2014

October 28, 2014

# Part I

# A. STATISTICS

• Numbers and percentages in each class.

See Table 1, page 1.

• Numbers of vivas and effects of vivas on classes of result.

As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

### • Marking of scripts.

The double unit dissertations and single unit dissertations were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A.

• Numbers taking each paper. See Table 7 on page 9.

			Number	r		Percentages $\%$					
	2014	(2013)	(2012)	(2011)	(2010)	2014	(2013)	(2012)	(2011)	(2010)	
Ι	45	(56)	(45)	(47)	(49)	45.92	(47.46)	(45.45)	(46.53)	(46.23)	
II.1	42	41	(36)	(37)	(37)	42.86	(34.75)	(36.36)	(36.63)	(34.91)	
II.2	11	(15)	(15)	(14)	(15)	11.22	(12.71)	(15.15)	(13.86)	(14.15)	
III	0	(4)	(3)	(1)	(5)	0	(3.39)	(3.03)	(0.99)	(4.72)	
F	0	(2)	(0)	(2)	(0)	0	(1.69)	(0)	(1.98)	(0)	
Total	98	(118)	(99)	(101)	(106)	100	(100)	(100)	(100)	(100)	

Table 1: Numbers in each class

#### B. New examining methods and procedures

The examination was conducted according to the same procedures as hitherto. However it should be noted that new Regulations came into force for the 2014 examination whereby

- (i) in order to proceed to Part C, a candidate must have achieved upper second class Honours or higher in Parts A and B together;
- (ii) candidates are required to offer eight 1.5 hour papers (or equivalent), rather than six.

[In the current Regulations the term 'unit' now relates to a paper based on a 16-lecture course, designated a 'half unit' in the previous Regulations.]

# C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

### D. Notice of examination conventions for candidates

The first notice to candidates was issued on 21st February 2014 and the second notice on 5th May 2014.

These can be found at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinationsassessments/examination-conventions, and contain details of the examinations and assessments. All notices and the examination conventions for 2014 examinations are on-line at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examinationconventions.

# Part II

#### A. General Comments on the Examination

The examiners would like to thank in particular Helen Lowe, Waldemar Schlackow and Charlotte Turner-Smith for their commitment and dedication in running the examination systems. We would also like to thank Nia Roderick, Jessica Sheard and Sandy Patel for all their work during the busy exam period.

We also thank the assessors for their work in setting questions on their own courses, and for their assistance in carefully checking the draft questions of other assessors, and also to the many people who acted as assessors for dissertations. We are particularly grateful to those—this year the great majority—who abided by the specified deadlines and responded promptly to queries. This level of cooperation contributed in a significant way to the smooth running of what is of necessity a complicated process.

The internal examiners would like to thank the external examiners Professor Jack Carr and Professor Andrew Thomason for their prompt and careful reading of the draft papers and for their valuable input during the examiners' meeting. In addition the Chairman would like to thank them for coming to Oxford in time for her to have a meeting with them the day before. This facilitated their full involvement in the examiners' meeting and led to constructive discussions on the Part C course and its assessment.

#### Timetable

The examinations began on Monday 2nd June and finished on Monday 16th June. This was made possible by having four papers running in parallel in many of the examination sessions.

#### Medical certificates and other special circumstances

The examiners considered the medical certificates relating the Part C examinations. Details of cases in which special consideration was required are given under reserved business.

#### Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related unit involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses (and also Michaelmas Term course papers submitted late). The cycle was repeated for the Hilary Term courses, with two examiners' meetings in the Easter Vacation; the schedule here was much tighter. Following the preparation of the Camera Ready Copy of the papers as finally approved, each assessor signed off their paper in time for submission to Examination Schools in week 1 of Trinity Term.

A team of graduate checkers, under the supervision of Helen Lowe, sorted all the marked scripts for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, each change was signed by one of the examiners who were present throughout the process. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the marks sheets.

#### **Determination of University Standardised Marks**

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B. The examiners were mindful that the changes in Regulations (reported in Part I, B) could be expected to result in class percentages somewhat different from those in recent years, particularly at the lower end.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students. We outline the principles of the calibration method and then give some information about this year's process.

The Department's algorithm to assign USMs in Part C was used in the same way as last year for each unit assessed by means of a traditional written examination. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers  $N_1$ ,  $N_2$  and  $N_3$  are first computed for each paper:  $N_1$ ,  $N_2$  and  $N_3$ are, respectively, the number of candidates taking the paper who achieved in Part B overall average USMs in the ranges [70, 100], [60, 69] and [0, 59], respectively.

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map  $R \to U$  (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points (100, 100),  $P_1 = (C_1, 72), P_2 = (C_2, 57), P_3 = (C_3, 37), \text{ and } (0, 0)$ . The values of  $C_1$  and  $C_2$  are set by the requirement that the proportion of I and II.1 candidates in Part A, as given by  $N_1$  and  $N_2$ , is the same as the I and II.1 proportion of USMs achieved on the paper. The value of  $C_3$  is set by the requirement that P2P3 continued would intersect the U axis at  $U_0 = 10$ . Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs. The examiners have scope to make changes, usually by adjusting the position of the corner points  $P_1, P_2, P_3$  by hand, so as to alter the map raw  $\rightarrow$  USM, to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional corners.

Table 2 on page 6 gives the final positions of the corners of the piecewise linear maps used to determine USMs from raw marks. For each paper,  $P_1$ ,  $P_2$ ,  $P_3$  are the (possibly adjusted) positions of the corners above, which together with the end points (100, 100) and (0,0) determine the piecewise linear map raw  $\rightarrow$  USM. The entries  $N_1$ ,  $N_2$ ,  $N_3$  give the number of incoming firsts, II.1s, and II.2s and below respectively from Part B for that paper, which are used by the algorithm to determine the positions of  $P_1$ ,  $P_2$ ,  $P_3$ .

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. These revised USM maps provided the starting point for a review of the scalings, paper by paper, by the full board of examiners.

Our general impression was that candidates had risen well to the challenge of taking eight units (as compared with the six hitherto required). There were very few instances where extremely low raw marks occurred, suggesting that almost all candidates had successfully mastered at least the basic bookwork across all their courses. It is worth noting that, unsurprisingly perhaps, the percentage of candidates offering a dissertation (as either a double or a single unit) was significantly up: 35% this year, as compared with 20%-24%in the four preceding years. The borderlines as finally agreed led to all candidates being awarded lower second class honours or higher, with a smaller percentage of II.2s than in any of the preceding four years (see Table 1). This reflects the higher entry requirement for Part C now in force.

Paper	$P_1$	$P_2$	$P_3$	Additional corners	$N_1$	$N_2$	$N_3$
C1.1a	(13.16, 37)	(22.9, 57)	(33.4, 72)		6	6	0
C1.1b	(11.89, 37)	(24, 57)	(35, 72)		3	3	0
C1.2a	(9.59,  37)	(24, 57)	(32, 72)		6	6	0
C1.2b	(12, 37)	(22, 57)	(34,72)		9	11	0
C2.1a	(10.34, 37)	(23, 57)	(35, 72)		12	4	0
C2.1b	(16, 37)	(24, 57)	(32, 72)		8	3	0
C2.2a	(7, 37)	(16, 57)	(29, 72)		16	8	1
C2.2b	(10, 37)	(27, 57)	(35.6, 72)		11	4	0
C2.3b	(4.65, 37)	(22, 57)	(33, 72)		10	5	0
C3.1a	(15.63, 37)	(27.2, 57)	(36.2, 72)		9	3	1
C3.2b	(12, 37)	(23, 57)	(33.2, 72)		6	1	2
C3.3b	(10.4, 37)	(16, 57)	(24, 72)		4	1	0
C3.4a	(18.84, 37)	(30, 57)	(38.8, 72)		8	1	0
C3.4b	(9.65,  37)	(28, 57)	(36, 72)		4	1	0
C4.1a	(10, 37)	(28, 57)	(36, 72)		5	3	1
C4.1b	(8, 37)	(18, 57)	(31, 72)		5	2	0
C5.1a					2	0	0
C5.1b					1	0	0
C5.2b	(28, 63)				3	1	0
C5.3b					2	0	0
C6.1a	(8.33, 37)	(23, 57)	(33, 72)		6	14	1
C6.1b	(25, 60)	(35, 70)			6	15	0
C6.2a	(15.22, 37)	(26.5, 57)	(41, 72)		5	2	0
C6.2a	(5, 37)	(19, 57)	(28, 72)		8	23	3
C6.3a	(6.26, 37)	(19, 57)	(31, 72)		13	26	2
C6.3b	(8.27, 37)	(19, 57)	(30, 72)		10	18	1
C6.4a	(8, 37)	(28, 57)	(37, 72)		8	17	0
C6.4b	(10.28, 37)	(28, 57)	(36, 72)		5	27	0
C6.5b	(4.37, 37)	(13, 57)	(27, 72)		4	8	0
C7.1b	(12.93, 37)	(27, 57)	(37, 72)		1	8	0
C7.2a	(15.51, 37)	(27, 57)	(40, 72)		6	9	1

