

# Examiners' Report: Honour Moderations in Mathematics and Philosophy 2012

## Part I

### A. STATISTICS

#### (1) Numbers and percentages in each class

See Table 1, page 1.

Table 1: Numbers in each class

	Number					Percentages %				
	2012	2011	2010	2009	2008	2012	2011	2010	2009	2008
I	6	7	8	5	5	40	38.89	36.36	22.73	26.3
II	6	10	12	16	11	40	55.56	54.55	72.73	57.9
III	0	1	1	1	2	0	5.56	4.55	4.55	10.5
Fail	3	0	1	0	1	20	0	4.55	0	5.3
Total	15	18	22	22	19	100	100	100	100	100

#### (2) Vivas

No vivas were given.

#### (3) Marking of Scripts

In Mathematics, all scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. There is an extensive checking process. In Philosophy, all scripts were single marked except for failing scripts, which were double-marked.

### B. New examining methods and procedures

No new methods were used.

### **C. Changes in examining methods and procedures currently under discussion or contemplated for the future**

The Mathematics department is currently going through the process of changing to Prelims for the 2013 examinations.

### **D. Notice of examination conventions for candidates**

The first Notice to Candidates was issued at the beginning of Trinity term.

These can be found at <http://www.maths.ox.ac.uk/content-47>, and contain details of the examinations and assessments. The course Handbook contains the full examination conventions and all candidates are issued with this at Induction in their first year. All notices and examination conventions are on-line at <http://www.maths.ox.ac.uk/notices/undergrad>.

## Part II

### A. GENERAL COMMENTS ON THE EXAMINATION

The Moderators would like to express their gratitude to:

- Nia Roderick, Helen Lowe, and Charlotte Turner-Smith, for all their work in running the examinations system for Honour Moderations in Mathematics and Philosophy.
- Waldemar Schlackow, for continuing to maintain and update the Examinations Database, without which the Moderators would have a huge amount of work to do to produce a Pass List.
- Dr Brian King, for his help with the philosophy examination, as an assessor.

Of the six Firsts, two were awarded on the basis of excellence in Mathematics, three were awarded on the basis of excellence in Philosophy, and one was awarded on the basis of excellence in both subjects. Three candidates failed the examination, two of the three passing in Mathematics, and none passing in Philosophy.

### Timetable

The examinations began on Monday 18th June at 2.30pm and ended on Wednesday 20th June.

### B. EQUAL OPPORTUNITIES ISSUES AND BREAKDOWN OF THE RESULTS BY GENDER

The breakdown of the final classification by gender is as follows:-

Class	Num	Gender	Percent
First	6	m	54.55
	0	f	0
Second	4	m	36.36
	2	f	50
Third	0	m	0
	0	f	0
Fail	1	m	9.09
	2	f	50

## C. DETAILED NUMBERS ON CANDIDATES' PERFORMANCE IN EACH PART OF THE EXAMINATION

### Paper 1: Pure Mathematics 1

Question	Maths and Philosophy		Single School	
	Mean	Std Dev	Mean	Std Dev
Q1	10.29	3.43	10.75	3.83
Q2	15.08	3.99	15.64	3.58
Q3	10.09	4.81	12.47	4.45
Q4	10.27	3.95	9.94	4.75
Q5	11.58	6.20	9.17	5.39
Q6	9.75	3.86	11.00	4.67
Q7	11.57	5.86	14.10	3.94
Q8	8.33	7.64	11.83	4.27

### Paper 2: Pure Mathematics II

Question	Maths and Philosophy		Single School	
	Mean	Std Dev	Mean	Std Dev
Q1	14.21	4.28	12.67	4.13
Q2	7.5	4.36	5.66	3.75
Q3	8.5	5.53	8.75	4.65
Q4	14.38	4.86	13.59	5.08
Q5	15.8	2.01	16.33	2.64
Q6	11.5	7.60	12.58	6.08
Q7	11	5.61	13.49	4.39
Q8	12.5	6.86	11.45	5.05

