Nonlinear Fokker-Planck equations modelling large networks of neurons

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General Prerequisites:
Good knowledge in Functional Analysis; basic knowledge about PDEs and distributions; notions in probability.

Course Term:
Trinity (weeks 6,7,8)

Course lecture information:
4 lectures (2 hours each)

Course Overview:
The course will be a detailed presentation of some nonlinear PDE models in neuroscience, from derivation and motivation to analytical results, with emphasis on the Nonlinear Noisy Leaky Integrate and Fire model.

Learning Outcomes:
Understanding how nonlinear non-local PDE models can help to understand the formation of complex activity in large networks of neurons and get knowledge into the abstract methods used to study this type of equation: tranformation to a Stefan problem, relative entropy methods, bifurcation theory...

Course Synopsis:
We will start from the description of a particle system modelling a finite size network of interacting neurons described by their voltage. After a quick description of the non-rigorous and rigorous mean-field limit results [1, 2, 9], we will do a detailed analytical study of the associated Fokker-Planck equation, which will be the occasion to introduce in context powerful general methods like the reduction to a free boundary Stefan-like problem [6], the relative entropy methods [2, 4], the study of finite time blowup [14, 10] and the numerical and theoretical exploration of periodic solutions for the delayed version of the model [3, 11]. I will then present some variants and related models, like nonlinear kinetic Fokker-Planck equations [12, 13] and continuous systems of Fokker-Planck equations coupled by convolution [7, 5, 8].

References

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