Table 2: Position of corners of piecewise linear function

Table 4 on page 6 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Av USM	Rank	Candidates with this USM or above	%
93	1	1	1.02
89	2	2	2.04
86	3	4	4.08
85	5	6	6.12
84	7	8	8.16
83	9	9	9.18

Table 4: Percentile table for overall USMs

82	10	12	12.24	
81	13	13	13.27	
80	14	16	16.33	
79	17	19	19.39	
78	20	20	20.41	
77	21	25	25.51	
76	26	26	26.53	
75	27	31	31.63	
74	32	36	36.73	
73	37	39	39.8	
71	40	41	41.84	
70	42	45	45.92	
69	46	51	52.04	

Paper	$P_1$	$P_2$	$P_3$	Additional corners	$N_1$	$N_2$	$N_3$
C7.2b	(9.71, 37)	(22, 57)	(36, 72)		5	5	0
C8.1a	(10.28, 37)	(28, 57)	(36, 72)		3	10	1
C8.1b	(7.24, 37)	(23, 57)	(34, 72)		4	21	1
C9.1a	(15.05, 37)	(26.2, 57)	(35.2, 72)		5	2	1
C9.1b	(14.76, 37)	(28, 57)	(36, 72)		8	2	1
C9.2a	(14.42, 37)	(32, 57)	(43, 70)		10	12	0
C10.1a	(11.95, 37)	(27, 57)	(41, 72)		6	11	0
C10.1b	(14, 37)	(22, 57)	(29.4, 72)		4	4	0
C11.1a	(9, 30)	(17, 46)	(28, 57)	(38, 70)	6	22	2
C11.1b	(12.93, 37)	(30, 57)	(42, 72)		6	15	1
C12.1a	(8.73, 37)	(20, 57)	(42, 72)		4	13	1
C12.1b	(11, 37)	(21, 57)	(36, 72)		3	12	0
C12.2a	(11.66, 37)	(20.3, 57)	(41, 70)		3	14	2
C12.2b	(16.14, 37)	(34, 57)	(44, 70)		5	14	1
MS1b	(7, 30)	(9.5, 37)	(19, 57)	(32, 72)	4	8	1
MS2b	(12.98, 37)	(22.6, 57)	(42, 70)		5	20	1
MS5a	(11.83, 37)	(26, 57)	(42, 70)		3	15	1
MS6a	(11.66, 37)	(20.3, 57)	(40, 72)		0	9	1
MS6b	(8.04, 37)	(20, 60)	(39, 70)		2	7	1

Av USM	Rank	Candidates with this USM or above	%
68	52	55	56.12
67	56	61	62.24
66	62	67	68.37
65	68	71	72.45
64	72	75	76.53
63	76	76	77.55
62	77	77	78.57
61	78	83	84.69
60	84	87	88.78
59	88	92	93.88
58	93	93	94.9
57	94	95	96.94
55	96	96	97.96
54	97	98	100

# B. Breakdown of the results by gender

Table 6, on page 8 shows the performances of candidates broken down by gender.

Class	Total		Mal	e	Female		
	Number	%	Number	%	Number	%	
Ι	45	45.92	38	50	7	31.82	
II.1	42	42.86	29	38.16	13	59.09	
II.2	11	11.22	9	11.84	2	9.09	
III	0	0	0	0	0	0	
F	0	0	0	0	0	0	
Total	98	100	76	100	22	100	

Table 6: Breakdown of results by gender

# C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included.

Paper	Number of	Avg	StDev	Avg	StDev
	Candidates	RAW	RAW	USM	USM
C1.1a	12	33	5.89	72	9.25
C1.1b	6	33.17	6.65	70.17	10.05
C1.2a	12	30.25	7.44	71.42	9.55
C1.2b	20	29.7	7.23	66.9	10.85
C2.1a	16	35.94	9.21	75.75	14.52
C2.1b	11	33.27	8.1	72.82	14.38
C2.2a	25	23.32	8.05	64.88	11.11
C2.2b	15	36.07	11.4	74.6	19.49
C2.3b	15	28.67	8.67	66.67	11.81
C3.1a	13	36.69	6.01	73.85	11.2
C3.2b	9	34.56	11.28	74.67	18.49
C3.3b					
C3.4a	9	41.56	4.19	79.89	9.6
C3.4b					
C4.1a	9	32.11	10.49	66.89	17.16
C4.1b	7	24.57	8.4	63.71	11.61
C5.1a					
C5.1b					
C5.2b					
C5.3b					
C6.1a	21	30.14	7.07	68.14	10.84
C6.1b	21	31.95	7.28	67.71	11.4
C6.2a	7	44.14	6.01	85	11.37
C6.2b	32	24.44	7.43	65.69	10.95
C6.3a	41	27.39	8.74	67.71	12.02
C6.3b	29	27.21	7.85	68.17	10.9
C6.4a	23	35.87	6.6	71.35	12.04
C6.4b	30	34.87	5.24	70.27	9.63
C6.5b	12	18.25	7.96	62	9.54
C7.1b	9	35	5.15	70	8.96
C7.2a	16	37.62	5.66	71.12	9.16
C7.2b	10	28.6	9.02	65	13.09
C8.1a	13	32.15	7.86	65.92	13.52
C8.1b	26	29.12	5.52	65.69	7.68
C9.1a	8	36.5	8.05	75.12	14.51
C9.1b	11	38.82	7.65	78.09	14.47
C9.2a	22	40.41	6.57	71.32	14.86
C10.1a	14	35.71	4.91	66.57	6
C10.1b	8	27.5	4.87	67.38	8.42

Table 7: Numbers taking each paper

Paper	Number of	Avg	StDev	Avg	StDev
	Candidates	RAW	RAW	USM	USM
C11.1a	29	34.17	11.46	68.9	18.6
C11.1b	20	37.9	5.65	68.6	9.49
C12.1a	18	28.89	6.72	65.78	7.08
C12.1b	14	28.57	9.48	64.57	13.43
C12.2a	19	36.32	8.49	71.32	11.14
C12.2b	19	41.42	5.43	68.89	9.7
MS1b					
MS2b	18	38.61	6.24	71.5	9.63
MS5a	10	39.3	5.93	70.2	8.72
MS6a					
MS6b					
C6					
CCS1					
CCS3					
Single Unit CD					
Dissertation					
Double Unit CD Dissertation	29	-	-	74.83	7.01
Double Unit OD Dissertation					

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included.

