4th Brooke Benjamin Lecture

The Enigma of The Transition to Turbulence in a Pipe

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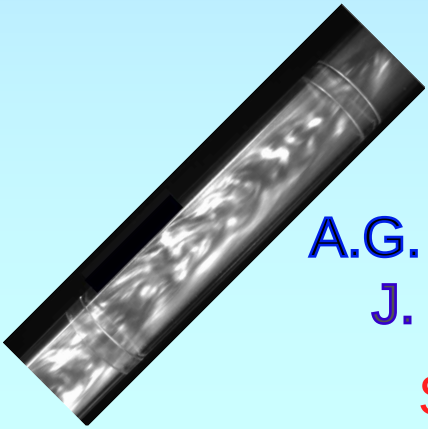
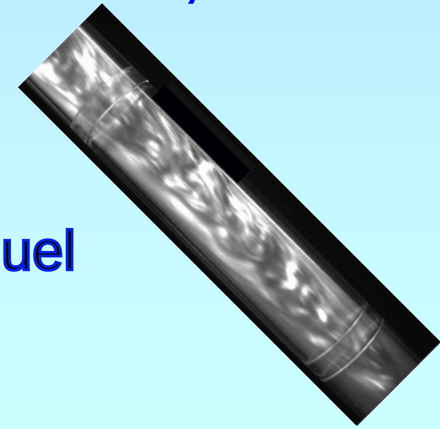




Joint work with:

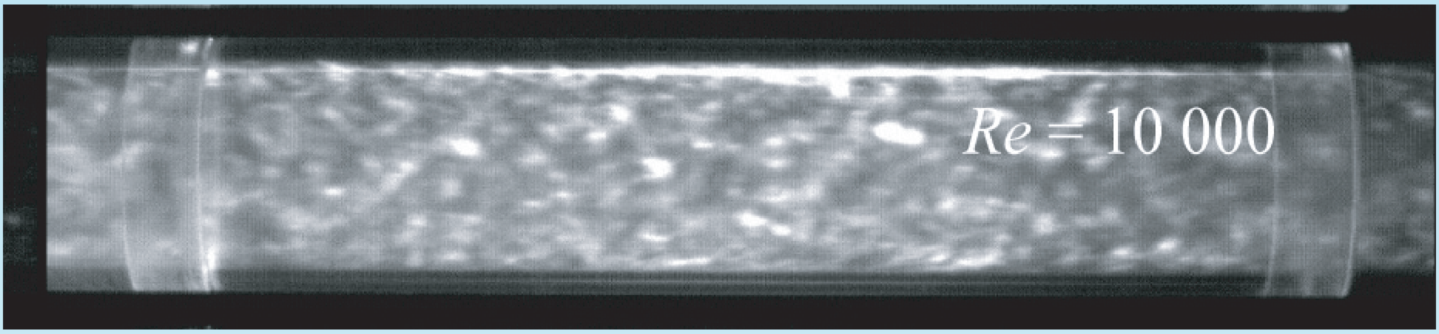
A.G. Darbyshire, B. Hof, A. Juel
J. Peixhino & Y Tasaka

