Examiners' Report: Final Honour School of Mathematics Part B Trinity Term 2016

October 20, 2016

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers					Percentages %				
	2016	(2015)	(2014)	(2013)	(2012)	2016	(2015)	(2014)	(2013)	(2012)
I	56	(48)	(49)	(54)	(57)	39.72	(32.88)	(31.01)	(34.34)	(34.34)
II.1	58	(69)	(78)	(78)	(79)	41.13	(47.26)	(49.37)	(49.68)	(47.59)
II.2	24	(25)	(21)	(21)	(21)	17.02	(17.12)	(13.29)	(13.38)	(12.65)
III	3	(3)	(9)	(2)	(5)	2.13	(2.05)	(5.7)	(1.27)	(3.01)
P	0	1	(1)	(2)	(3)	0	(0.68)	(0.63)	(1.27)	(1.81)
F	0	0	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
Honours (unclassified)	0	(0)	(0)	(0)	(1)	0	(0)	(0)	(0)	(0.6)
Total	141	(146)	(158)	(157)	(166)	100	(100)	(100)	(100)	(100)

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**

  As in previous years there were no vivas conducted for the FHS of Mathematics Part B.

- **Marking of scripts.**

  The following were double marked: whole unit BE Extended Essays, BSP projects, and coursework submitted for the History of Mathematics course, the Mathematics Education course and the Undergraduate Ambassadors Scheme.

  The remaining scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

  See Table 5 on page 14.

B. New examining methods and procedures

There were no changes to examining methods or procedures.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

Three changes to examining procedures have been agreed for next year. Firstly, the length of time allowed for Mathematics unit papers will increase from 1.5 hours to 1.75 hours. Statistics papers will also increase to 1.75 hours, and Computer Science papers to 2 hours.

Secondly, BEE Extended Essays and BSP Structured Project written reports will now be marked by the supervisor and one assessor, rather than by two assessors.

Thirdly, candidates taking Part A from 2016 onwards take either 9 or 10 papers in Part A, that is, they must take papers A0, A1, A2, ASO, and five or six out of A3-A11. If a candidate takes 9 papers, paper A2 counts as a double unit and the remaining papers as single units in the Part A USM average. If a candidate takes 10 papers, the two lowest scoring papers from A3-A11 count as half a unit each in the Part A average. In both cases, the classification in Part B depends on the sum of 40% of the Part A average and 60% of the Part B average.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 16 February 2016 and the second notice on 28 April 2016.

All notices and the examination conventions for 2016 are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chairman would particularly like to thank Helen Lowe for administering the whole process with extraordinary efficiency, and also to thank Nia Roderick, Charlotte Turner-Smith, and Waldemar Schlackow.

In addition the internal examiners would like to express their gratitude to Professor Blackburn and Professor Higham for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

Standard of performance

The standard of performance was broadly in line with recent years. In setting the USMs, we took note of

- the Examiners' Report on the 2015 Part B examination, and in particular recommendations made by last year's examiners, and the Examiners' Report on the 2015 Part A examination, in which the 2016 Part B cohort were awarded their USMs for Part A;
- a document issued by the Mathematics Teaching Committee giving broad guidelines on the proportion of candidates that might be expected in each class, based on the class percentages over the last five years in Mathematics Part B, Mathematics & Statistics Part B, and across the MPLS Division.

Having said all this, the proportion of first class degrees awarded was higher than usual (see Table 1 on page 1). This year we awarded 39.72% firsts, compared to an average of 33.76% in Mathematics Part B over 2011-15, and an MPLS average of 36% firsts over 2010-14. This was not intended by the examiners, but resulted from a combination of several factors:

- The proportion of firsts at Part A in 2015 (36.17%) was higher than usual (average 33.5% over 2011-2015). The USM algorithm produces an initial scaling function for each paper with as many (or more) first class marks as there were candidates sitting the paper with first class marks in Part A. So if the examiners broadly follow the algorithm, the proportion of firsts in Part B will be similar to that in Part A.

- The examiners determine USMs for Mathematics and for Mathematics and Statistics simultaneously, and so consider both cohorts together. This year the proportion of firsts in Mathematics and Statistics was low (20%), so the combined average is close to the MPLS divisional average.
- After the USMs were fixed, three candidates were promoted from II.1 to I because of considering the Strong Paper Rule and Factors Affecting Performance applications.

The proportion of candidates in classes II.2 and below is in line with recent Part B and MPLS divisional averages. Hardly any candidates performed very poorly.

Setting and checking of papers and marks processing

Requests to course lecturers to act as assessors, and to act as checkers of the questions of fellow lecturers, were sent out early in Michaelmas Term, with instructions and guidance on the setting and checking process, including a web link to the Examination Conventions. The questions were initially set by the course lecturer, in almost all cases with the lecturer of another course involved as checkers before the first drafts of the questions were presented to the examiners. Most assessors acted properly, but a few failed to meet the stipulated deadlines (mainly for Michaelmas Term courses) and/or to follow carefully the instructions provided.

The internal examiners met at the beginning of Hilary Term to consider those draft papers on Michaelmas Term courses which had been submitted in time; consideration of the remaining papers had to be deferred. Where necessary, corrections and any proposed changes were agreed with the setters. The revised draft papers were then sent to the external examiners. Feedback from external examiners was given to examiners and to the relevant assessor for response. The internal examiners at their meeting in mid Hilary Term considered the external examiners' comments and the assessor responses, making further changes as necessary before finalising the questions. The process was repeated for the Hilary Term courses, but necessarily with a much tighter schedule.

Camera ready copy of each paper was signed off by the assessor, and then submitted to the Examination Schools.

Except by special arrangement, examination scripts were delivered to the Mathematical Institute by the Examination Schools, and markers collected their scripts from the Mathematical Institute. Marking, marks processing and checking were carried out according to well-established procedures. Assessors had a short time period to return the marks on standardised mark sheets. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the mark sheets.

All scripts and completed mark sheets were returned, if not by the agreed due dates, then (with the exception of one paper) at least in time for the script-checking process.

A team of graduate checkers under the supervision of Helen Lowe sorted all the scripts for each paper for which the Mathematics Part B examiners have sole responsibility, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way, errors were corrected with each change independently verified and signed off by one of the examiners, who were present throughout the process. A small number of errors were found, but they were mostly very minor and hardly any queries had to be referred to the marker for resolution.

Standard and style of papers

At the beginning of the year all setters were asked to aim that a I/II.1 borderline candidate should get about 36 marks out of 50, and that a II.1/II.2 borderline script should get about 25 marks, and emphasising the problems caused by very high marks.

This year three papers (B3.2, B4.1, B6.2) turned out to be too easy. This causes problems with determining USMs at the top end. For two of these papers the algorithm's initial recommendation was to map raw marks of 49/50 and 49.8/50 to a USM of 72. In the third 7 out of 48 candidates received full marks, and so automatically received a USM of 100.

Setting papers that are significantly too easy (and marking such papers generously) is undesirable from the point of view of fairness. Such papers generate more USMs than usual in the range 80-100 from candidates with close to full marks. An undergraduate who has the good fortune to take an easy paper and score highly will typically receive a rather higher USM than he or she would otherwise have done – perhaps a USM of 100 – and this can easily push an otherwise high II.1 candidate into the first class.

There were no obvious problems this year caused by papers that were more difficult than expected.

We **RECOMMEND** that in the letters to setters in future years, setters should be advised that only the very best students should be able to score full marks in a paper, to avoid distortion of the scaling process. This could be achieved by having sufficiently difficult final parts in questions.

Timetable

Examinations began on Monday 30 May and finished on Friday 17 June.