# Paper C1.1a: Model Theory

Question	Mean Mark		Std Dev	Numb	er of attempts
	All	Used		Used	Unused
Q1	15.58	15.58	2.87	12	0
Q2	17.64	18	3.83	10	1
Q3	14.50	14.5	2.12	2	0

# Paper C1.1b: Gödel's Incompleteness Theorems

Question	Mean Mark		Std Dev	Number of attempts		
	All	Used		Used	Unused	
Q1	15.75	15.75	1.26	4	0	
Q2	16.83	16.83	5.49	6	0	
Q3	17.5	17.5	2.12	2	0	

# Paper C1.2a: Analytic Topology

Question	Mean Mark		Std Dev	Numb	er of attempts
	All	Used		Used	Unused
Q1	7			0	1
Q2	14.58	14.58	5.42	12	0
Q3	15.67	15.67	3.77	12	0

Paper C1.2b: Axiomatic Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.56	15.56	4.36	9	0
Q2	14.53	15.13	3.66	16	1
Q3	13.31	14.13	5.64	15	1

# Paper C2.1a: Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.06	21.06	2.93	16	0
Q2	17	17	9.49	4	0
Q3	14.17	14.17	6.81	12	0

# Paper C2.1b: Representation Theory of Symmetric Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.9	16.9	4.65	10	0
Q2	16.22	16.22	3.31	9	0
Q3	17	17	7.94	3	0

# Paper C2.2a: Commutative Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.67	11.67	6.68	24	0
Q2	11.96	12.39	3.82	23	1
Q3	4.75	6	2.87	3	1

# Paper C2.2b: Homological Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.58	18.58	4.66	12	0
Q2	18.7	18.7	5.96	10	0
Q3	16.38	16.38	9.53	8	0

# Paper C2.3b: Infinite Groups

Question	Mean Mark		Std Dev	Numb	er of attempts
	All	Used		Used	Unused
Q1	12.13	12.93	5.38	14	1
Q2	14.17	14.17	6.91	6	0
Q3	16.4	16.4	4.12	10	0

Paper C3.1a: Algebraic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.91	16.91	3.42	11	0
Q2	19.54	19.54	3.78	13	0
Q3	18.5	18.5	2.12	2	0

# Paper C3.2b: Geometric Group Theory

Question	Mean Mark		Std Dev	Numb	er of attempts
	All	Used		Used	Unused
Q1	19.33	19.33	5.1	9	0
Q2	17	17	6.88	7	0
Q3	9.00	9.00	2.83	2	0

# Paper C3.4a: Algebraic Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	22.11	22.11	1.76	9	0
Q2	18.86	18.86	3.29	7	0
Q3	21.5	21.5	0.71	2	0

Paper C4.1a: Functional Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.88	15.88	5.49	8	0
Q2	17.33	17.33	7.03	6	0
Q3	13.4	14.5	5.94	4	1

# Paper C4.1b: Linear Operators

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.6	10.6	4.04	5	0
Q2	12.43	12.43	4.79	7	0
Q3	16	16	4.24	2	0

# Paper C6.1a: Solid Mechanics

Question	Mean Mark		Std Dev	Numb	er of attempts
	All	Used		Used	Unused
Q1	20.15	20.15	3.51	20	0
Q2	9.83	17.33	8.77	3	3
Q3	9	9.37	5.59	19	1

# Paper C6.1b: Elasticity and Plasticity

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.4	16.4	3.73	20	0
Q2	14.46	14.46	5.24	13	0
Q3	15.8	17.22	7.41	9	1

# Paper C6.2a: Statistical Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.25	19.25	2.75	4	0
Q2	22.4	22.4	4.22	5	0
Q3	24.	24	1	5	0

# Paper C6.2b: Networks

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.59	11.59	4.96	32	0
Q2	12.61	12.93	4.42	30	1
Q3	5.4	11.5	5.81	2	3

Paper C6.3a: Perturbation Methods

Question	Mean Mark		Std Dev	Numb	er of attempts
	All	Used		Used	Unused
Q1	13.84	14.5	5.38	30	2
Q2	8.95	10.22	5.94	18	3
Q3	14.46	14.82	5.01	34	1

Paper C6.3b: Applied Complex Variables

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	8.53	8.53	1.46	17	0
Q2	13.75	15	5.97	22	2
Q3	16.53	16.53	4.98	19	0

# Paper C6.4a: Topics in Fluid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17	17	3.82	17	0
Q2	18.29	18.29	4.01	17	0
Q3	18.75	18.75	2.90	12	0

Question	Mean Mark		Std Dev	Numb	er of attempts
	All	Used		Used	Unused
Q1	17.53	17.53	3.27	30	0
Q2	14.33	15	3.85	9	3
Q3	17.13	18.33	5.39	21	2

Paper C6.4b: Stochastic Modelling of Biological Processes

# Paper C6.5b: Mathematical Mechanical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.5	9.5	3.63	10	0
Q2	12.88	14.43	7.04	7	1
Q3	3	3.29	0.94	7	3

# Paper C7.1b: Quantum Theory and Quantum Computers

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.88	18.29	4.39	7	1
Q2	17.11	17.11	4.73	9	0
Q3	12.67	16.5	6.81	2	1

Paper C7.2a: General Relativity I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.5	17.0	3.39	14	0
Q2	18	18	3.56	4	0
Q3	20.36	20.36	4.52	14	0

# Paper C7.2b: Relativity II

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	9.78	9.78	3.87	9	0
Q2	17.4	17.4	4.09	10	0
Q3	24	24		1	0

# Paper C8.1a: Mathematical Geoscience

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12	12	3	3	0
Q2	14.92	14.92	4.62	12	0
Q3	18.45	18.45	3.62	11	0

Paper C8.1b: Mathematical Physiology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.11	17.11	3.92	9	0
Q2	14.4	14.4	3.12	25	0
Q3	13.5	13.5	3.59	18	0

# Paper C9.1a: Modular Forms

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	19.33	19.33	3.27	6	0
Q2	14.4	14.4	3.85	5	0
Q3	18.5	20.8	7.84	5	1

# Paper C9.1b: Elliptic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17	17	7.48	5	0
Q2	18.44	20.5	7.45	8	1
Q3	19.78	19.78	4.02	9	0

Paper C9.2a: Analytic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	22.1	22.1	2.79	20	1
Q2	18.21	18.21	5.74	14	0
Q3	19.2	19.2	5.31	10	0

# Paper C10.1a: Stochastic Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.71	18.71	3.15	14	0
Q2	7	-	5.66	0	2
Q3	17	17	3.70	14	0

Paper C10.1b:	Brownian	Motion	$\mathbf{in}$	Complex	Analysis
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Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.67	11.60	3.78	5	1
Q2	4.4	5.67	2.88	3	2
Q3	18.13	18.13	3.18	8	0

Paper C11.1a: Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.76	14.27	5.7	15	2
Q2	18.04	18.04	6.69	24	0
Q3	18.11	18.11	5.16	19	0

# Paper C11.1b: Probabilistic Combinatorics

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	14.83	14.83	2.04	6	0
Q2	17.14	17.14	5.35	14	0
Q3	21.45	21.45	2.7	20	0

Paper C12.1a: Numerical Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.73	12.86	3.83	14	1
Q2	15.06	15.06	4.43	18	0
Q3	17.25	17.25	6.02	4	0

Paper C12.1b: Continuous Optimization

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15	15	2.83	14	0
Q2	12.92	14	8.32	12	1
Q3	11	11	5.66	2	0

Paper C12.2a:Approximation of Functions

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.58	18.58	3.93	19	0
Q2	14.85	15.3	5.11	10	3
Q3	17.33	20.44	7.45	9	3

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.88	19.33	3.46	15	1
Q2	22.06	22.06	2.86	17	0
Q3	20.33	20.33	6.68	6	0

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.13	18.13	4.41	15	0
Q2	20.56	20.56	3.41	16	0
Q3	14.71	18.8	7.45	5	2

#### Paper MS2b: Stochastic Models in Mathematical Genetics

# Paper MS5a: Probability and Statistics for Network Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.56	21.56	2.74	9	0
Q2	16.43	19.4	5.62	5	2
Q3	17	17	4.6	6	0

### D. Recommendations for Next Year's Examiners and Teaching Committee

# Changes in Regulations with effect from the 2014 examination

The guidance note on class percentages supplied by the Teaching Committee was based solely on historic data, and took no account of how the percentages might be expected to change as a result of the higher entry hurdle for Part C and by the increased workload. It would have been helpful if data from previous cohorts had been available to indicate how much cross-over normally occurs between classes in Part B and Part C for members of a given cohort.

Three candidates awarded II.2s in Part B had been granted special dispensation to proceed to Part C, but otherwise all candidates for Part C had been awarded firsts or upper seconds last year. This was reflected in the final class list in that no thirds, passes or fails were awarded this year and only two candidates had an average USM below 55. This outcome may be compared with the data in Table 1, showing that in each of the previous four years between 3 and 6 candidates candidates obtained third class honours in Part C or failed.

### The number of courses in Part C

The 2014 examiners were very aware that the total number of Mathematics courses in Part C has been rising steadily over recent years:

Academic Year	Number of Courses
2008-9	32
2009 - 10	33
2010-11	34
2011 - 12	36
2012 - 13	41
2013-14	45
2014 - 15	47

These figures leave out of account the courses in Statistics and in Computer Science candidates may also offer.