Supported by EPSRC

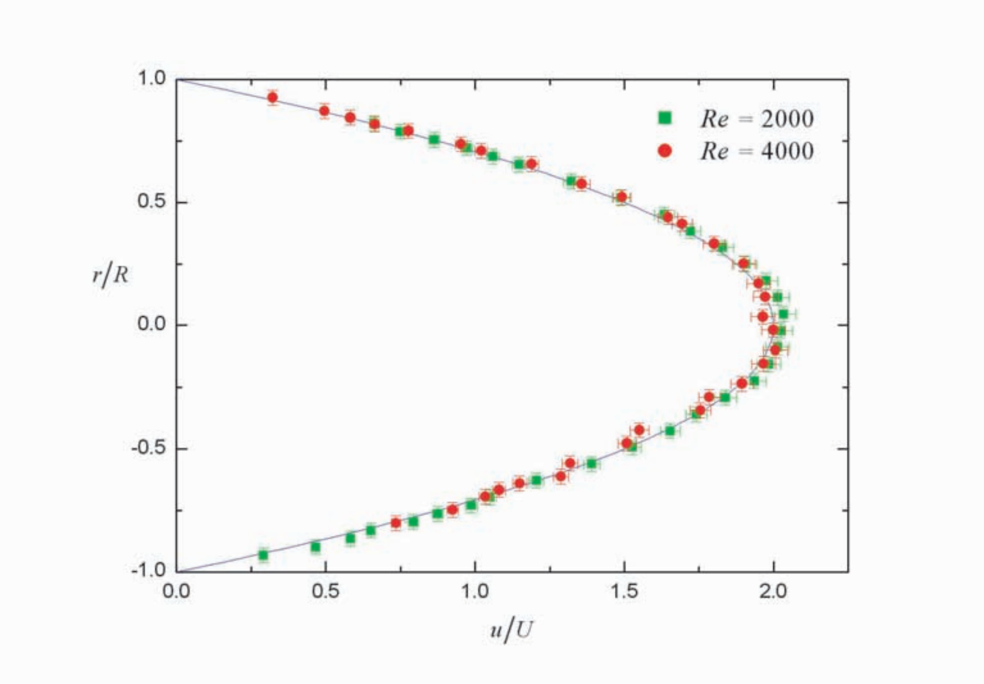
The Enigma

All theory suggests that Poiseuille flow is linearly stable i.e. **laminar** flow should be the norm.

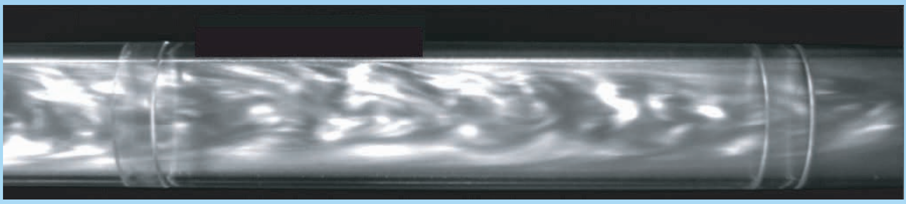
In practice most pipe flows are **turbulent** even at modest flow rates.



Transition between **Poiseuille** flow



and **Turbulence**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is a **Finite** Amplitude Instability

It is a challenging scientific problem, unresolved in the ~ 125 years since **Reynolds**' work.

Practical interest

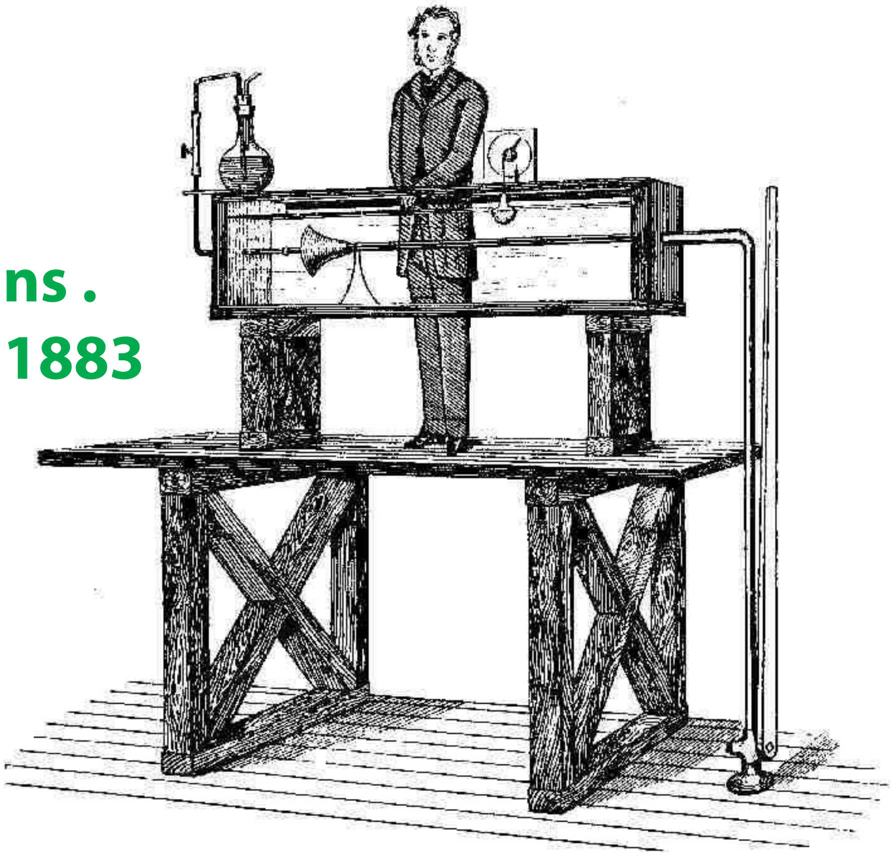
Most pipe flows are turbulent in practice.