### Paper 3: Elements of Deductive Logic

Maths and Philosophy		MathsPhil and PhysPhil	
AvgUSM	StdDevUSM	AvgUSM	StdDevUSM
55.3	19.8	52.9	18.3

### Paper 4: Introduction to Philosophy

AvgUSM	StdDevUSM
65.8	4.6

## D. COMMENTS ON INDIVIDUAL PAPERS

See the Mathematics report for reports on the following papers:

Paper 1: Pure Mathematics I

Paper 2: Pure Mathematics II

### Report on Elements of Deductive Logic

The statistics in the following table, and the comments on individual questions, are based on the answers of both Mathematics and Philosophy and Physics and Philosophy candidates.

Statistics on individual questions:

Question	1	2	3	4	5	6	7	8
Number of Answers	36	29	22	27	8	11	22	13
Average	14.8	12.0	14.5	14.7	15.5	12.4	11.2	9.0
St. Dev.	5.4	4.6	6.3	7.1	5.6	4.4	3.4	6.8

The average marks for this paper were low. Despite this, 6 of the 15 mathematics and philosophy candidates obtained USMs of 70 or above. It is also noteworthy that, as detailed in what follows, many of the errors that led to low marks were indicative of a lack of a proper understanding of central material, rather than of unusually difficult questions. Three Mathematics and Philosophy candidates failed to obtain the passing mark of 30.

### Comments on individual questions:

1. (*Compactness*)

Most candidates obtained full marks for part (a), suggesting that last year's examiners' report had been read. In their answers to part (b), several candidates thought that they could appeal to a structure that models  $\Gamma_n$ , directly on the basis that  $\Gamma_n$  was (assumed to be) finitely satisfiable. This, of course, is a result not established until part (c), the argument for which relies on the result to be established in part (b). Part (c) was also not well done in general. Many answers suffered from a lack of precision, suggesting the candidates had only a partial grasp of the material. For example, when asked to specify a structure that satisfies  $\Gamma^+$ , many candidates gave descriptions true of such a structure, but in such a way that failed to allow them to prove that such a structure existed (e.g., "Let  $\mathcal{A}$  be such that  $|\phi|_{\mathcal{A}} = T$  for every  $\phi$  in  $\Gamma^+$ "). Some candidates, while offering a lengthy answer to this part of the question, failed to specify a structure at all.

Two general strategies were employed in answer to part (c): some candidates sought to prove the result by an induction on the complexity of

sentences in  $\Gamma^+$ , exploiting  $\Gamma^+$ 's analogue of negation completeness; others sought to establish it by reductio. Two common mistakes by those offering a proof by induction were: failure to consider  $\perp$  as a base case; and failure to realise that their induction technique required one to prove that a sentence is true in  $\mathcal{A}$  if *and only if* that sentence is in  $\Gamma^+$ . A number of answers included attempts at multistep proofs at least partially relevant to the induction step, but with little or no explanation, suggesting the material had been learned by rote and not fully understood.

Those employing the reductio strategy had less work to do. Weaker answers using this method were marred by a failure, having made the assumption that there was some sentence in  $\Gamma^+$  not satisfied by  $\mathcal{A}$ , to specify carefully enough the finite subset of  $\Gamma^+$  that could then be argued to be unsatisfiable.

Several answers to part (e) attempted to prove the result without appeal to compactness.

2. (*Expressive Adequacy*)

This question was relatively popular but part (c) was surprisingly poorly answered. Answers to part (a) were, in general, better than answers to parts (b)–(d), but, even here, some answers were marred by very imprecise language (for example, a confusion of truth functions and sentences that express them). Very few candidates succeeded in getting close to the full marks available for part (c)(ii). In answer to part (c)(i), more than one candidate failed to realise that there are four 1-place truth functions.