We greatly welcome the inclusion in Part C of courses close to research frontiers and in new areas, and the list being regularly refreshed. Nevertheless we have concerns about the steady increase in the number of courses.

It would not be appropriate for the examiners to argue the academic pros and cons of having a very large number of courses on offer, and we focus our comments primarily on the issues such a menu raise for assessment. First and foremost, there are clear concerns about quality assurance: maintaining appropriate control and ensuring comparability of standards across the full spectrum of courses. The examiners this year have benefited from good cooperation by the assessors, with most papers submitted on time and in generally good shape, We were duly grateful for this. But we note that, notwithstanding, a heavy workload was imposed on the board of examiners (5 internal, 2 external) and on the staff of the Academic Office in getting all the papers set, checked, vetted and revised as necessary; the examinations mounted and the scripts marked and checked; and USMs determined, paper by paper. Every paper results in substantial work, for the assessor, the checker, the examiners, and for the administrative staff. We draw attention to the number of Mathematics papers in the 2014 examination offered by few or very few candidates: six papers had no more than 5 candidates, and two papers had 1 or 2 candidates; All these papers were in the areas of Geometry or of PDE's (two of the latter will not be offered next year). Papers with very low take-up pose particular issues as regards comparability. The large number of courses on offer also allows candidates, should they choose to do so, to specialise narrowly in their fourth year. We do not have evidence of excessive specialisation in practice, but we do believe that the course should be set up so as to discourage this.

In relation to lecturers/assessors we note the considerable pressures that examining imposes on faculty members, and it is not clear that the benefit of offering so many courses for Part C outweighs the disadvantages in terms of use of manpower and academic resources (provision of teaching, in terms of small intercollegiate classes, or even in tutorials, for courses with low undergraduate numbers is also labour intensive). We note also the increase this year to 35% in the proportion of Part C candidates offering a dissertation also has resulted in considerable additional burdens in terms of supervision, administration and assessment.

#### Issues in relation to new courses

Two newly introduced courses, C6.2b Networks and C6.5b Mathematical Mechanical Biology, gave rise to complaints from candidates forwarded by the Proctors Office. In both cases the examiners were able to respond that the complaints, in relation to the appropriateness of the questions set, were without basis. Nonetheless general issues do arise which we recommend should be addressed by the Teaching Committee.

In the case of C6.5b the lecturer realised by the time the course began that the published synopsis had been over-ambitious, and announced that only two of the five topics listed would in fact be covered, in the lectures, problem sheets and examination. The Synopses form a Supplement to the Course Handbook, to which the Examination Regulations refer, and as such contain the official syllabus for each course. We therefore do not regard it as adequate that amendments to a syllabus should de facto be made via an announcement in a lecture. We suggest that all lecturers giving new courses, who may have good reason

to need to vary a synopsis published long in advance, should be advised that the Teaching Committee (and the Subject Panel Convenor) should be informed, so that an official announcement can be issued and course documentation updated.

### **Courses in Numerical Analysis**

The examiners were concerned that the examination questions in this area seemed not to be as challenging as those set on other disciplines within Mathematics. They believe this to be a recurrent issue, and not specific to this year's papers. A number of possible reasons can be suggested: the need to draw on elementary material, for example from Part A or from Prelims-level analysis; greater predictability of what will be set in exam questions than occurs in other areas; some questions rather short; questions lacking a genuinely tough tail part. We have particular concerns about C12.2a. Approximation of Functions, in which both raw marks and USMs were conspicuously high this year.

Following a recommendation by the Examinations Committee in its review of the 2012 examinations it is now standard practice that in Michaelmas Term Subject Panels are asked to review the examination performance in the courses in Parts B and C in their areas; they are provided with the summaries of marks for these courses (from Section D) and the assessors' reports, as well as the corners finally used in the algorithm (Table 2). We recommend that additionally Panels should be asked to pay particular attention to courses which had been given for the first time. On this occasion we request that C6.2b and C6.5b should be reviewed, especially in relation to the way their synopses are presented. We also request that the examiners' general concerns in relation to Numerical Analysis papers be conveyed to the Panel.

The student complaints about C6.2b and C6.5b, and the comments we received from the lecturers in response, lead us to wonder whether these courses—and perhaps some other Part C mathematical modelling courses too—are not well suited to assessment by traditional written examinations. We note in particular a comment that problem sheet questions on C6.2b are much longer than examination questions can be, and so are different in style. We also wonder whether there might be scope for the introduction into Part C of an analogue of the Part B Structured Projects Option: specific projects involving techniques of numerical computation (but with less formal supervision than at Part B level).

We are very pleased that the numbering system for Part C papers has been rationalised for 2014/5.

### Chairman's note

After the examiners' meeting, and so not recorded above, the examiners have had some discussion by email about the structure of Part C arising out of their concerns about the number of courses. This has revealed positive support from four of the examiners for a pattern whereby candidates would take fewer, but longer, courses. The other examiners have not expressed views on this.

At present candidates offer 8 units, a unit comprising a 16-lecture course or equivalent. One may question whether 16-lecture blocks are appropriate at Part C level. Longer courses would allow subjects to be studied in greater depth, as befits M-level, and would be likely to be more suitable for graduate students, in particular those on CDT programmes. Fewer, longer, courses would reduce fragmented choices on the one hand and near-overlaps between courses on the other. More effective use could be made of teaching resources (fewer very small classes) and an examination with fewer papers would definitely be much easier to operate both administratively and in terms of quality control. In addition the everincreasing menu of 16-lecture courses is contributing to difficulties with lecture timetabling and avoidance of clashes.

It is worth remembering that the pattern of courses of 16 lectures dates from a bygone era. It used to be the case that a high proportion of established posts within the Faculty were CUF lecturerships, carrying a lecturing obligation of 16 lectures per year. UL contracts stipulate a higher number of lectures and are operated much more flexibly. I note also that Cambridge Part III has a mixture of 16-lecture(2-unit) and 24-lecture (3-unit) courses, with 19 units required in total. A similar mixed pattern might suit Oxford too, to accommodate the needs of the various subject panels.

The main arguments against restructuring Part C would be (i) that the present requirement of 8 units based on a unit having the weight of 16 lectures has run for only one year so far; (ii) Joint Schools. On (i) it may be noted that some re-organisation of Part C will be necessary next year in any case when the cohort who have taken Prelims and the restructured Part A reach the fourth year. Accommodating the Joint Schools is always a problem (I recall the recent contortions in relation to Part C Mathematics & Philosophy) and MMathPhys (first examination in 2016) introduces an additional complication. Nonetheless, a structure analogous to that in Cambridge might be viable.

#### E. Comments on papers and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. Some data to be found in Section C above have been omitted.

#### C1.1a: Model Theory

Overall all three questions proved to be harder this year compared to last year in particular and to previous years on average. This was an intended consequence of putting more 'unseen' problems into the exam questions.

Although one could find a simple solution to any of the problems, the unfamiliar formulations scared the candidates away from looking for solutions. In particular, probably because of this reason only 4 candidates attempted question 3.

But bookwork questions also caused troubles to many candidates. It looks as if those students glossed over some delicate points in the bookwork material.

Every candidate except one attempted question 1, but most could not handle the unseen part and many had gaps in writing down the standard material. Only one scored above 21.

#### C1.1b: Gödel's Incompleteness Theorems

The exam was sat by 10 candidates: 2 Mathematics & Computer Science, 2 Mathematics & Philosophy and 6 Mathematics students. The average raw mark was somewhat lower than expected. All 10 candidates attempted Question 2; 6 also attempted Question 1, and 4 Question 3.

Question 1

1(a): First part: bookwork, generally well done. Second part: deceptively easy, poorly done.

1(b) First part: bookwork, generally well done. Second part: one or two sentences would have sufficed for full marks; not well answered.

1(c): Candidates mostly failed to appreciate that the same general style of argument as for 1(b) would work.

Question 2

2(a): Generally, though not uniformly, well done.

2(b): Mostly bookwork, generally very well done.

2(c): Very similar to a problem sheet question, but a bit trickier because of the multiple inductions involved. Poorly answered. Some candidates proved that PA proves the commutativity of numerals (rather than variables) under addition.

