MULTIVALUED DIR-MINIMIZING FUNCTIONS

General Prerequisites: Solid knowledge of functional analysis and Sobolev spaces. Acquaintance with variational methods in PDEs and some basic geometric measure theory is also recommended.

For Sobolev spaces: chapters 2 and 7 in the book by Gilbarg-Trudinger [2] should suffice. Alternatively, a mastery in the material of the courses C4.3 (Functional Analytic Methods in PDEs) and/or C4.6 (Fixed Point Methods for Nonlinear PDEs). For Geometric measure theory: familiarity with chpaters 6 and 7 in the book by Krantz-Parks [3] would be helpful, although this material will most probably be reviewed according to necessity.

Course Lecture Information: 4h lectures + 4h reading groups.

Course Overview: The course will serve as an introduction to the theory of multivalued Dir-minimizing functions, which can be viewed as harmonic functions which attain multiple values at each point. These objects play a fundamental role in Almgren's big regularity paper [1] is one of the most remarkable achievements in geometric measure theory. The basic text on which the material of the course is based on is the memoir "Q-valued functions revisited" [5] by De-Lellis-Spadaro, which drastically simplifies the original paper by Almgren. Other related works are [4] and [6].

Course Synopsis: The space of unordered tuples. The notion of differentiability and the theory of metric Sobolev in the context multi-valued functions. Multivalued maximum principle and Holder regularity. Estimates on the Hausdorff dimension of the singular set of Dir-minimizing functions. If time permits: mass minimizing currents and their link with Dir-minimizers. Additional topics (in the expense of some of the topics suggested above) may be included according to how things evolve.

Aimed at: Postgraduate students interested in geometric measure theory and its link with elliptic PDEs.

References

[1] F. J. Almgren, Jr. *Almgren's big regularity paper.* Volume 1 of World Scientific Monograph Series in Mathematics. World Scientific Publishing Co. Inc., River Edge, NJ, 2000.

[2] D. Gilbarg and N. S. Trudinger. *Elliptic Partial Differential Equations of Second Order.* Springer-Verlag GmbH Germany. 2001.

[3] S. G. Krantz and H. R. Parks. *Geometric Integration Theory.* Birkhäuser Boston, MA. 2008.

[4] C. D. Lellis and E. N. Spadaro. *Q-valued functions and approximation of minimal currents.* PhD Thesis. 2009.

[5] C. D. Lellis and E. N. Spadaro. *Q-valued functions revisited.* Memoirs of the American Mathematical Society. Volume 211 . Number 991. 2011.

[6] E. N. Spadaro. *Complex varieties and higher integrability of dir-minimizing q-valued functions.* Manuscripta mathematica, ISSN 0025-2611, Vol. 132, N. 3-4, 2010, 415-429.