3. (*Duality*)

This question was reasonably well answered. Some of the better answers could have been improved by more careful presentation. This was most in evidence in answers to part (c)(i), where more effort in clearly setting out what was to be proved before ploughing through proofs would have been desirable. Candidates who risked losing marks by not listing the rows of their truth tables in (b)(i) in the canonical order should be grateful to the markers for their fastidiousness. More serious mistakes included, when taking the dual of a sentence, forgetting the need to take duals of subsentences, rather than only changing the main connective. Some candidates thought that, if  $\phi$  and  $\psi$  are jointly unsatisfiable, then both are false in any structure. Some candidates even got (a)(ii) wrong, with predictable knock-on consequences.

4. (*Interpolants*)

This question was generally well done. Part (c)(ii) would have been a more interesting question if additional restrictions had been put on  $\phi'$  and  $\psi'$  by requiring that  $\phi \vDash \phi' \vDash \chi \vDash \psi' \vDash \psi$ . As it was, candidates could obtain full marks via the rather easier route of specifying (and proving the relevant properties of)  $\phi'$  and  $\psi'$  such that  $\phi' \vDash \phi \vDash \chi \vDash \psi \vDash \psi'$ . Combinations involving both possibilities were also acceptable answers. In fact there were well argued examples of all four options. In contrast, some

attempts at (c)(ii) were very poor, involving unthinking attempts to prove the interpolation theorem via an inductive proof. Almost no one explicitly considered the vacuous case where  $\phi \vDash \psi$  has no interpolant.

The quality of answers to part (b) was also mixed. Several candidates made non-trivial logical mistakes. Some candidates moved from the true claim that if  $\mathcal{A}(X) = T$  then  $|\phi|_{\mathcal{A}} = |\phi[\top/X]|_{\mathcal{A}}$ , to the not necessarily true claim that  $\phi \vDash \phi[\top/X]$ . Others mistakenly thought that the fact that, for any structure  $\mathcal{A}$ , either  $|\phi|_{\mathcal{A}} = |\phi[\top/X]|_{\mathcal{A}}$  or  $|\phi|_{\mathcal{A}} = |\phi[\perp/X]|_{\mathcal{A}}$  meant that, for any structure  $\mathcal{A}$ ,  $|\phi|_{\mathcal{A}} = |(\phi[\top/X] \vee \phi[\perp/X])|_{\mathcal{A}}$ . (This mistake was even made by one candidate who had considered explicitly the case where  $\phi$  is sentence letter and had noted that in this case  $(\phi[\top/X] \vee \phi[\perp/X])$  is a tautology.) Several answers to (b)(iii) stuck rather rigidly to the argument format that had used in (b)(ii) (and had been learned from the lectures), rather than making the more natural move of considering the case in which  $\phi$  is False.

While part (a) was, in general, well done, there were a few very weak answers, including one involving the claim that if  $\phi \vDash \psi$  then,  $\phi$  and  $\psi$  have the same truth value in all structures.

5. (*Completeness*)

This was a relatively unpopular question, but it was done well by those who attempted it, getting the highest average mark. The fact that the question departed from bookwork rather more than most of the other meta-logical questions probably explains both features. Marks were lost in part (a), through failure to provide a rigorous enough proof. Some of those who did set out explicitly a proof by induction on complexity failed to realise that an inductive hypothesis about the truth of sentences in structures (rather than the satisfaction of formulas relative to assignments over structures) did not (without ugly contortions that were not provided) suffice to deal with the cases involving quantified formulas. In their answer to part (b)(ii), some candidates forgot to deal with the case of proofs of sequents with premises not in  $\mathcal{L}_U$ .

6. (*Argument Formalization; Definite Descriptions*)

Many answers to this relatively straightforward question were surprisingly weak. Some candidates lost marks in part (a) by taking too many short-cuts, providing formalisations that were not sufficiently faithful (e.g., all reference to naturalists/naturalism was sometimes excised). More than one candidate, in their answer to part (b), opted for the less natural option of formalizing the argument by giving the definite description wide scope, and yet claimed that, so formalised, the argument was invalid. One candidate provided a (correct) counter-example to their formalisation for part (b) that had the novel feature of ascribing to the empty set the property of coming into existence at some time or other.