Question 3

3(a): Bookwork, generally very well done.

3(b): (i)&(ii) Not well answered. Candidates did not seem to have revised the logic GLS for the exam.

3(c): (i) Bookwork, very well done. (ii) Candidates found this hard.

3(d): Bookwork, generally well done.

3(e): Poorly done, perhaps because candidates had run out of time by this stage.

#### C1.2a: Analytic Topology

The questions on this paper seemed to be more demanding than anticipated. There were only very few attempts on Question 1 and most of these seem to have been aborted early on.

There were some very good answers on Questions 2 and 3 with each part of each question being completed by at least one candidate.

Bookwork was done mostly correctly, however a number of candidates seemed to have problems understanding what they were asked to do.

In Question 2(a), some only showed uniqueness of the maximal Hausdorff compactification, whilst others only showed that the greatest Hausdorff compactification must have the Stone-Čech property.

In Question 3(b), when trying to show that  $X_M$  is normal, a large number of candidates took  $X_M$ -closed disjoint subsets of  $X_M$  and claimed (with erroneous proofs) that  $\overline{C}^X \cap D =$  $\emptyset = C \cap \overline{D}^X$  in order to apply part (a). However, taking X = M = [0, 1] and C, D dense, co-dense in [0, 1] shows that this is incorrect. The correct way would have been to consider  $C' = C \setminus M$  and  $D' = D \setminus M$ , apply the above reasoning to C', D' and deal with points from M (which are isolated in  $X_M$ ) separately.

### C1.2b: Axiomatic Set Theory

The general standard of bookwork and its direct applications was high. However, candidates had difficulties applying their knowledge in new, creative ways to solve the harder last parts. The three most common mistakes were:

- leaving out 'easy' parts of the bookwork;
- in Question 1(d) for  $x \in H_{\kappa}$  it was not enough to show that  $|\mathcal{P}(x)| < \kappa$  but to use regularity of  $\kappa$  to show that  $|TC(\mathcal{P}(x))| < \kappa$ ;
- in 2(b) [Replacement] and 3(a) [defining *Def*] and (c) [Separation], a number of candidates failed to take the appropriate relativization of the formulae in question.

#### C2.1a: Lie Algebras

Question 1 was found relatively straightforward, as had been intended. It was noticeable though that students spent a lot of time on it, and had a lot less time for their other question. Two common gaps were that some students forgot to check in (a) that the given subset is a linear subspace, and in (f) some failed to mention that since g is semisimple, its Killing form is nondegenerate (which was needed to apply the quoted result about dimensions).

Question 2 only got a few attempts. Only two people gave substantially complete proofs for the non-reducibility of some representations in characteristic p.

Question 3 was also quite straightforward, with people with enough time on their hands achieving reasonable results. One common source of difficulty was remembering how best to denote the weights of the Lie algebra sl(n).

#### C2.1b: Representation Theory of Symmetric Groups

Question 1. Bookwork in part (a) caused no problems. Solutions to bookwork in part (b) often were incomplete. Relevant theorems were partly not cited, or students had the right idea but had problems to write it down correctly. Only few students attempted part (c), with some problems with the induction beginning.

Question 2. Some students had gaps in their bookwork knowledge in part (a). The first part of (b) was solved fine, but only few students understood how to get the last part of (b), partly involving guessing.

Question 3. Only a few students attempted this question, but those who did, did often very well, only having minor mistakes, or missing out on parts of the argument at the very end of the question.

#### C2.2a Commutative Algebra

Question 1. Mostly bookwork, well done by some candidates. Only one managed the final bit (giving counterexamples).

Question 2. (a) and (b) bookwork, mostly well done. Unfamiliar concept in (c) flummoxed most candidates, and only one got very far with it. Small error in (c)(iii): should stipulate  $x \neq 0$  (two candidates spotted this).

Question 3. Only two candidates attempted this. (a) was bookwork and done O.K. The rest was too hard and they got nowhere with it.

Overall: Few candidates could cope with an unseen problem.

#### C2.2b: Homological Algebra

Question 1. Parts (a) and (b): High standard. No completely correct solution, but some near perfect ones. Many students spent too much time struggling with )c) and (d) not allowing sufficient time to finish other parts of the exam.

Question 2. High standard of answers. Several perfect and close to perfect solutions.

Question 3. High standard of answers. Several perfect solutions.

#### C2.3b: Infinite Groups

Question 1. This was the most popular question. Nobody could give a complete answer to (d), but several decent attempts. Part (c) was the most technically challenging with many wrong arguments.

Question 2. Surprisingly few correct answers for (b). Few candidates spotted how to solve (d) using orders of  $a^b$  and  $a^e$ .

Question 3. Another very popular question. Common errors in (c) was to assume that H is normal in G.

#### C3.1a: Algebraic Topology

Question 1. This was a relatively straightforward but long question. The computations were not always carried out well with candidates getting lost in the algebra. They should have recognised that the left delta-complex represents a projective plane while the right one represents a Möbius strip. The second part of the question was generally done well though none of the candidates realised that the first homology group of  $X_{\bullet}$  is a cyclic group of order four.

Question 2: This question was done relatively well and in retrospect seems to be a little too easy even though marks were lost when candidates were somewhat imprecise in the second half of part (a). Many students recognised part (b) as Brouwer's fix-point theorem but had trouble to finish the suggested line of proof or to prove it with by a different argument. Marks were lost for simply stating that the map  $\tilde{g}$  was the identity when restricted to the boundary sphere.

Question 3. This question covered the latter part of the course and required a good understanding of the main theorems, the Universal Coefficient Theorem and the Poincaré Duality Theorem. Most marks were lost in the first part of the problem when proving the Universal Coefficient Theorem. It was pleasing to see that the students had no trouble at all proving the application in part (b).

### C3.2b Geometric Group Theory

Question 1. This was a basic question about algorithmic questions in groups and free groups attempted by all candidates. Most candidates missed the point of the last bit of (a)(i), enumerating the elements of G rather than the words in S. Part (a)(ii) was standard bookwork. In (a)(ii) some candidates failed to deal with the case of elements not in the subgroup. In the second part, (b)(i) and (b)(ii) were generally well done but in (b)(ii) some candidates failed to provide a complete argument for the rank of C.

Question 2. This was a question on amalgamated products and actions on trees. Some candidates failed to state Kurosh's theorem correctly and did not manage to sketch a proof of this using actions on trees (bookwork). Many candidates did well in part (c), the first part of which was bookwork. Some argued using residual finiteness rather than modifying the argument of the first part. Only two managed to show that the kern2e is not finitely generated.

Question 3. This question was on the geometric part of the course dealing with quasiisometries and hyperbolic groups. They generally did well on the first part but some had difficulties with the second part—they correctly tried to apply thinness of triangles but failed to obtain the required inequalities.

### C3.3b: Differentiable Manifolds

Question 1. Most candidates used a basis of eigenvalues for the functions f, g and avoided any discussion of manifolds.

Question 2. Nobody understood part (c), or used the given property of trace. Only one candidate discussed the target space of O(3) in (d)(ii). Several thought that the cohomology of O(3) mapped to that of Gl(n) rather than the other way round.

Question 3. Most candidates knew what they had to do here.

### C3.4a: Algebraic Geometry

All questions involved relatively long bookwork.

Question 1. The last part, on describing the orbits of the given action, turned out to be more difficult than expected.

Question 2. The main difficulty was the calculation of the Hilbert polynomials in the last part of the question. No complete solution for part (c)(ii) was given.

Question 3. This was the least popular question though those who attempted it scored very well. The bookwork was less involved, the question rather focused on calculation with blow ups in the last part which went very well.

#### C3.4b: Lie Groups

Question 1. Everyone attempted question 1 perhaps because it looked familiar. The trickiest part is (e) since the topology of the subgroup is not necessarily the subspace topology. Some candidates did not realise the previous parts build up to solve (e).

Question 2. Nobody remembered at the end of (f) to use the Weyl invariance of characters they had proved in (d).

Question 3. Nobody attempted part (f) perhaps these candidates did not remember the definition of root under time-pressure. In part (c), only one candidate actually used part (b) as the question suggested, the others got stuck without the help of (b).

#### C4.1a: Functional Analysis

*Question 1.* This question, was fairly straightforward no candidate got full marks. In particular it was surprising that only one candidate was able to give an example of a closed subspace of a Banach space which is not topologically complemented.