If flow could be kept **laminar**
–> tremendous energy saving.

My Approach

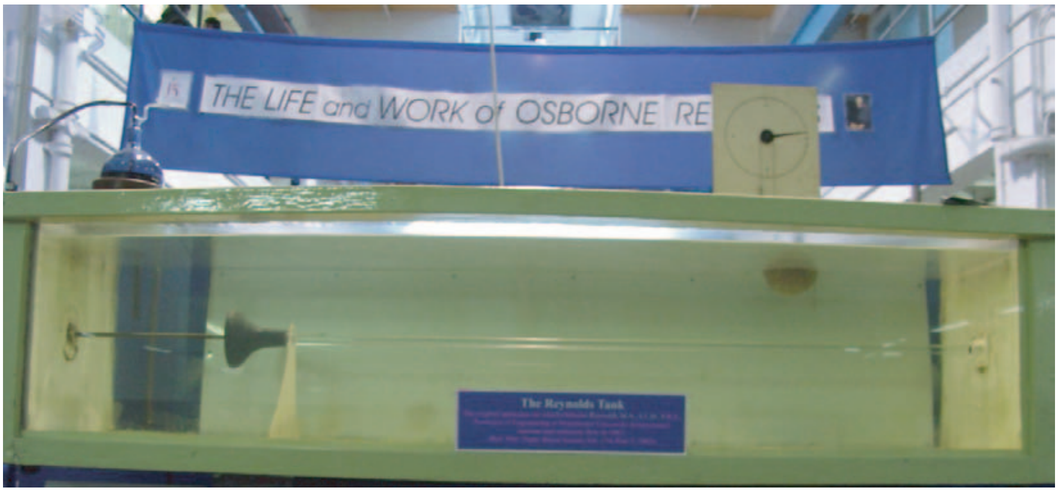
Use experimental physics allied with modern theory to make progress in understanding.