Consultation with assessors on written papers

Assessors were asked to submit suggested ranges for which raw marks should map to USMs of 60 and 70 along with their mark-sheets, and a large majority did so. When the mark-sheets were received, in advance of the examiners' final meetings, we calculated the raw marks that the standard algorithm would propose to map to 60 and 70, and compared them with the assessor's suggestions. When the proposals by the assessor and the algorithm were out of line, the chairman invited the assessor to respond to the algorithm's proposal, and compiled a list of the responses, which were read out in the examiners' final meeting when the USM scaling functions for those papers were determined. Most assessors were happy with the algorithm, but six assessors suggested adjustments to it, including one strong disagreement.

Determination of University Standardised Marks

We followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part A performances of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part A average USMs in the ranges $[69.5, 100]$, $[59.5, 69.5)$ and $[0, 59.5)$, respectively.

The algorithm converts raw marks to USMs for each paper separately. For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the number of I and II.1 candidates in Part A, as given by N_1 and N_2, is the same as the I and II.1 number of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs, and the Examiners may then adjust them to take account of consultations with assessors (see above) and their own judgement. The examiners have scope to make changes, either globally by changing certain parameters, or on individual papers usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm. They also have the option to introduce additional corners. For a well-set paper taken by a large number of candidates, the algorithm yields a piecewise linear map which is fairly close to linear, usually with somewhat steeper first and last segments. If the paper is too easy or too difficult, or is taken by only a few candidates, then the algorithm can yield anomalous results—very steep first or last sections, for instance, so that a small difference in raw mark can lead to a relatively large difference in USMs. For papers with small numbers of candidates, moderation may be carried out by hand rather than by applying the algorithm.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held ahead of the first plenary examiners' meeting to assess the results produced by the algorithm, to identify problematic papers and to try some experimental changes to the scaling of individual papers. This provided a starting point for the first plenary meeting to obtain a set of USM maps yielding a tentative class list with class percentages roughly in line with historic data.

The first plenary examiners' meeting, jointly with Mathematics & Statistics examiners, began with a brief overview of the methodology and of this year's data. Then we considered the scaling of each paper, making provisional adjustments in some cases. The full session was then adjourned to allow the examiners to look at scripts. This was both to help the external examiners to form a view of overall standards, and to answer questions that had arisen on how best to scale individual papers; for instance, to decide whether a given raw mark should correspond to the I/II.1 or II.1/II.2 borderline, an examiner would read all scripts scoring close to this raw mark, and make a judgement on their standard.

The examiners reconvened and we then carried out a further scrutiny of the scaling of each paper, making small adjustments in some cases before confirming the scaling map (those Mathematics & Statistics examiners who were not Mathematics examiners left the meeting once all papers with significant numbers of Mathematics & Statistics candidates had been considered).

Table 2 on page 11 gives the final positions of the corners of the piecewise linear maps used to determine USMs.

At their final meeting on the following morning, the Mathematics examiners reviewed the positions of all borderlines for their cohort. For candidates very close to the proposed borderlines, marks profiles and particular scripts were reviewed before the class list was finalised.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

On the BSP Structured Projects 'peer review'

Several assessors expressed dissatisfaction with the 'peer review' part of the BSP Structured Projects. Concerns were expressed both about the questions the students had been asked to address in writing their reviews, and about the mark scheme for assessing the reviews. We **RECOMMEND** that this issue should be considered by the appropriate Committee.

Factors affecting performance

Under the new procedures, a subset of the examiners had a preliminary meeting to consider the submissions for factors affecting performance in Part B. There were no Part 12 submissions, and eleven Part 13 submissions which the preliminary meeting classified in bands 1, 2, 3 as appropriate. The full board of examiners considered the eleven cases in the final meeting, and the certificates passed on by the examiners in Part A 2015 were also considered. All candidates with certain conditions (such as dyslexia, dyspraxia, etc) were given special consideration in the conditions and/or time allowed for their papers, as agreed by the Proctors. Each such paper was clearly labelled to assist the assessors and examiners in awarding fair marks. Details of cases in which special consideration was required are given in Section E.2.

Table 2: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	Additional Corners	N_1	N_2	N_3
B1.1	(19.3,37)	(33.6,57)	(45.6,72)		12	18	10
B1.2	(14.94,37)	(26,57)	(41,72)	(49,95)	27	22	13
B2.1	(14.25,37)	(24.8,57)	(34,72)		15	4	3
B2.2	(10.91,37)	(22,57)	(34,72)		13	6	3
B3.1	(14.71,37)	(25.6,57)	(37.6,72)	(48,92)	22	12	4
B3.2	(12.24,37)	(27,57)	(41,69)	(47,84)	8	10	2
B3.3	(11.72,37)	(23.5,57)	(38.4,72)	(47,90)	12	10	2
B3.4	(15.45,37)	(26.9,57)	(37.4,72)		16	11	2
B3.5	(7.99,37)	(20,57)	(39.4,72)		22	14	4
B4.1	(10.86,37)	(21,57)	(44.4,72)		24	20	3
B4.2	(9.25,37)	(22,57)	(34,72)		20	14	1
B5.1	(11.49,37)	(20,57)	(35,72)		6	15	6
B5.2	(13,40)	(24.7,57)	(38.2,72)		21	25	15
B5.3	(19.24,37)	(33.5,57)	(41,72)		14	10	12
B5.4	(14.36,37)	(25,57)	(40,72)		14	10	9
B5.5	(14.94,37)	(26,57)	(41,72)		12	30	14
B5.6	(13.73,37)	(23.9,57)	(34.4,72)		15	20	12
B6.1	(14.13,37)	(24.6,57)	(36.6,72)		9	9	3
B6.2	(15.22,37)	(28.5,57)	(44,70)	(49,92)	5	5	2
B6.3	(12.87,37)	(24,57)	(36,72)		3	9	8
B7.1	(13.33,37)	(23.2,57)	(32.2,72)		8	13	6
B7.2	(10.51,37)	(18.3,57)	(31.8,72)		4	7	3
B8.1	(11.6,37)	(22,57)	(41,72)	(47,88)	14	19	1
B8.2	(9.48,37)	(26,59)	(39,72)		8	7	0
B8.3	(14.94,37)	(26,57)	(41,72)	(48,88)	12	26	12
B8.4	(8.67,37)	(15.1,57)	(33.6,72)		4	17	6
B8.5	(14.71,37)	(25.6,57)	(37.6,72)		19	28	12
SB1	(17.52,37)	(30.5,57)	(53,72)		6	21	10
SB2a	(8.5,37)	(14.8,57)	(35.8,72)		4	28	8
SB3a	(11.89,37)	(20.7,57)	(37.2,72)		20	46	10
SB3b	(9.71,37)	(20,57)	(36,70)		10	29	6
SB4a	(14.82,37)	(25.8,57)	(43,72)		7	24	14
SB4b	(9.82,37)	(17.1,57)	(36.6,72)		5	18	8

Table 3 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Table 3: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
90	1	1	0.71
89	2	2	1.42
88	3	3	2.13
87	4	4	2.84
86	5	5	3.55
84	6	6	4.26
83	7	7	4.96
82	8	9	6.38
81	10	13	9.22
80	14	15	10.64
78	16	17	12.06
77	18	22	15.6
76	23	25	17.73
75	26	29	20.57
74	30	33	23.4
73	34	39	27.66
72	40	44	31.21
71	45	47	33.33
70	48	55	39.01
69	56	62	43.97
68	63	66	46.81
68	63	66	46.81
67	67	73	51.77
66	74	76	53.9
65	77	81	57.45
64	82	88	62.41
63	89	94	66.67
62	95	100	70.92
61	101	107	75.89
60	108	114	80.85
59	115	117	82.98
58	118	121	85.82
57	122	123	87.23
56	124	130	92.2
55	131	131	92.91
54	132	133	94.33
53	134	135	95.74
50	136	138	97.87
46	139	139	98.58
42	140	140	99.29
41	141	141	100

B. Equal opportunities issues and breakdown of the results by gender

Table 4: Breakdown of results by gender

Class	Total		Female		Male	
	Number	%	Number	%	Number	%
I	56	39.72	10	25.64	46	45.1
II.1	58	41.13	17	43.59	41	40.32
II.2	24	17.02	10	25.64	14	13.73
III	3	2.13	2	5.13	1	0.98
P	0	0	0	0	0	0
Total	141	100	39	100	102	100

Table 4 shows the performances of candidates broken down by gender. The examiners were concerned to discover, after the class lists were agreed, that the percentage of male candidates awarded first class degrees was not far from double the percentage of female candidates awarded first class degrees, and that the percentage of female candidates awarded II.2s and below was more that double the percentage of male candidates in the same range. We would like to bring this year's very significant gender discrepancy to the attention of the department, which we know is already well aware of this issue.