7. (*Predicate structures; Formalization*)

Despite only testing Michaelmas term work, the lowest average marks were gained on this question and on question 8. This did not seem to reflect the inherent difficulty of the questions. It seemed more likely to be because candidates with a good grasp of the material covered in the whole course opted for other questions. One common error in answers to (a)(iv) was to mistake the property of not containing  $\neg$ ,  $\wedge$  or  $\vee$  for the property of containing some connective other than  $\neg$ ,  $\wedge$  or  $\vee$ . Answers to part (a) betrayed a widespread failure to note that not all elements of the domain were sentences. Curiously, several candidates guilty of this went on to correctly diagnose the problems with the suggested formalizations in (b). Several erroneously claimed no alternative formalization was possible. Others offered  $\exists y \exists z Ryzx$  as a formalization of “ $x$  is a sentence”, overlooking the case of sentence letters. As all  $\mathcal{L}_1$  contradictions are complex sentences containing connectives, this predicate could be used straightforwardly in a successful answer to (b)(i), but full marks were given only if the candidates explained why (which some did). One particularly ingenious answer attempted a formalization along the lines of: “ $x$  entails its own negation.” A common error in the formalizations offered for (b)(ii) was to overlook the case where  $x = y$ .

8. (*Formalization, Natural Deduction, and Counterexamples*)

This question obtained the lowest average. The passage to be translated for part (a) was reasonably involved, and this did make the task of providing a successful translation and natural deduction proof far from trivial. Nevertheless, one candidate succeeded in gaining 21 marks for this question. Many candidates committed the opposite error to that common in answers to question 6, making their translations unnecessarily complex. (For example, several predicate letters, rather than one, were used to translate certain predicates that occurred in the passage only in logically equivalent combinations.) One candidate used the natural deduction rules for double arrow from the pre-publication version of Halbach’s lecture notes. They were not penalised for this.

## Report on Introduction to Philosophy

**Marks:** 70+, 4 scripts; 60–69, 10 scripts; 50–59, 1 script.

All 15 candidates passed this paper. Six attempted 2 questions from the Frege section, and one attempted 3. Some questions from each section went unanswered – two from general philosophy, on perception and dualism, and two on Frege’s *Grundlagen* (‘laws of number’ and ‘criterion of identity’). Interestingly, there were no answers on perception from the 27 Physics and Philosophy candidates either, though the question itself seemed attractive enough (‘Must sense-data form part of any adequate account of perception?’). Perhaps tutors were anticipating its removal from the General Philosophy menu?

On the remaining questions, numbers of answers were as follows:



Question	1	2	3	4	5	7	9	11	12	13	14
Answers	7	6	5	2	6	11	7	5	2	6	3

The general standard of answers was good, though the general adoption of a standard reading list and the very recent teaching for the Frege component gave a rather standard ‘template’ feel to many even of the better answers, often with a sense of the material only being half-digested. The examiner could be much more confident that material had been understood than that candidates had anything of their own to say, and the occasional script that adopted an independent tone and approach was most refreshing (and encouraging for future work) even where it did not quite translate into marks in the 70s.

### Comments on more popular individual questions:

#### General Philosophy

1. *If the belief that the future will resemble the past isn't based on reason, ought we to regard successful predictions about the future as lucky?*

A tendency to repeat tutorial essays on induction, and to under-play the importance of an analysis of ‘lucky’. The best answers concentrated quickly on the conditions under which one might correctly describe a prediction as ‘lucky’; one line of argument asked whether, if inductive reasoning is based on an underlying truth that the future will resemble the past, predictions could possibly be described as lucky, whilst if there is no underlying truth there is no basis for probabilistic assessment and thus no basis for an ascription of luck at all.