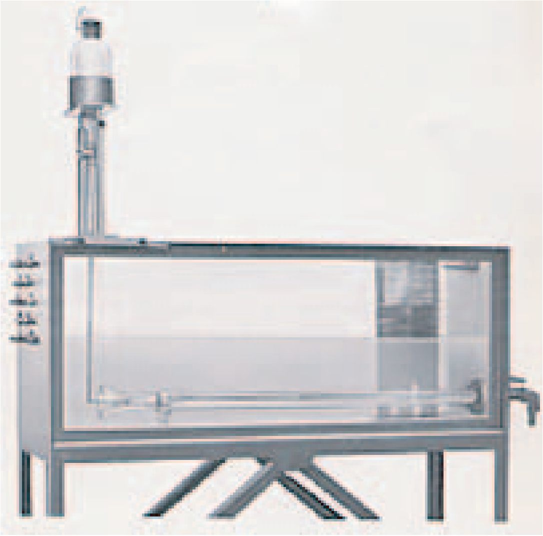
Reynolds' Experiment



Phil. Trans .
Roy. Soc. 1883

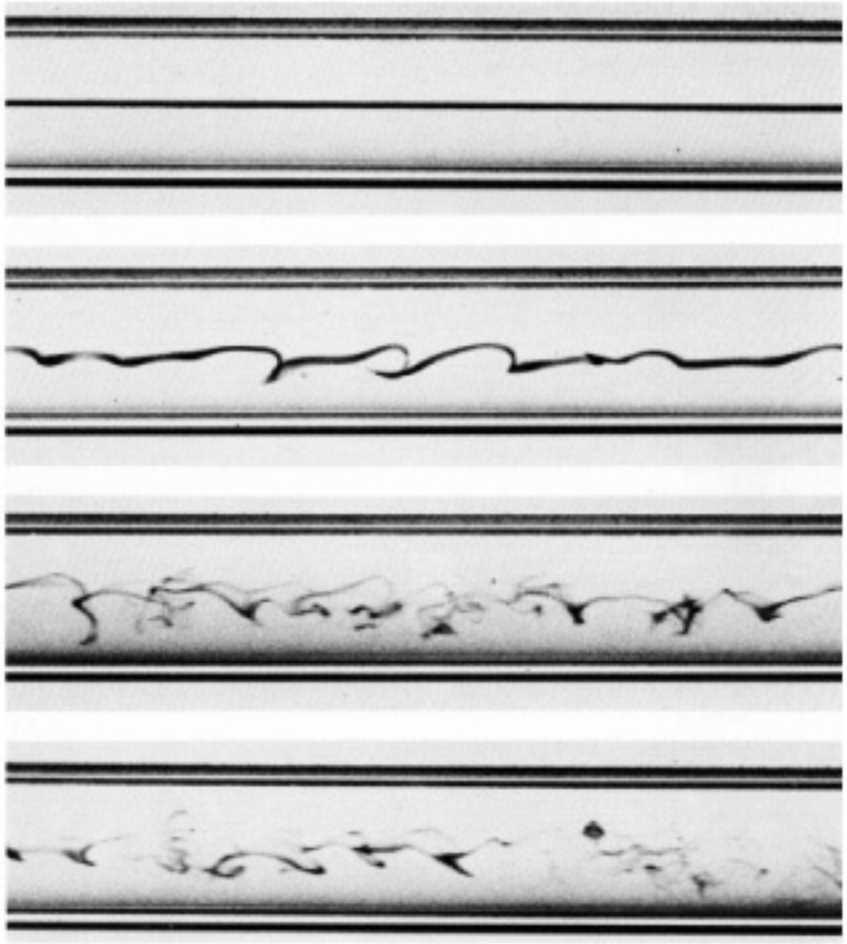
Manchester Engineering Dept.





Maruto Testing Company
Tokyo.

Reynolds' Experiment



Van Dyke (1982) *An album of fluid motion, Parabolic press*

- Reynolds found : "turbulence" above

$$Re_c = 2000$$

- In careful experiments laminar flow up to

$$Re_c = 13000$$

- $2000 < Re < 2700$ "flashes" "puffs"

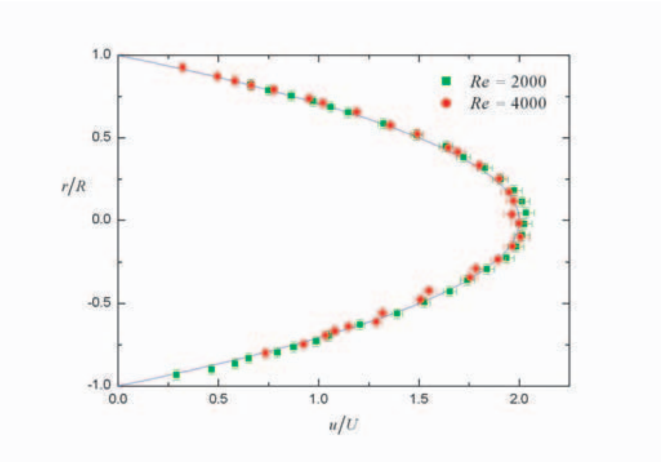
$Re > 3500$ "slugs"