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 5.

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
B1.1	40	37.48	8.74	62.18	13.58
B1.2	61	35.44	9.9	68.54	15.87
B2.1	22	34.91	8.09	73.95	13.95
B2.2	21	33.95	9.27	73.67	14.33
B3.1	38	37.45	6.62	73.68	10.32
B3.2	20	40.45	8.86	75.3	14.44
B3.3	24	36.5	8.89	73.67	13.24
B3.4	28	36.93	6.39	73.04	12.31
B3.5	41	35.12	10.98	73.15	14.91
B4.1	48	39.23	8.78	73.6	13.38
B4.2	36	32	5.96	70.39	8.53
B5.1	23	27.48	6.46	64.26	7.85
B5.2	61	29.98	9.68	63.08	14.02
B5.3	37	36.65	6.7	65.19	13.43
B5.4	34	33.21	7.79	65.09	10.13
B5.5	53	33.3	6.28	64.72	8.07
B5.6	49	29.59	5.47	65.06	8.65
B6.1	24	31.12	8.49	65.25	13.8
B6.2	15	36.4	8.86	67.07	13.47
B6.3	19	26.37	6.45	59.68	9.67
B7.1	27	27.78	6.55	64.15	11.54
B7.2	14	26.36	6.15	66.07	7.63
B8.1	31	36	8.28	70.65	11.21
B8.2	12	35.25	8.69	71.08	12.52
B8.3	37	33.68	10.06	64.76	16.19
B8.4	26	23.54	8.32	63.62	9.79
B8.5	55	33.25	6.93	67.64	11.28
C7.3	15	33.53	8.38	70.67	12.69
SB1	3	-	-	-	-
SB2a	11	22.45	6.41	61.18	8.06
SB3a	51	29.59	7.12	65.35	9.03
SB3b	21	33.86	6.39	70.38	8.29
SB4a	27	31.63	10.09	61.59	15.33
SB4b	16	25.62	7.45	62.31	9.01
CS3a	3	-	-	-	-
CS4b	2	-	-	-	-
BO1.1	5	-	-	-	-
BO1.1X	5	-	-	-	-
BN1.1	7	-	-	65.86	5.55
BN1.2	7	-	-	66.29	2.29
BEE	8	-	-	75.88	8.89
BSP	15	-	-	66.07	6.52
B5.1o	1	-	-	-	-
122	1	-	-	-	-
127	1	-	-	-	-

Individual question statistics for Mathematics candidates are shown below for those papers offered by no fewer than six candidates.

Paper B1.1: Logic

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.49	18.49	4.06	39	0
Q2	18.88	18.88	5.49	40	0
Q3	23	23		1.00	0

Paper B1.2: Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.27	14.79	5.42	38	3
Q2	18.75	19.04	5.69	52	1
Q3	17.8	19.06	6.22	32	3

Paper B2.1: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.42	17.42	3.89	19	0
Q2	17.5	18.27	5.16	11	1
Q3	16.86	16.86	5.52	14	0

Paper B2.2: Commutative Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.67	12.67	5.06	18	0
Q2	20	20	4.2	19	0
Q3	18.67	21	7.76	5	1

Paper B3.1: Galois Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.46	19.46	3.5	37	0
Q2	16.26	17.39	5.04	23	4
Q3	18.28	18.94	4.34	16	2

Paper B3.2: Geometry of Surfaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.41	18.41	5.61	17	0
Q2	20.78	20.78	5.24	9	0
Q3	20.75	22.07	4.93	14	2

Paper B3.3: Algebraic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.96	17.96	5.66	24	0
Q2	18.82	18.82	5.24	22	0
Q3	15.5	15.5	7.78	2	0

Paper B3.4: Algebraic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.45	16.45	4.84	11	0
Q2	19.38	19.38	3.73	21	0
Q3	17.84	18.58	5.13	24	1

Paper B3.5: Topology and Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.4	12.63	5.72	24	1
Q2	17.42	17.67	6.08	30	1
Q3	21.68	21.68	3.93	28	0

Paper B4.1: Banach Spaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.64	16.78	3.64	27	1
Q2	22.03	22.38	3.4	32	1
Q3	18.95	19.3	6.03	37	1

Paper B4.2: Hilbert Spaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.62	15.62	3.88	26	0
Q2	15.4	15.4	3.44	25	0
Q3	17.19	17.19	3.39	21	0

Paper B5.1: Stochastic Modelling and Biological Processes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.36	18.36	3.17	22	0
Q2	9.91	10.7	4.76	10	1
Q3	8.33	8.64	5.15	14	1

Paper B5.2: Applied PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.07	15.07	5.62	55	0
Q2	13.64	14.24	5.57	55	4
Q3	17.15	18.08	5.98	12	1

Paper B5.3: Viscous Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.43	18.43	1.99	37	0
Q2	19.21	19.57	5.26	28	1
Q3	12.9	14	5.97	9	1

Paper B5.4: Waves and Compressible Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.22	16.75	6.49	16	2
Q2	17.26	17.63	4.48	30	1
Q3	15.09	15.09	4.4	22	0

Paper B5.5: Mathematical Ecology and Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.85	15.85	3.94	46	0
Q2	17.34	17.34	3.98	53	0
Q3	15.63	16.71	4.87	7	1

Paper B5.6: Nonlinear Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.41	15.85	4.77	33	1
Q2	13.23	13.23	2.58	43	0
Q3	16.27	16.27	3.91	22	0

Paper B6.1: Numerical Solution of Differential Equations I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.93	13.62	4.81	13	1
Q2	16.5	16.5	4.24	24	0
Q3	14.83	15.82	7.46	11	1

Paper B6.2: Numerical Solution of Differential Equations II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.93	20.93	3.75	15	0
Q2	17.33	19.6	6.31	5	1
Q3	13.4	13.4	7.12	10	0

Paper B6.3: Integer Programming

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.2	11.93	4.69	14	1
Q2	11.33	12.13	4.21	8	1
Q3	14.81	14.81	3.58	16	0

Paper B7.1: Classical Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.58	14.58	4.01	26	0
Q2	13.68	13.68	3.62	19	0
Q3	10.82	12.33	5.12	9	2

Paper B7.2: Electromagnetism

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.21	14.21	4.46	14	0
Q2	12.17	12.17	3.13	6	0
Q3	11	12.13	3.65	8	2

Paper B8.1: Martingales through Measure Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.93	18.93	4.89	27	0
Q2	15.69	16.14	5.45	28	1
Q3	20	21.86	5.98	7	1

Paper B8.2: Continuous Martingales and Stochastic Calculus

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.5	15.5	0.71	2	0
Q2	16.08	16.73	5.26	11	1
Q3	17.83	18.91	5.95	11	1

Paper B8.3: Mathematical Models of Financial Derivatives

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.62	15.62	5.08	26	0
Q2	16.53	17.47	6.52	30	2
Q3	16.33	17.56	6.28	18	3

Paper B8.4: Communication Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.08	13.08	5.35	26	0
Q2	9.95	10.53	4.50	19	2
Q3	7.92	10.29	4.52	7	5