Question 2. The question went well for the majority who attempted it. Marks were mainly lost on the last part of (b) concerning the closed range of the operator T + K.

*Question 3.* The bookwork part (a) seems to have gone down well with those who tried, and also the first part of (b) went well for the majority. However, nobody was able to exhibit an example showing that the result fails without completeness despite it being very close to an example discussed in lectures. Apart from one good attempt part (c) did not attract much attention.

#### C4.1b: Linear Operators

This was a new course for the lecturer and for Oxford (apart from some overlap with a course given by a different lecturer in 2000–2005). The candidates found the questions quite long and harder than expected.

Question 1 included material from the first few, and the last few, lectures. Some of the candidates had difficulty even with the early material on basic theory of closed operators in part (a), and most of them were uncomfortable with semigroup generators in (b) and (c).

Question 2. Several candidates coped competently with the standard material, but nobody made much progress with part (c).

Question 3 was the least popular but most successful question. Part (a) was familiar, but complicated, material, and the question was attempted only by candidates who had made an effort to understand that material.

#### C5.1a: Methods of Functional Analysis for PDEs

Question 1. Bookwork has been done well. However, in the second part of question 1, justification of the formula for the gradient turned out to be difficult for all candidates attempting the question.

Question 2. No problem with bookwork. Difficulties occurred in the second part of the question, in particular in showing that the energy identity holds true globally. Surprisingly, a sharper estimate from below for the first eigenvalue of the Laplace operator with the Dirichlet boundary condition has been derived by all candidates attempting the question.

Question 3. This question is more difficult than the previous ones conceptually. However, it has been done well by all candidates attempting it.

#### C5.1b: Fixed Point Methods for Nonlinear PDEs

Question 1. Not attempted.

Question 2. The problem is an amalgamation/elevation of the material presented over two weeks during the course, with a bit of a twist.

Question 3. This question is a standard one on monotone operator and was handled very well.

#### C5.2b: Calculus of Variations

Question 1. This question tested basic knowledge of the relationship between a function's magnitude and the integrability of its derivative. The methods required advanced calculus and basic Sobolev space theory for functions in one dimension.

Question 2. This question tested basic understanding of weak and strong local minimisers in the calculus of variations. The second part required computing solutions to simple ordinary differential equations, and using those solutions to ascertain if the necessary conditions from the first part had been met. The computations did not appear to present too many difficulties. Most marks were lost in sub-part (iii) of part (b), and most likely stemmed from the nonlinear nature of the resulting equation.

Question 3. The first part of this question required the derivation of the weak form of the Euler–Lagrange equations, and its associated integrated form. From this, the students were asked to show that the a priori weak solution has  $C^3$  regularity. The students who did not answer this correctly, appeared to have some difficulties with writing the correct form of the integrated Euler–Lagrange equations, and hence could not properly start the bootstrap procedure for obtaining regularity. The second part of the question test the socalled direct method of the calculus of variations, and required that students understand the concepts of coercivity and lower semicontinuity. Some difficulties were encountered in both deriving the Euler-Lagrange equations and in establishing correct lower bounds for the integral functional.

#### C5.3b Hyperbolic Equations

All the candidates had a good understanding of the course material. Some of them have difficulties with Question 1(b). This is an extension of the standard Riemann problem. It does need students equipped with standard DE techniques. Question 2 should be standard material. Some students dropped crucial details of the proof.

Question 3 is an interesting application of the standard energy approach. No-one progressed beyond (a).

#### C6.1a: Solid Mechanics

Question 1. Students did very well on this question with most of them succeeding to answer most of the question.

Question 2. Only a few of the students tried this question as it was more theoretical. If the material was well understood the question was easy (not much computation) and those students did very well.

Question 3. This question was poorly done unfortunately despite the large number of similar examples done in problem sheets and in the consultation classes. Most students could not find the centrifugal acceleration of a point on a uniformly rotating cylinder (which is standard material from previous years). This disabled them from finishing the question as they needed the acceleration term in the Cauchy equation.

#### C6.1b: Elasticity and Plasticity

Question 1. This was the most popular question and generally quite well answered. Most were able to follow the bookwork needed for part (a), although some key steps were often muddled or bluffed. For the end of part (b), very few candidates showed convincingly that W > 0 unless  $\mathcal{E} = 0$ , and several attempted to prove the wrong implication (i.e. only if, rather than if). Part (c) was a very straightforward application of the identity ( $\star$ ) proved in part (a). However, many candidates instead tried to follow the energy argument given in lectures for the steady problem, which was irrelevant here.

Question 2. This question was quite popular but generally found rather difficult. Parts (a) and (b) were all bookwork, but many candidates struggled to reproduce key steps. In part (c), few managed the basic analysis needed to demonstrate convincingly when the string starts to make contact with the obstacle, and even fewer succeeded in evaluating the contact set.

Question 3. This question was the least popular but attracted the best solutions. Alarmingly, in part (a), no-one knew how to maximize a linear function of  $\sin(2\theta)$  and  $\cos(2\theta)$ ! Parts (b) and (c) were similar to previously seen problems, and those candidates who had properly learned and understood the material were able to generalise well the approach given in the lectures.

#### C6.2a: Statistical Mechanics

I had imagined the spectrum of material in this new course would provide a stern test for the students, but they were well prepared and found the questions easy.

#### C6.2b: Networks

Question 1. Many students had trouble with part (a), though this was supposed to be easy. Most students completed part (b)(i). Part (b)(ii) was more challenging, and question (b)(iii) also gave many people trouble, though there were some good solutions. Perhaps this question was slightly too hard overall, but in the main answers were OK apart from for part (a).

Question 2. This question had one unfortunate typo (corrected during the exam), which was taken into account while marking. In part (a) many people just wrote down the functional form of a generating function without including definitions of the quantities used, which was not enough for full marks. Many people did well with part (b). Part (c) was mixed: some did well, and others had trouble. Many people completed the first part of (d), but few people were able to get expressions for u and v, and not that many people even brought up the idea of an excess degree distribution, despite the idea appearing on several similar problems on problem sheets. Part (e) had the unfortunate use of the term "explicit" which I can imagine students would use to mean "closed-form". In fact, the answer is not in closed form and nobody got the final answer, although a few students did recognize the procedure they needed to follow. This possible misunderstanding was taken into account when marking. There were a range of answers for part (f), including some nicely nuanced ones. Many people did well on the final part.

Question 3: Very few students attempted question 3, which predominantly covered the last part of the course (i.e. predominantly in the last 4 hours worth of lectures and problem sheet 4). I don't feel that question 3 was harder than the others, but perhaps the students found it intimidating.

#### C6.3a: Perturbation Methods

Question 1. The bookwork on Laplace's method in part (a) was well done. Marks were lost for failing to show that (i) the local contribution dominates over the global one or (ii) the expansions are self consistent. In part (b) nearly all candidates identified the correct steepest descent contour and about a third chose a parametrisation that enabled them to apply part (a). Marks were lost for failing to justify the deformation to the steepest descent contour or for the subsequent application of Laplace's method.

Question 2. The bookwork on boundary layer theory in part (a) was done poorly. Marks were lost for failing to derive the correct boundary layer equations (by Taylor expanding the coefficients p and q), for failing to solve correctly the leading-order outer problem or for failing to apply correctly Van Dyke's matching rule. While nearly all candidates identified the correct dominant balance in part (b), the ensuing WKB analysis was dealt with poorly by all but a few candidates and only a handful suggested a strategy for dealing with a turning point.

Question 3. The bookwork on the nonuniformity of the regular perturbation solutions in part (a) was well done on the whole. Marks were lost for failing to demonstrate that the coefficient of the secular term is nonzero. A surprising number of candidates failed to use the solution to part (a) to inform their multiple-scales analysis in part (b), which caused them to run into problems with the nonlinear differential equation in part (b)(ii).