Wygnanski and Champagne, *J. Fluid Mech.*, 59, 281-351

`Turbulent' Puffs

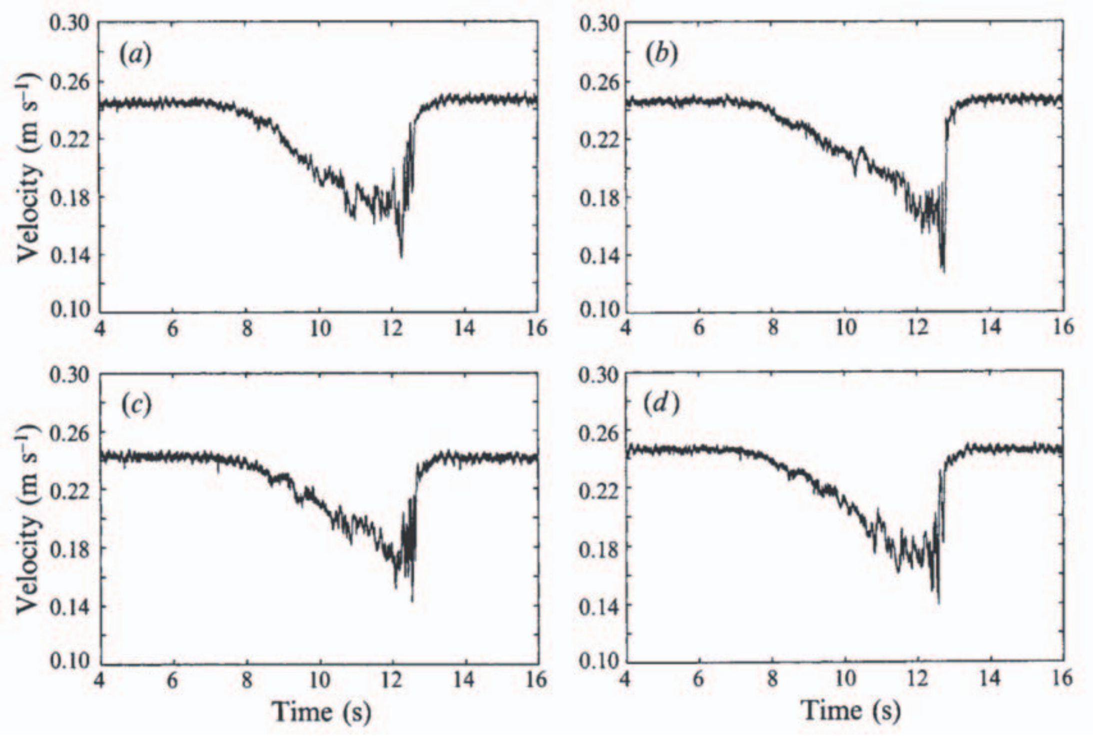
Exist in Re range ~1800 to 3000

Puffs travel at ~0.9u

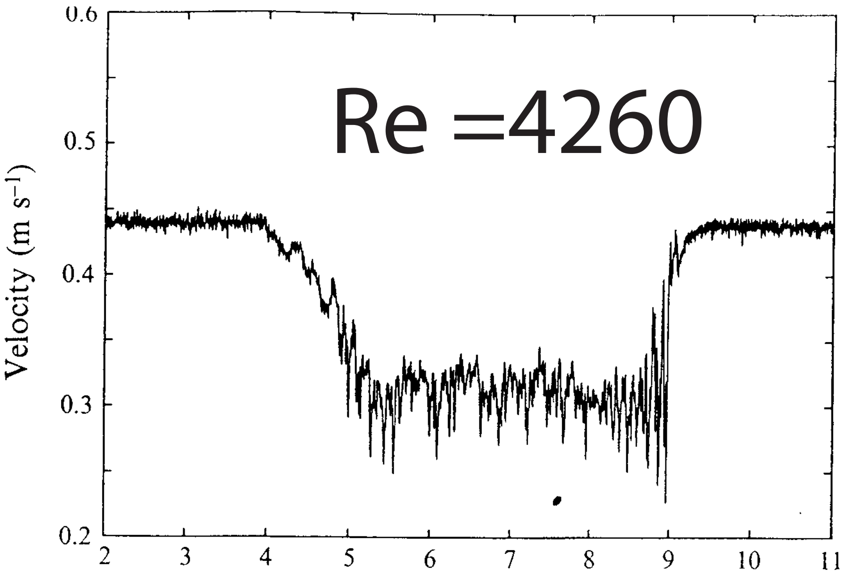