Paper B8.5: Graph Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.98	17.25	4.63	40	2
Q2	15.36	16.15	5.03	41	3
Q3	16.27	16.45	3.76	29	1

Paper C7.3: Further Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.07	17.07	5.39	15	0
Q2	16	16	3.74	14	0
Q3	7.75	23	10.31	1	3

Paper SB2a: Foundations of Statistical Inference

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.73	11.3	4.05	10	1
Q2	9.33	9.33	3.87	9.	0
Q3	16.67	16.67	3.79	3	0

Paper SB3a: Applied Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.47	14.16	4.43	38	5
Q2	13.83	14.02	3.82	47	1
Q3	18	18.35	3.2	17	1

Paper SB3b: Statistical Lifetime-Models

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.37	17.37	3.08	19	0
Q2	14.13	14.13	6.15	8	0
Q3	16	17.87	5.39	15	3

Paper SB4a: Actuarial Science I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.12	18.12	5.39	25	0
Q2	11.42	11.91	6.05	22	2
Q3	19.86	19.86	3.8	7	0

Paper SB4b: Actuarial Science II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13	13	5.61	16	0
Q2	12.93	13.29	2.94	14	1
Q3	5.33	8	6.11	2	1

Assessors' comments on sections and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. The examiners have not included assessors' statements suggesting where possible borderlines might lie; they did take note of this guidance when determining the USM maps. Some statistical data which can be found in Section C above have also been removed.

B1.1: Logic

Question 1: Almost all candidates chose this question and performed quite well on it. In part (b), some candidates failed to explain how an assignment is extended from variables to terms and then how this induces a valuation on formulas. In part (c), the fact that $\mathcal{L}$-structures and $\mathcal{L}'$-structures are the same was not always recognized (the two languages only differ on the logical symbols, but in structures only the non-logical symbols are interpreted). The very last bit of part (d) on how to modify assignments was only seen by half of the candidates.

Question 2: Again, almost all candidates chose this question and performed even better on it. In part (a), the fact that if the assumption in **A4** was omitted the deductive system would no longer be sound, should have been illustrated by an example which was not always done. In part (b), some candidates assumed that propositional tautologies could be used without proof, but this is not granted by the question. In fact, the challenge of this part was, to string together the propositional and the predicate calculus proofs of the Deduction Theorem. In part (c), the derivation of axiom **A5** in the new system seemed hard, but a good many candidates managed also this.

Question 3: Only 4 candidates chose this question. This may be on the one hand due to the fact that this question was about the most advanced material in the course, but also because the term *elementarily equivalent* which was defined in lecture 15 and which can also be found in most logic textbooks, unfortunately didn't make it into the online lecture notes or online slides of lectures. Obviously, some candidates knew the term, but most candidates seemed hesitant to approach a question with an unknown term. The phenomenon that two structures are elementarily equivalent (i.e., that they have the same theory) does appear in the online lecture notes, and the definition could easily have been read off from part (b) of the question, but it is very understandable that if part (a) contains an unknown term one is not even tempted to continue to look at part (b).

B1.2: Set Theory

Question 1:Part (a) was generally well done, though too many did not define $X \prec Y$ correctly and/or did not check in (iii) that there could be no bijection. Few students managed (b) part (ii) which was surprising given that similar questions have often appeared on past exams. Part (c) was managed quite well though often aspects (mostly the uniqueness) was neglected, and part (iv) was quite challenging.

Question 2: Part (a) was generally well done. In (b) some did not mention that a class must be defined by a suitable formula, and some found it a challenge to come up with the right set to apply Foundation in (ii) and, especially, (iii). Part (c) was generally well done, and most easily showed by example that the order need not be total.

Question 3: In part (a), many found (ii) more challenging than necessary and used Foundation (OK but unnecessary: better to use transitivity), while (iii) was generally well done. Part (b) was generally well done overall, even if proofs were sometimes too sketchy and not many got all parts right. Part (c) was generally well done.

B2.1: Introduction to Representation Theory

Question 1: Very popular question, however few students managed to complete the last part by observing that the action of A on a simple module is uniquely determined by the action of the diagonal matrices.

Question 2: Another popular question. Few students realized in (d) that if the semisimple algebra A has a simple module L of dimension n then the dimension of L must be at least n^2.

Question 3: Fewer students attempted this, there were several complete answers while most candidates managed parts (a) (b) and (c) only.

B2.2: Commutative Algebra

Question 1: This was a very popular question, but many candidates struggled with parts (c) and (e). In part (c) very few people found counterexamples like Z_{2Z} and instead tried to prove the statement is true. In part (e) one can easily use part (d) generalized to more than two ideals.

Question 2: This was the most popular questions with lots of complete solutions.

Question 3: Fewer candidates tried this question but those who did submitted almost complete solutions. The last part follows easily by realizing that the algebraic closure of k is an integral extension of k and then applying the going-up theorem and part (c).

B3.1: Galois Theory

Question 1: This was the most popular question and there were several excellent solutions. The majority of the mistakes where in the explicit computation of the action of the elements of the Galois group on the splitting field of the polynomial in part (c), and in part (b) when finding the values of a for which the field L/K is a normal extension.

Question 2: This was the hardest question. There were some excellent solutions however there were many mistakes in part (b).

Question 3: This was the least popular yet the most successful for students. There was excellent work presented.

B3.2: Geometry of Surfaces

Most students did well (many did extraordinarily well) even though the exam was not easy. Most students chose Q1 and Q3, almost half of the students attempted Q2. All questions went well on average, with high averages on all three questions (an average point higher on Q3). Students did surprisingly well on Q2(d), remembering a clever choice of local coordinates from the lecture notes (so almost nobody used/needed the Hint).

Common difficulties were: Q1(c) (students sometimes erroneously assumed the map was holomorphic and used Riemann–Hurwitz), Q1(d) (students tried defining the new Riemannian metric without using $D\varphi$), Q3(b) (a wrong sign in the second entry of the normal vector often caused incorrect second fundamental forms), Q3(d) (students did not know how to write down the formula for a surface of revolution, sometimes putting only a $\cos x$ factor in the first coordinate, but not putting the $\sin x$ factor in the second).

B3.3 Algebraic Curves

Question 1 was done by all candidates. There were many good answers; the average mark was just over 18. In part (a) of the question, the most common pitfall was to incorrectly argue (or omit the argument altogether) why the Hessian curve cannot share a component with the original curve in (ii). In part (b), in (i) the most common source of problems was failing to realize that the vanishing of all derivatives puts a condition on λ; there were also assorted computational mistakes. In (ii) the most common mistake was to divide by a factor which could a priori be zero, being nonzero precisely because of nonsingularity and (i).

Question 2 was done by most candidates. Again, there were many good answers; the average mark was almost 19. (a) and (b) were mostly done competently. The most common issue was the computation of intersection numbers in (c) which becomes very cumbersome with the resultant definition; some candidates also failed to identify one or other of the special values of μ.

Question 3 was done by only two candidates. One of the attempts was essentially complete, while the other had many gaps.

B3.4: Algebraic Number Theory

The examination appears to have been at a reasonable level, perhaps a little on the easy side. Rather, it was possible to avoid the harder questions and still accumulate a substantial number of marks.

Q1. (a) This question was straightforward for almost all who attempted. A few used Galois Theory rather than the minimal polynomial. One student applied Galois theory in an erroneous manner.

(b) This question was also easy.

(c) This appears to have been a genuinely difficult problem. Only two students submitted a complete solutions. Perhaps more hints would have been appropriate.

(d) Applying part (c) to this problem caused little difficulty.

Q2. (a) A surprising number of students struggled to produce a clean proof of this theorem, even though this problem was bookwork. Perhaps there is still some confusion working with quotient rings, or perhaps the proof was too long. A few students had trouble putting down a clean statement as well.