#### C6.3b: Applied Complex Variables

Question 1. While the bookwork on the Schwarz-Christoffel map in part (a)(i) was well done, the conformal mapping problem in part (a)(ii) was done poorly, with many attempts failing to map the problem to the Z-plane or from there follow the hint. While the sketch of the potential plane in part (b)(i) was well done, the assessor was surprised to find no correct sketches of the hodograph plane, which relies on combining elements from two previously seen examples (the teapot flow covered in lectures and the channel flow covered in a problem sheet question). Many attempts at the hodograph plane failed because the candidates stuck to their (incorrect) claim that the liquid velocity is zero at the corners, despite the resulting hodograph plane failing to preserve the orientation of boundary points. The subsequent conformal mapping of the potential plane was reasonably well done, though a surprising number of candidates failed to apply correctly the Möbius map. Partial credit was given for applying successfully the log map to the incorrect hodograph plane and for the subsequent Schwarz-Christoffel mapping in part (b)(i).

Question 2. The bookwork in part (a) was well done, though a number of candidates neglected to take account of the arbitrary function H when deriving the density in part (a)(ii). There were only a handful of solid attempts to part (b) despite (i) and (ii) being covered in lectures or in a problem sheet question; only two attempts dealt successfully with (iii) and the tail. Part (c) was better done than anticipated.

Question 3. The bookwork in part (a) and the derivation of the Wiener–Hopf equation in part (b)(i,ii) were both very well done. There were many solid solutions to part (b)(iii) and part (c), though only a handful were able to manipulate the expressions efficiently and accurately, and derive thereby a solution of the integral equation.

#### C6.4a: Topics in Fluid Mechanics

Question 1. This question was generally done well. Common pitfalls included: not realising that only the magnitude of the pressure jump across the interface due to surface tension was given (so that the integrated conservation of mass equation had to have a fudged minus sign to obtain the given governing pde) and only determining the wavenumber of fastest growing instability but not the wavelength requested. Only a few candidates were sufficiently careful to obtain the appropriate inequalities on M for stability and realise where the restriction  $h_0 < (3/2)^{1/2}$  came from.

Question 2. This question was generally well done. The bookwork in part (a) was almost universally well done and candidates generally dealt well with finding the equations for the similarity solution. The result in part (c) was obtained by a good number of candidates but none gave a compelling physical interpretation of it (namely that the vertical rate of change of the heat flux integrated across the plume is equal to the heat flux into the fluid from the wall).

Question 3. Candidates tackled this question reasonably well on the whole. Some failed to realise that  $w_0 = 0$  is an important part of the geostrophic equations both since it ensures that the flow is two-dimensional and simplifies the material derivative. In part (b), candidates were generally able to deal with the algebra that leads to the conservation of potential vorticity though a surprising number ignored the advice to "show explicitly any commutation of the operator  $D_0/Dt$  with partial derivatives ... that you require". Again the physical interpretation in part (c) was not done well though some candidates realised that when  $\beta = 0$  the Rossby waves are advected with the mean flow while when  $\beta > 0$  they move westwards relative to the mean flow.

#### C6.4b: Stochastic Modelling of Biological Processes

Question 1:. Parts (a)–(d) tested material from Lectures 1, 2 and 3. In part (a), most candidates used general results G(1,t) = 1 and G(0,t) = p(0,t) and then they argued that  $p(0,t) \equiv 0$  for the specific chemical system studied in this question, because A(0) = 100 and the state with no molecules cannot be reached. A few candidates incorrectly claimed that G(0,t) = 0 for any chemical system. This incorrect argument leads to the correct answer G(0,t) = 0 for part (a), but would give incorrect answers for general chemical systems. Part (b) was mostly correct. Candidates derived the PDE for G(x,t) from the chemical master equation. Most errors came from incorrect formulations of the chemical master equation. Part (c) was fine as well. There were two correct approaches to part (d). The majority of candidates used a (relatively) quicker approach by making use of  $G_s$  (derived in part (c)) and obtained the stationary distribution, mean and variance by differentiating  $G_s$ . Some candidates started from the (stationary) chemical master equation again and solved it for the stationary distribution, mean and variance without using  $G_s$ . The last part (e) of this question tested material on the chemical Fokker–Planck equation covered in Lecture 7. Mistakes in part (e) were mostly made in solving the chemical Fokker-Planck.

Question 2. Most candidates successfully solved parts (a)–(c) which tested material covered in Lecture 11. Mistakes were usually made in parts (d) and (e) of this problem. The difficulty in part (d) stemmed from the fact that some candidates tried to compute unnecessary quantities. The question was to compute the stationary value of  $V_s$ , i.e. there was no need to solve any time dependent problems. The quickest approach was to derive a system of three linear equations for variances and solve them for  $V_s$ . Only a few candidates attempted to solve part (e). Here, the main idea is to use the backward Kolmogorov equation and modify the argument from Lecture 6 (computation of mean escape times for diffusing particles) to take into account that molecules are produced with rate  $\sigma$  and degraded with rate  $k_2$ .

**Question 3.** Most candidates successfully solved parts (a)–(c) which tested material from Lectures 5 and 13. In part (d), some candidates made mistakes in the derivation of the ordinary differential equation (ODE) for the average mean-square displacement. The most

common mistake appeared in part (e) in the derivation of the ODE for the variance of the follower's position  $\langle X_F^2(t) \rangle$ . This ODE had  $\langle X_F(t)X_L(t) \rangle$  on the right hand side. The correct approach is to derive and solve another ODE for  $\langle X_F(t)X_L(t) \rangle$ , and then substitute its solution to the ODE for  $\langle X_F^2(t) \rangle$ . Several candidates incorrectly assumed that  $\langle X_F(t)X_L(t) \rangle = \langle X_F(t) \rangle \langle X_L(t) \rangle$ . This assumption would be justified if  $X_L(t)$  and  $X_F(t)$ were independent. However, the model studied in Question 3 was describing a situation when one animal (follower) is following another (leader). Therefore, the independence of their positions cannot be justified. In fact, one can explicitly show, using results from parts (c) and (d), that  $\langle X_F(t)X_L(t) \rangle \neq \langle X_F(t) \rangle \langle X_L(t) \rangle$ .

#### C6.5b: Mechanical Mathematical Biology

Question 1. Students found this question more challenging than anticipated despite the fact that it followed closely a derivation given in the course. Part 1a was well done for most students. In part b the students did mostly well but they lost some time in proving standard equalities for integrals of trig functions. They did not seem to have a good grasp on the equipartition theorem which stopped them from obtaining the expression they needed for parts (c) and (d).

Question 2. This question was mostly done well. Most candidates were able to write the energy for the system. The minimisation of the energy with respect to the radius was fine but candidates had a hard time with the other free variables and lacked the understanding to interpret the result.

Question 3. This question was poorly done unfortunately despite the large number of similar examples done in problem sheets and in the consultation classes. It seems that all the candidates assumed that the elastic stretch and elastic growth are constant in the entire interval rather than realising that they are only constant in each sub-interval. That crucial mistake created a chain of mistakes that invalidated the rest of the answer.

#### C7.1b: Quantum Theory and Quantum Computers

Question 1 was attempted by almost everyone and seemed to have worked well, with a good spread of solutions.

Question 2. There were many good attempts at this question, with only (d), the most novel part, presenting serious difficulties. A number of candidates, however, claimed in (b) that the function had n different values that could be tested, not  $2^n$ .

Question 3. Very few attempts—one excellent, understanding the connection between (a)(iii) and (c), others less convincing although (b) was well done. In (a), candidates typically neglected to mention the self-adjoint condition.

#### C7.2a: General Relativity I

Question 1. Most students had some difficulty with part (e): they gave the correct definition for proper time, but were unable to compute the proper time for the trajectory in part (c). Question 2. Most students got an incorrect result for part e, which asked students to compute the Einstein tensor for  $AdS_2$ . This was mostly due to calculational errors.

Question 3. Overall, most students did well on this question. Some had difficulty with part (b), which asked them to show that photons are not deflected by gravity in the Newtonian limit. This was mostly due to not recalling the definition of the Newtonian limit. Most students had some minor difficulty with part (e), which asked them to solve the Einstein equation for dust in the Newtonian limit.

#### C7.2b: Relativity II

Question 1. The setter was very surprised by the low marks for this question. In part (a) many did not use the Euler–Lagrange equations to compute the Christoffel symbols, and some candidates who computed them from the definition made computational errors. Few were able to argue why some components of the Riemann tensor vanish without computing them explicitly, and only a few discussed Einstein's equations in the vacuum (given the curvature, this part was particularly easy). Part (a) was considered a standard computation in general relativity, with a metric which depends only on one of the coordinates. There was only one good attempt to part (b) even though it was similar to the plane wave solutions discussed in the lectures.