2. *I believe  $P$ , and  $P$  is true. I am justified in believing  $P$ ; the belief was formed through an entirely reliable route. However, I do not know that I am thus justified. Does it follow that I don't know  $P$ ?*

Generally good accounts of internalist and externalist views of justification, and some reasonably nuanced discussions of why one might prefer one approach to the other (with ultimate verdicts divided equally). There was a disappointing tendency among weaker candidates to mention arguments rather than state and use them – ‘because of Gettier’s arguments’ is not a substitute for actually arguing, and there was rather too much ‘name dropping’ of this sort.

3. *Is scepticism based on a mistake?*

Two possible mistakes were high on candidate’s lists, both generally premised on the identification of scepticism with external-world scepticism, a restriction which was usually implicit but in a few cases boldly made. GE Moore’s attempt to defuse the external-world sceptic’s central argument was widely reported; Putnam’s ‘brain-in-vat’ paper also much discussed, though not always well understood. Similar views seem to have been held by a philosopher called ‘Puttenham’, possibly Putnam’s stunt double. Descartes was not the first sceptic, in spite of an explicit claim to the contrary; but if Descartes is being read, the restriction of scepticism to external-world sciep-

ticism is surprising.

5. *Considerations of functional continuity allow us to call an object the same car even when many parts have been changed; parallel considerations of an organic kind suffice to determine that someone is the same human being. Are there any good reasons not to think of 'same person' in an analogous way?*

Some of the better answers expended time and thought on the notion of 'function' in relation to personhood; weaker ones repeated material from tutorial essays without much thought to its relevance. Some of it was relevant, but the examiner was left to do the sifting, when by rights that is a task for the candidate.

7. *Is there more than one sense of 'could not have done otherwise'? Does your answer have any bearing on discussions of human free action?*

Typical answers distinguished a 'deterministic' denial of alternative possibilities from a claim based upon external constraint, with better answers offering an analysis of constraint and weaker ones leaving this task to one side. There was disappointingly little reflection on character as something for which one might be held responsible, and thus a dimension of the subject relating to 'I could not have done otherwise' was missing, though one candidate brought in Luther ('Here I stand; I can do no other') as a relevant case. The reading list seemed to have exercised its own measure of constraint on the ability of candidates to imagine a broader range of considerations opened up by the question.

## Frege

9. *'[Hume's] opinion that numerical equality or identity must be defined in terms of one-one correlation... raises at once certain logical doubts and difficulties which ought not to be passed over without examination.' [FREGE] What are these doubts and difficulties? May they be met?*

Good accounts of the role of this passage in the argument of Grundlagen, and of the way in which 'doubts and difficulties' feed into subsequent arguments. The extent to which doubts and difficulties offered were always logical doubts and difficulties was not generally raised; the 'Julius Caesar' problem, which many candidates saw as the most obvious focus for their answers, may or may not be logical (which suggests quite a good question). Better candidates saw the extent to which some difficulties at least are the product of others of Frege's commitments too, with the context principle looming large.

11. *Can Mill's account of arithmetic be defended from Frege's criticisms?*

Generally good accounts both of Mill's and Frege's positions, and an account of an attempt to rescue Mill, which met with varying degrees of favour from candidates, though not always with much evidence to back up their agreement/disagreement. This was a question where most answers

seemed compromised a little by lack of distance from the material, though the material was clearly in place.

13. *Why does Frege think of numbers as objects? Are they?*

A rather ‘set-piece’ question which may well have been a tutorial essay for most candidates, and produced a gratifyingly broad range of responses. There was little enthusiasm for grammatical arguments based on the noun-role of number-words, but on the whole candidates’ intuitions ultimately ran with Frege’s here, though one answer argued explicitly for an adjectivalist account, and did so very well.

## **E. RESERVED BUSINESS**

None.

## **F. NAMES OF MODERATORS**

- Dr Oliver Pooley (chair for Honour Moderations)
- Dr Dave Leal
- Dr Robin Knight
- Dr Neil Laws