(b) Nearly all students solved this without difficulty.

(c) This problem was also straightforward.

(d) Here, one direction was easy, but very few students came up with a correct counterexample, indicating that the relationship between the different notions of primes and divisibility is still a source of conceptual confusion.

(e) Many students managed to solve this, although I had predicted some difficulty. Coming up with a list of primes with norm 3 and 7 is easy. However, choosing the correct decomposition requires some thought about membership of elements in ideals, which many students understood quite well.

Q3. (a) This bookwork problem was straightforward.

(b) Most students were familiar with this type of problem. However, there was some expository confusion in writing down a complete answer, About half of the students who attempted this problem wrote down the full computation correctly.

(c) This problem was also familiar, although minor errors of argument, e.g., regarding divisibility or coprimeness, cropped up here and there.

(d) Only one student solved this problem correctly. However, I had intended this problem to be a challenging one, requiring some creative thinking.

B3.5 Topology and Groups

The exam successfully tested students across the syllabus. The standard of answers were impressively high on average, suggesting that they understood the course well.

Question 1: The first part consisted of elementary questions about the fundamental group and homotopies. Although elementary, they were not all straightforward. In particular, 1(a)(iii) eluded many people. This required a proof that homotopic loops give the same conjugacy classes in the fundamental group.

The second part of the question required the students to show that there is a single homotopy class of maps $S^2 \to S^3$ and a single homotopy class of maps $S^2 \to S^3 \times S^3$. The former question was covered in the problem sheets, and requires the use of the Simplicial Approximation Theorem. Most students did it well, but some unfortunately tried to use the fundamental groups of these spaces, and this led to fallacious reasoning. Fortunately, most students saw that the second question follows quickly from the first one.

The average mark on this question was the lowest of the three. However, I do not think that it was harder than the other two. Instead, as it was based on material earlier in the course, it was a question that weaker students tended to aim for.

Question 2: Part (a) was bookwork, and was done almost uniformly well. Part (b) required students to understand push-outs of groups and to be confident with group presentations. Fortunately, it too was done well. A few students could not see the required push-out in (b) (i) or could not see the required relation in (b) (iii).

Question 3: This question was based on the last chapter of the course, and so was the most theoretically advanced of the three questions. Fortunately, it was surprisingly popular and done impressively well on average. It was very gratifying that most students could determine the free generating sets of the finite index subgroups and also provide good justifications for why $\tilde{X}_2$ corresponds to the required kernel whereas $\tilde{X}_1$ does not. I am delighted that so many students had mastered this material, which represents an important cross-over between topology and group theory.

B4.1: Banach Spaces

Overall this exam is on the easy side and the general level of answers was quite high.

Question 1: First question turned out to be the most difficult and least popular. Most of mistakes were minor, the only persistent major mistake was the claim that point-wise or uniform convergence implies convergence in the Holder norm. In part (d) many failed to notice that $f'(0)$ is not necessarily equal to 0 and so part (c) can not be applied directly.

Question 2: This was the easiest question. As in the previous years, there were quite a few functions that are supposed to be bounded linear functionals but are not linear. Also surprisingly many stated the Hahn–Banach theorem for Banach spaces.

Question 3: This was the most popular question. The bookwork part was done very well, most of the candidates got full mark for parts (a) and (b). In part (c) many candidates treated x in $xf(x)$ as a scalar factor. In (c) (ii) many could not come up with an example showing the lower bound on $\|T\|$. In part (d) many candidates claimed that coordinate-wise convergence implies convergence in the norm.

B4.2: Hilbert Spaces

Question 1 (Orthogonal projections)

In (b) it was disappointing that so many candidates ignored part (a) (statement of the Riesz Representation Theorem) and gave the familiar derivation from the Projection Theorem of the existence and properties of a projection operator, or even went back to the Closest Point Theorem (with proof); by taking (a) as the starting point one gets a shorter argument, as those who did this discovered. Those who gave voluminous answers to (b) did not thereby lose marks but would have had little time to devote to part (c).

In (c)(iii) several candidates failed to realise that the answer didn't come directly from (c)(i) and that the key point was to show that w belongs to $K \cap L$, for which (c)(ii) provides a stepping stone.

Question 2 (Equivalent and inequivalent norms; Closed Graph Theorem and Uniform Boundedness Theorem)

A pot pourri of results, with equivalence of norms as a unifying theme, testing a range of skills and knowledge from across the course and drawing on basic facts from B4.1 too. Except for the very last part, which was intentionally challenging, the question was generally answered successfully.

In (a)(ii), (iii) almost all candidates knew what was called for but a few tripped up in presenting examples.

Part (a)(i) was rarely handled economically (despite similar CGT examples in notes and a problem sheet). A number, deviously, went via the Inverse Mapping Theorem because they started from the identity map whose continuity was a consequence of the condition given in the question. Many candidates stated CGT without specifying how 'closed' was being interpreted.

Part (b) concerned UBT and two applications. The first application, though new, was generally handled well. Almost all appreciated that the embedding of Z into Z'' was needed; several cited Riesz Representation Theorem here, not having realised Z was not a Hilbert space. Only a couple of candidates saw how to attack (b)(iii) and many didn't offer an attempt.

Question 3 (Spectral theory)

In preparation for parts (b) and (c) the bookwork component of this question covered basic material from B4.1 [spectral theory did not feature on this year's paper] and recalled in B4.2. This part was poorly answered by many, with sloppy reasoning and scanty justification.

Parts (b) and (c)(i) (very easy) and (c)(ii) (which got progressively harder) gave candidates a chance to show off their facility in working with inner products and operators. In (c)(ii) the majority made at least some progress with the calculations, but almost without exception those who reached the appropriate quadratic expression failed to realise that a "$B^2 - 4AC$" argument was then needed to get the required conclusion (a method seen in the course in a different context). Many candidates who got stuck with the inequalities didn't proceed further.

[A typo right at the end of (c)(ii) (the final λ should have been M) wasn't queried during the exam. It didn't affect candidates' ability to give a valid answer to (c)(iii).]

B5.1: Stochastic Modelling and Biological Processes

Question 1: Almost all candidates attempted this question, and overall it was well answered. The techniques required were standard, and so most marks were lost for algebraic mistakes. These could have been avoided in some cases by recognising that the reactions are all zeroth or first order and so the mean equations are identical to the reaction rate equations.

Question 2: About half the candidates attempted this question, and most found the second half of the question difficult. The bookwork in part (a) was well answered. In part (c), very few candidates used the results from (a) to derive an expression for the mean square displacement of the particle (which was the most simple approach).

Question 3: About half the candidates attempted this question, and most found it difficult. In part (a) many candidates did not write down the correct boundary condition at $x = L_2$, and very few could calculate $p(x, t)$ explicitly (many tried separation of variables, very few wrote down the fundamental solution to the heat equation and tried to adapt it using the method of images). In part (b) most candidates recapitulated the argument from the lecture notes, without adapting it specifically to the problem at hand. Very few candidates attempted part (c), and only a fraction of those included the correction term.

B5.2: Applied PDEs

Question 1: Most students attempted this question. A common error was not considering the possibility of singularities forming due to the function c that would affect the domain of definition. Getting the right picture for the shocks required understanding that characteristics emerge from the data curve.

Question 2: Also attempted by most students. Parts a and bi, bii were mostly done well, though there were some conceptual misunderstandings in introducing a jump discontinuity in a. Part biii caused the most trouble, with almost no students obtaining an expression for φ only involving x and y.

Question 3: This problem was attempted by very few students, though people who did attempt it did quite well adapting previously seen ideas with Green's function for elliptic equation to 3D. A variety of successful approaches were taken in part biii.

B5.3: Viscous Flow

Question 1.

