Fully funded PhD studentship: October 2024-September 2028

Mathematical problems in deep neural networks: Causes and solutions

Joab Winkler (j.r.winkler@sheffield.ac.uk)
Department of Computer Science, The University of Sheffield

Background Deep neural networks (DNNs) are used in many applications because they can learn complicated features in data, segregate data into clusters that are defined by specified properties, and automate some tasks performed by humans. These applications include medical diagnosis, image processing and natural language processing. A DNN consists of a large number of neurons, the parameters of which are calculated by a learning algorithm. The input to this algorithm is a large quantity of training data with known outputs. If this training is successful, the DNN should return the correct result, with a high level of confidence, when new data are applied. This desirable situation may not, however, occur and thus problems arise. Three common problems in DNNs are vanishing and exploding gradients, and adversarial examples. The implications of these problems may be serious because of the use of DNNs in, for example, autonomous vehicles, the diagnosis of tumours and the detection of financial fraud. It is therefore imperative that their causes be identified, such that remedial action be taken and DNNs return the correct result. These problems arise because of instabilities in the algorithm that propagates data through a DNN.

Aims

- To identify the causes of instabilities in the equation that defines the propagation of data in DNNs, and the conditions for stable and unstable propagation of data.
- To propose methods and solutions of the equation that defines the propagation of data and thus distinguish between stable and unstable regimes in DNNs.
- To test these methods and solutions on practical examples and compare the results with the results from established methods.

Methods

The propagation of data through a DNN will be represented as a dynamical system and three methods of analysis will be used:

- The discretisation of a non-linear differential equation: The eigenvalues of the Jacobian matrix in this discretisation are related to the stability of a DNN.
- Dynamical systems: Boundaries between stable and unstable regimes in DNNs will be considered because their traversal by data may cause exploding gradients.
- Linear algebra: Stability measures of the computations in a DNN will be developed.

Candidate requirements Candidates must have, or expect to obtain, a 2.1 or 1st class degree in Mathematics, or a dual degree, Mathematics and Engineering, Mathematics and Physics, or Mathematics and Computer Science. A strong interest in numerical analysis and computational linear algebra are essential because the project involves theoretical development and software implementation.

Interested candidates are strongly encouraged to contact Dr Winkler (j.r.winkler@sheffield.ac.uk) to discuss their interest in, and suitability for, the project prior to submitting their application.

The award will fund the full (UK or overseas) tuition fee and UKRI stipend (currently £18,622 per annum) for 3.5 years, and a research grant to support costs associated with the project.

Closing date for applications 24 January 2024