Question 2. There were some very good attempts to this question. However, the assessor was concerned that only a couple of candidates were able to explain what an event horizon was in part (a).

Question 3. The assessor was surprised by the lack of attempts to this question.

#### **C8.1a:** Mathematics of Geoscience

Question 1. Probably this question should have been even simpler.

Question 2. There were four parts to question 2. Part (a): straightforward mass and momentum derivations and generally quite well done. Part (b): non-dimensionalisation—again generally well done but numerous careless algebraic mistakes. Part (c): stability analysis—not as well done as expected since candidates had seen this done many times in lectures and problem sheets. Numerous careless algebraic mistakes. Part (d): more difficult component with only 4 candidates having a serious attempt at answering and these 4 all scored the highest marks overall.

Question 3. Straightforward, and reasonably well done. Few got anywhere with the last part requiring some thought.

#### C8.1b: Mathematical Physiology

Overall, the questions were found to be more difficult than anticipated, but this was common among all the questions. Question 1. This was not a popular question, but it was executed well by those who attempted it. The bookwork sections had very good answers in general, though students had varying degrees of success with the final part. Many of those that recognised that most of the earlier relations held with only minor adjustment made it to the final step of rewriting both the internal and external potentials in terms of the transmembrane potential but no student managed this final step.

Question 2. The bookwork was executed well. Student attempts at the eliminations required for deducing the ODE for  $s_e$ , the bulk of the last section of the question, were varied. They gradually petered out in most cases even though there were enough algebraic relations to systematically perform the eliminations and these simplified given epsilon was small.

Question 3. The bookwork was sometimes disappointing with weak students failing to linearise a time-delayed equation in particular. Those that made it past this hurdle often presented suitable reasoning for the absence of eigenvalues with positive real parts, though the attempts at the final part tailed off to varying degrees.

#### C9.1a: Modular Forms

Question 1. This was generally well done, although many students had difficulty identifying the elliptic point in (b) and computing the genus.

Question 2. Candidates found this question challenging, and although (a) was bookwork it was not always done correctly. Parts (b)(ii) and (c)(iii) also caused some difficulties.

Question 3. This was done very well by most candidates who attempted it, although some who had perhaps not prepared as fully for this part of the course did get lower marks. Part (c) was somewhat tricky, but some candidates did do well on it.

#### C9.1b: Elliptic Curves

Question 1. This problem was essentially standard and students mostly did well. However, part (b) was a source of some difficulty. That is, the assumption implies that the prime factorization of the number must have all prime factors occurring with an even power. However, the number could be negative. In this case, you need also to show that the number must be positive using the assumptions.

Question 2. Parts (a) and (b) were mostly fine, except some of the definitions and statements could have been cleaner. Part (c) was also standard bookwork, except the precise divisibility property, using the fact the the map [p] is a homomorphism, could have been more clearly stated. Part (d) was done reasonably by most students. Part (e) was a bit more challenging, but many good answers were submitted, including a method I didn't expect, that is, bounding the torsion (say by reduction mod p) and showing that some rational point couldn't belong to the torsion subgroup.

Question 3. Parts (a), (b), (c) were all standard bookwork and the students did all right. As with some of the other problems, the exposition could have been cleaner. Part (d) was also fairly straightforward, except some students were a bit sloppy with the reduction to the case where z is not divisible by p.Part (e) was rather challenging. The rank 1 bound assuming the congruence condition was done all right by a good portion of the students.

However, finding the point of infinite order when p = 5 may have been a bit too difficult under examination conditions.

#### C9.2a Analytic Number Theory

Three minor typos were announced during the exam.

Question 1. Easy and popular. A few candidates had problems with the final part (e).

Question 2. In part (b) some candidates gave inadequate proofs of convergence. A few candidates got the Euler product wrong and then went off the rails. Others could not locate the zeros of  $1 - p^{1-s}$ .

Question 3. (a) and (b) were well done, but some candidates fell down on the final part.

#### C10.1a: Stochastic Differential Equations

The exam was generally well done with most students able to show a good knowledge of the basic material of the course.

Question 1. This was a question about Brownian motion and the first three parts were standard and most students obtained most of the marks for these sections. However a number of people failed to differentiate correctly to find the density of the hitting time of a. The final section was more challenging and nobody managed to produce a perfect solution.

Question 2. Only two attempts, one almost perfect.

Question 3. A question with a substantial bookwork component which ensured that most were able to get a reasonable way in to the question. Few people could prove that the stochastic integral of a bounded process against Brownian motion was a martingale. The tail required a careful discussion to obtain the result which few candidates were able to provide.

#### C10.1b: Brownian Motion in Complex Analysis

Question 1. Surprisingly many students did not mention arc-length when they tried to compute the density with respect to the arc-length. Part (b)(ii) turned out to be surprisingly challenging given that most of the necessary ingredients were obtained in the previous parts.

Question 2. This is a mostly computational question similar to several computations done in the course. This question turned out to be very unpopular. For some reason many students worked under the assumption that the real-valued function F is complex differentiable.

Question 3. This is mostly bookwork that was done generally well.

#### C11.1a: Combinatorics

The first part of Question 1 was generally answered well; some candidates found the later parts of the question more difficult, and there were very few perfect answers to 1(d). Questions 2 and 3 were generally done well.

#### C11.1b Probabilistic Combinatorics

Question 1. This was the least popular question, and was generally not successfully handled. In particular, students found the asymptotics too hard (under exam conditions).

Question 2. For part (a), many could repeat chunks of proof from lectures. In part (b)(i), some saw quickly that  $\frac{1}{2}(Y_i+r) \sim Bin(r, \frac{1}{2})$ , but others found this confusing. Part (b)(ii) on the LLL was usually very well handled. In part (c)(i), some students gave succinct answers, some made assertions; and few gave a good answer to (ii).

Question 3. This turned out to be a little too straightforward. It was very well done; with most students giving excellent answers for part (a) and part (b)(i), and many students also handled part (b)(ii) well.

#### C12.1a : Numerical Linear Algebra

Question 1. Overall good results with marks generated on each sub-question, though with few parts solved without flaw. Difficulties in flop count in part (d).

Question 2. Generally high scores with all questions well solved with the exception of the bound 8 in part (a).

Question 3. High marks for those that attempted the question though few did. Common errors in backward stability.

#### C12.1b : Continuous Optimisation

The great majority of candidates answered Questions 1 & 2, presumably because they simply solved them in the order they were listed.

Question 1 was attempted by all candidates. The book work was generally well solved, while the harder parts of the problem gave the stronger students the chance to distinguish themselves.

Question 2 was attempted by 13 candidates, some of whom struggled with even the easy parts of the problem, while 43% of the candidates achieved more than 70% of the maximum raw mark.

Question 3 was only attempted by two candidates, although there was no indication that this problem was more challenging than the first two questions.

#### C12.2a Approximation of Functions

The examination proceeded straightforwardly. All candidates attempted question 1, on Chebyshev polynomials and accuracy of approximations. There was an even split between attempts on questions 2, on best approximations and equioscillation, and 3, on the Weierstrass approximation theorem and interpolatory quadrature.

#### C12.2b: Finite Element Methods for Partial Differential Equations

Question 1. The question concerned the finite element approximation of a two point boundary problem. Several candidates had difficulties proving the inequality in the first half of the question and the weak differentiability of u' in the last part.

Question 2. The question was generally well done. A recurring error in several scripts was the use of the Poincaré–Friedrichs inequality in a setting where it is not applicable.

Question 3. The question was attempted by a small number of candidates, but the solutions were overall of high quality.

#### Statistics Units

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

MS1b: Statistical Data Mining and Machine Learning

MS2b: Stochastic Models in Mathematical Genetics

MS5a: Probability and Statistics for Network Analysis

MS6a: Modern Survival Analysis

MS6b: Advanced Simulation Methods

#### **Computer Science**

Reports on the following courses may be found in the Mathematics and Computer Science examiners' report.

Quantum Computer Science

Categories, Proofs and Processes

Automata, Logic and Games

# F. Comments on performance of identifiable individuals and other material which would usually be treated as reserved business

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