- Part (a): the book work aspects of this question were very well done.
- Parts (b)(i) and (b)(ii) were well done.
- In part (b)(iii) many candidates struggled to solve for $\hat{u}(\hat{y}, \hat{t})$. Hence few candidates were able to show that the stress on the plate is $\pi/4$ out of phase with the velocity far from the plate as $\hat{y} \to \infty$

Question 2.

- Part (a): the book work aspects of this question were very well done.
- In part (b)(i) some candidates were unable to deduce the form of $g(x)$. Some candidates struggled to give the form of the boundary conditions for $f(Y)$ and $H(Y)$.

Question 3.

- It was pleasing to see the majority of candidates giving correct answers to 3(a).
- Some candidates were unable to give the correct physical interpretation of the last three conditions in 3(b).
- Part (c) was well done in general.
- The majority of candidates struggled to show that no steady state solutions exist if $Q > Q_{max}$.

B5.4: Waves and Compressible Flow

Q1: The first two parts were well done, though many students did more than was required in part (b), starting from the compressible Euler equations rather than the wave equation. Some students started part (c) well, but most got lost with the algebra in determining the equation solved by λ, in many cases due to poor structure of their solutions. There was a sign error in the equation that λ satisfies (the cosine term should be negative), but only one candidate made it far enough to realise this. Many students found the approximate solution corresponding to membrane modes in part (d), but no one accounted for the approximate solutions $\lambda = n\pi$, corresponding to modes of the gas.

Q2: This question was the most popular and was generally done well, especially parts (a) and (b). No candidates gave entirely satisfactory answers to part (c), with many wrongly guessing that the two cases would correspond with $c_p < c_g$ and $c_p > c_g$. The realisation that the first case requires $k \ll 1$, and hence $c_p \approx c_g$ was made by just one candidate.

Q3: All candidates had the marks for part (a), there were some good answers to part (b) and a few good answers for part (c), which was the most different from previously seen examples. A surprising number of candidates misread $h_R < h_L$, which led to some confusion and to some unphysical sketches in part (d). Only a couple of candidates were able to show graphically that the solution for h_- in part (b) was unique, and a common mistake in part (c) was to assume that it was the 'positive' characteristics emanating from the origin in the expansion fan.

B5.5: Mathematical Ecology and Biology

Question 1: By and large, this question was quite well done, but most candidates needlessly threw away marks by not putting arrows on their cobweb diagrams. The very last part required candidates to know that a complex number can be written in trigonometric form. Only one candidate seemed to know this.

Question 2: There was an obvious typographical error at the beginning and in part (b) in that K was not included in the list "A, B, C and D". This was caught near the start of the examination and an announcement made. It did not seem to affect anyone.

This question was very well done. A number of candidates talked about activation and inhibition in explaining the model when it was a predator-prey system and not a chemical system. Surprisingly, in the stability analysis a number of candidates solved the eigenvalue problem $(\lambda - 1)(\lambda + ab) = 0$ by multiplying out to obtain a quadratic in λ and then using the formula for solving a quadratic and invariably getting the wrong answer. Candidates still really do not know how to do phase planes despite the fact that it is covered in a number of courses.

Question 3: This was attempted by only a handful of students. Most did not get (d)(ii) (they should have pointed out that the analysis ignored the boundary conditions). Overall, reasonably well done.

B5.6: Nonlinear Systems

Qn 1: Reasonably well answered overall. Only one or two candidates were able to find the approximate location of the homoclinic orbit. Many people assumed initial conditions for part (b), where they should have been kept general.

Qn 2: Many candidates did not identify the first bifurcation in (a)(i) as a saddle-node. A few candidates got as far as (c)(i). No-one got (c)(ii).

Qn 3: Reasonably well answered by those candidates, who attempted this question.

B6.1: Numerical Solution of Differential Equations I

Question 1: Solution of second order ODE by rewriting as system of two first order ODEs. First three parts done generally very well, very few really high marks as most did not see that last part reduced to an eigenvalue problem.

Question 2: Multistep method, attempted by all. Methods well understood, generally well done with cluster of high marks but very few really high marks. Algebra in middle part of question proved too demanding in time available.

Question 3: PDE question, stability, maximum principle, boundary conditions. Bookwork parts generally well done, discretisation of boundary condition not all that well done. Some very high marks and some who attempted three questions with only a few marks.

B6.2: Numerical Solution of Differential Equations II

Question 1: was addressed by all candidates. Most of them had no difficulties to specify the matrix system and to establish its truncation error and convergence order. Some candidates had problems in reasoning why the matrix was invertible and why the maximum principle could not be straightforwardly employed to prove the higher convergence order for the modified stencil. All in all, questions of this type have been around in previous papers for several years and this was reflected in the relatively good performance of the candidates.

Question 2: corresponded to two main theorems from the textbook. Only a few candidates managed to follow all steps of the proof. Nevertheless, the majority was able to get the estimate for the particular case in subquestion a), representing most of the score.

Question 3: contained a first part of bookwork. It was rather well done, except for the last point, where no one realised that no conflict arises with the theorem regarding high order linear schemes. Also, for scheme iv) many did not realised that the scheme had only one possible value of λ to be TVD. Only a handful of candidates attempted to give a proof of the

Lemma.

B6.3: Integer Programming

The students performed very well in this exam. The vast majority showed good understanding of the basic concepts, with a good proportion of them also mastering more advanced techniques.

B7.1: Classical Mechanics

Question 1 is on Lagrangian mechanics. Part (a) required candidates to prove Noether's theorem, and was generally well answered. Part (b) is a fairly typically question on Lagrangian mechanics, involving two masses connected by a taut string, with one mass sliding on a smooth horizontal table, and the other hanging vertically through a hole in the table. A number of candidates did not correctly implement the constraint imposed by the string, although many gave largely complete answers to parts (i) and (ii). Very few candidates correctly linearised the problem in part (iii): a common error was to assume that $\dot{\theta}$ is constant, instead of P. Some candidates went on to gain marks in the unrelated part (iv).

Question 2 is on rigid body dynamics. Part (a) required candidates to derive an expression for the kinetic energy of a rigid body in terms of the inertia tensor and angular velocity, while part (b) asked candidates to derive the Lagrangian for the Lagrange top. Both are bookwork, and were generally well answered. A fair number of candidates got bogged down in showing (c)(i), usually through making algebraic errors (and often via an overly long route). Very few candidates made much progress with part (ii), although this can be answered using only the information given in (c)(i). Again, this is a linearisation problem, with some care needed since $\theta = 0$ is a coordinate singularity.

Question 3 is on Hamiltonian mechanics, specifically focusing on canonical transformations. This was the least popular question. Part (a) required candidates to write Hamilton's equations in terms of Poisson brackets, and give the definition of a canonical transformation. Both are bookwork, and were generally well answered. Answers to part (b) were, with a few exceptions, disappointing. Many candidates were not computing the correct Poisson brackets, and/or made errors in the computations (almost always involving problems with indices). Part (b) is the linearisation of part (c)(i), the latter having been covered in lectures (hence counts as bookwork). Candidates that noticed this generally scored well. A couple of candidates correctly identified the transformation in (c)(ii) as a rotation, although very few made it this far.

B7.2: Electromagnetism

Q1: This question was attempted by all students (being the first one, this is common). I think the level was adequate.

Q2: I think the level of this question was adequate.

Q3: As a small remark, in part (a) some students stated the conditions on source terms, without giving a proof. Maybe the phrasing "give reasons for your answer" should have been changed to "prove your statement", or something like that. However, a very similar question has been used previous years, with exactly the same phrasing as this year, but in those years the students did understand fully what was being required. In any case, credit was given for the precise statement without a proof.

B8.1: Martingales Through Measure Theory

Question 1. All candidates except six of them attempted this question. Most candidates did well on the conceptual parts, while a few candidates lose their marks for proving the coincidence of two measures in part (b) and part (c), either because of incomplete arguments or not quoting the theorem to justify their proofs.

Question 2. Again this is the question most of candidates attempted, all except two. While many candidates lose a few marks for part (a) (ii) (iii), and part (b) (iii). Part (a) (ii) looks so obvious but few candidates argued correctly. Most candidates had difficulty to split the proof of an "if and only if" statement into a proof of the "if" part and a proof of the "only if", and failed to produce complete arguments. On the other hand, most students know how to argue with Borel-Cantelli lemma. Part (b) (iii) seems the most challenging part of the paper, few candidates argued correctly, though I saw some excellent proofs.

Questions 3. Only 11 candidates attempted this question. I guess that only because the first two questions are manageable to most candidates, so most of them chose the first two. This is rather standard one too and at the same level of the first two questions. A few candidates lose their marks on the L^p version of the martingale convergence theorem.

B8.2: Continuous Martingales and Stochastic Calculus

Question 1 – only two students attempted this question and they also happened to attempt the remaining two questions. This limited evidence does not allow to judge what the common difficulties might have been. I note however that despite having the right ingredients the two students did not manage to obtain the correct ODE in the last part of the problem.

Question 2 – all students attempted to solve this question. The standard of answers was good. Several students failed to realise the Brownian motion was started in x and not in zero. This led to incorrect definition and, quickly, obviously false computations. The last part of part (d) caused some difficulties with several students making silly mistakes when invoking simple conditional probability.

Question 3 – all students attempted to solve this question. The standard of answers was good and was (very) marginally better than for Question 2. Significant part of students tried to give a standalone definition of Brownian motion without characterising it as a Gaussian process with a given covariance function and continuous paths (despite being asked for this). This then led to troubles in part (e) where it was much easier to use the latter than the former definition.

B8.3: Mathematical Models of Financial Derivatives

Question 1: This was a new question this year, replacing a question on the binomial method which had appeared, in various forms, for roughly the previous decade. The question consisted of two parts:

1. defining an Itô stochastic integral and computing its mean and variance;

2. solving the stochastic differential equation

   $$dr_t = \kappa(\alpha - r_t)\,dt + \sigma\,dW_t$$

   and finding its mean and variance as $t \to \infty$.

Most people who attempted the question got the first part correct (it was mainly bookwork). Somewhat fewer took the hint that started the second part and let $z_t = r_t - \alpha$. Fewer still realised that it was enough to express the solution in terms of the integral $\int_0^t e^{\kappa(s-t)}\, dW_s$ and then use the first part of the question to compute the variance. About 75% of candidates attempted the question.

Question 2: This was a question which started out with a European digital put problem to solve. Most people who attempted the question got this part out. Then the strike of the digital put was set equal to the share price at time $T_1 < T$, so $K = S_{T_1}$, and the question asked for the option price after T_1 and before T_1 (in that order). The after T_1 part was trivial, the before T_1 part stumped a few (the point being that for $t \leq T_1$ the option price does *not* depend on the share price). Finally, with the strike set equal to S_{T_1}, an up-and-out barrier was also introduced, set equal to $2\,S_{T_1} > S_{T_1}$ and again the question was to find the price after and before T_1.

The biggest problem was that students failed to realise that the option price was a constant at time T_1 in the second two cases.

Question 3: Involved a perpetual American option with a slightly non-standard payoff. The aim was to show that the smooth pasting conditions do not always hold. The first part assumed that the smooth pasting conditions did hold, and most people who attempted the question got this part correct; ultimately this involved solving a quadratic equation and showing one root was negative and one was greater than one.

The second part of the question asked the student to first show that smooth pasting was only possible under certain circumstances, which a good many did. The rest of the second part asked students to turn this condition back a 'financial' condition (which simply involved substituting values into a quadratic equation and noting that the quadratic had to be positive and increasing at these substitutions) and then stating and justifying where the optimal exercise boundary was in these cases. This was less well done.

The biggest problem with this question was the attempted use of explicit solutions to a quadratic equation in the second half of the question, rather than simply substituting values back into the equation (and noting that when doing so the quadratic was both positive and increasing).

B8.4: Communication Theory

Question 1: Many students picked up the bulk of their points in (a)(i),(ii) and (b)(i) (worth 9 pts collectively) and a smattering of points elsewhere. Few students correctly guessed that (a)(iv) was "false," and many wasted a great deal of time trying to prove it true. Perhaps as a result, a number of students did not seriously attempt (b)(ii). None of the students succeeded with the approach to (b)(ii) via Jensen's inequality, as suggested by the hint; but several students succeeded with Lagrange multipliers. In fact, for many the hint was a red herring, leading to hasty applications of Jensen's that could not produce a sharp bound.

Question 2: Most students had no difficulty with (a). For (b)(i), a number of students successfully cited Chebyshev's inequality or a law of large numbers to prove convergence for each individual letter, but few made clear arguments that frequencies of all letters would converge jointly in probability. Only one student made a complete argument for (b)(ii), and most either didn't attempt it or seemed confused. Every student attempted (c), and they achieved varying results. Many offered (sometimes incomplete) constructions, without much in the way of proof that their construction should be possible. Surprisingly, most students began (c) with a rendition of the construction of a code tree with a given length sequence, as in the proof of Kraft's inequality, rather than beginning with the code proffered by the question. Part (c) was time consuming, but in terms of students' results, it was the most enlightening problem on the exam.

Question 3: Most of those students earned the bulk of the 7 pts available on (a)(i) and (b)(i). Students had mixed results on (a)(ii); some succeeded by careful computation and guess-and-check, but few, in any, found natural examples indicating strong intuition about when the decoders differ. Few students made serious attempts on (b)(ii), and none of these had much merit; perhaps the problem could have been separated out from (b) and broken into parts. Despite this, several students gave mostly complete answers to (b)(iii), missing only that the Hamming code could be improved upon by taking a coset.

B8.5: Graph Theory

Q1 was generally done well, although many candidates forgot to check the inductive hypothesis in part (c). In Q2, a surprising number of candidates confused the vertex and edge forms of Max-Flow Min-Cut. Q3 was generally done well, although few candidates spotted the idea of adding a vertex to the unbounded face in parts (e) and (f).

BEE, BSP and BOE essays and projects

Mark reconcilation was handled for essays and projects as part of the same exercise. Some assessors did not make the deadline for submitting marks — which added to the amount of time and effort involved in reconciliation — so the procedure was handled on a rolling basis once initial suggested marks were received.

In the case of mark discrepancies between the assessors, both assessors were contacted. If they agreed sufficiently, as set out in the guidelines, then they were invited to indicate if they wanted to discuss things with each other but that otherwise the automatic reconcilation procedure was applied. If they differed sufficiently from each other (e.g., if the marks crossed class boundaries or didn't have ranges that overlapped with each other), then they were required to reconcile marks with each other. In all cases, if the supervisor suggested a mark that differed in a substantial way, the assessors were informed and were invited to include the supervisor in their discussions if they wished.

Most cases were handled easily, though there was one case in which the assessor marks were very far apart and could not be reconciled, so a third assessor was needed. Overall, things went smoothly, though recent rule changes necessitated the examiner in charge of the reconciliation procedure to send very many more e-mails than used to be necessary, and it is rather unfortunate that recent changes in the reconciliation procedures have complicated the procedure in this way and made the examiner's task more arduous.

Additionally, I note that despite instructions indicating that supervisors may give text in support of their marks that it seems that most elected not to do so. I find it rather surprising that it isn't the other way around, as I would expect that given this text in the instructions given to supervisors that one ought to expect that most of them would include a small bit of text in support of their suggested marks. Finally, I note that the rules are being changed next year to a much better set of rules (namely, assessment from supervisor and one other assessor) that will automatically address the above issues in the current rules.

BO1.1: History of Mathematics

Both the extended coursework essays and the exam scripts were blind double-marked. The marks for essays and exam were reconciled separately. The two carry equal weight when determining a candidate's final mark.

The paper consisted of two halves, which carry equal weights. In section A ('extracts'), the candidates were invited to comment upon the context, content and significance of two samples of historical mathematics (from a choice of six). Out of seven candidates, one person answered each of questions 1 and 3; two people answered question 2, whilst three people opted for each of questions 4 and 6. Question 5 was the most popular question, with four people having attempted this one.

The unpopularity of questions 1 and 3 is not particularly surprising: the latter (a general remark from Jacob Bernoulli, prefacing his law of large numbers) related to material seen only briefly in a single lecture. The background needed for question 1 (Viète on symbolic notation) had a slightly more prominent place within the course, but the particular extract given was a difficult one to interpret, making this one of the harder questions on the paper.

The material for question 2 (John Wallis on indivisibles) formed part of the main thrust of the lecture course, and where this question was chosen, it was done well. The same cannot be said, however, of questions 4 and 6. In both cases, candidates tended to read more into the extract than was actually there. Thus, for example, question 4 (Fourier series) sparked comments on continuity from some candidates, although this is only tangentially related to the extract given. Question 6 concerned Cantor's definition of a set, and was thus an invitation to write a little about the emergence of set theory; however, some candidates instead wrote about Cantor's role in the development of a rigorous notion of real number in the late nineteenth century, with only the merest nod to the notion of a set.

For the most popular question in section A (question 5: Galois' definition of a group), the answers were of a range of qualities. Unfortunately, some candidates fell into the trap of writing of 'group theory' almost as something that already existed at the time of Galois' work. The context for this extract (e.g., Lagrange's use of permutations in connection with the study of polynomial equations) was a little muddled in some answers.

Some candidates organised their answers to questions from section A under the three headings 'context', 'content', 'significance', with material mostly being distributed correctly. Amongst those who did not take this approach, there was a slight tendency to produce answers that were not very well structured, with ‘context’ and ‘significance’ jumbled up together. In some cases, the candidate was so keen to write down what they knew about the broader topic at hand that they failed to comment fully on the extract provided.

In section B of the paper, candidates were invited to write an essay, taken from a choice of three. Here, question 8 was the most popular option (four candidates), with question 9 attracting two responses, and question 7 just one. As for question 2 in section A, the popularity of question 8 (on changing attitudes to rigour in mathematics) is probably due to the fact that it related to one of the main themes of the lecture course. Answers to this question ranged in quality, but all candidates displayed at least a reasonable understanding of the topic.

The unpopularity of question 7 (on Descartes’ contributions to mathematics) may lie in the fact that the work of Descartes that was covered in the lecture course was not very wide-ranging, and tended to be overshadowed by discussions of other figures. Question 9 (on the validity of the ‘pure’/‘applied’ distinction prior to the twentieth century) was a challenging one.

An unfortunate feature of some exam scripts was the inclusion of (mostly slight) mathematical errors. Although this is a paper on the *history* of mathematics, and the need to present *detailed* mathematics arises only occasionally, it is important that candidates do still endeavour to get it right. Some errors of historical fact did also appear in some scripts, along with the inclusion of some irrelevancies. Nevertheless, it was clear that candidates had learnt something of the history of mathematics.

For the coursework essays, the candidates were invited to write about some aspects of the background, content, reception, etc., of Newton’s *Principia*. The seven essays were satisfyingly different, covering a broad range of topics related of the *Principia*, and were, moreover, written in a range of styles. There was evidence of a great deal of wider reading in the sources cited, although these sources were not always handled as critically as they might have been — the better candidates did this very competently though. The referencing was done well in five out of the seven essays.

BN1.1: Mathematics Education

The assessment of the course is based on:

- Assignment 1 (Annotated account of a mathematical exploration) 35%
- Assignment 2 (Exploring issues in mathematics education) 35%
- Presentation (On an issue arising from the course) 30%

Each component was double-marked. Where a significant difference between marks awarded by the two assessors arose, this was discussed in more detail before agreeing a mark.

There were 9 students on the course this year, all of whom also went on to study for the BN1.2 (Undergraduate Ambassador Scheme) in Hilary Term. This was the first time the course has run for several years, and generally we were impressed by the quality of the candidates. Indeed, four of the nine candidates were awarded an overall USM greater than 70.

BN1.2: Undergraduate Ambassadors Scheme

The assessment of the course is based on:

- A Journal of Activities (20%)
- The End of Course Report, Calculus Questionnaire and write-up (35%)
- A Presentation (and associated analysis) (30%)
- A Teacher Report (15%)

The Journal and Course Report were double-marked. Where a significant difference between marks awarded by the two assessors arose this was discussed in more detail before agreeing a mark.

There were 9 students on the course this year, all of whom had previously studied for the BN1.1 course in Mathematics Education in Michaelmas Term. All students engaged well with the practical aspects of the course leading to many first class marks being awarded in these areas. Pleasingly, and improving on last year, several candidates were awarded first class marks for their reflective writing too. There were three candidates who achieved an overall mark of 70 or more, again an improvement on the past two years. All other candidates achieved upper second class marks. While my sense is that strong candidates were attracted to the course, a stronger group identity and collaborative approach was also evident this year that perhaps is associated with ways of working that are afforded by candidates following both the BN1.1 and BN1.2 courses.

Statistics Options

Reports of the following courses may be found in the Mathematics & Statistics Examiners' Report.

SB1 Applied Statistics

SB2a: Foundations of Statistical Inference

SB3a: Applied Probability

SB3b: Statistical Lifetime Models

SB4a: Actuarial Science I

SB4b: Actuarial Science II

Computer Science Options

Reports on the following courses may be found in the Mathematics & Computer Science Examiners' Reports.

OCS1: Lambda Calculus & Types

OCS2: Computational Complexity

Philosophy Options

The report on the following courses may be found in the Philosophy Examiners' Report.

122: Philosophy of Mathematics

127: Philosophical Logic

E. Comments on performance of identifiable individuals

Removed from the public version of the report.

F. Names of members of the Board of Examiners

- **Examiners:**
  Prof. Simon Blackburn (External)
  Prof. Philip Candelas
  Prof. Des Higham (External)
  Prof Dominic Joyce (Chair)
  Prof. Frances Kirwan
  Dr. Neil Laws
  Prof. Irene Moroz
  Prof. Mason Porter

- **Assessors:**
  Prof Luis Fernando Alday
  Mr Nick Andrews
  Prof Ruth Baker
  Prof Sir John Ball
  Prof Charles Batty
  Prof Dmitry Belyaev
  Prof Coralia Cartis
  Prof Gui-Qiang Chen
  Prof Xenia de la Ossa
  Prof George Deligiannidis
  Dr Paul Dellar
  Prof Jeffrey Dewynne
  Prof Arnuad Doucet
  Prof Cornelia Drutu
  Dr Noah Forman
  Prof Eamonn Gaffney
  Dr Kathryn Gillow
  Prof Ben Green
  Dr Ian Griffiths
  Dr Cameron Hall
  Prof Ben Hambly
  Prof Roger Heath-Brown
  Prof Ian Hewitt
  Dr Christopher Hollings
  Prof Peter Howell
  Dr Jenni Ingram
  Prof. Minhyong Kim

Prof Jochen Koenigsmann
Prof. Jan Kristensen
Prof Marc Lackenby
Prof Alan Lauder
Prof Philip Maini
Prof. Lionel Mason
Dr Gary Mirams
Prof Derek Moulton
Dr Peter Neumann
Prof Nikolay Nikolov
Prof Harald Oberhauser
Prof Jan Obloj
Prof Jonathan Pila
Prof Hilary Priestley
Prof Zhongmin Qian
Prof Alexander Ritter
Dr Ricardo Ruiz Baier
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Dr Gabriel Stylianides
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Prof Dominic Vella
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