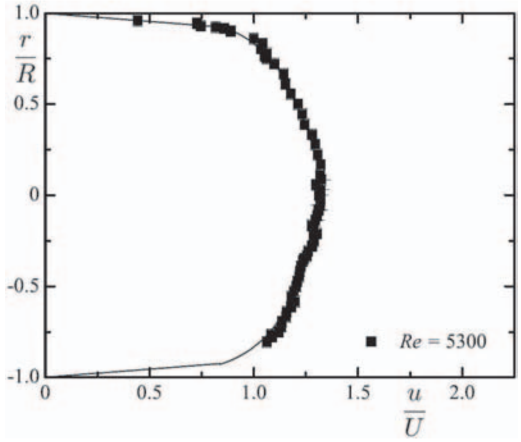
Initial Profiles

Nonuniqueness in puffs at Re=2269



Slugs (Re =4,000)





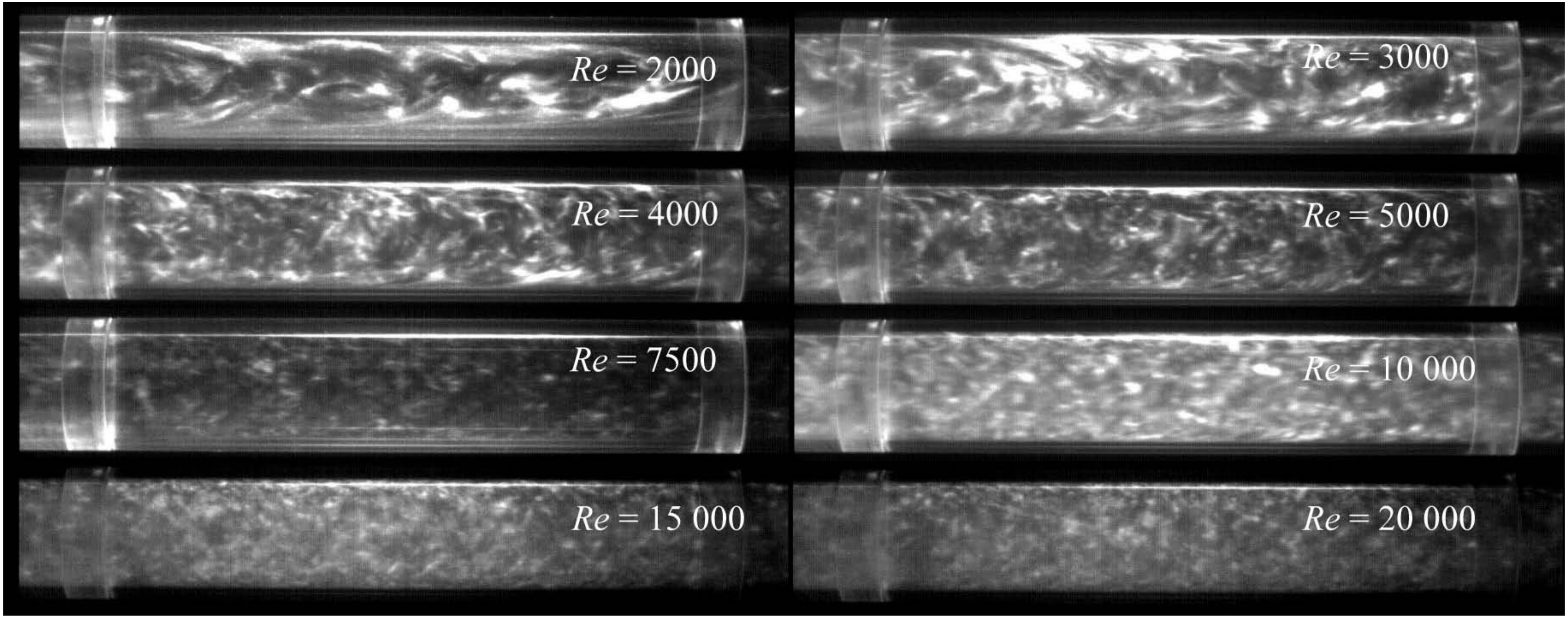
Mean Profile

Axial velocity vs time (centre)

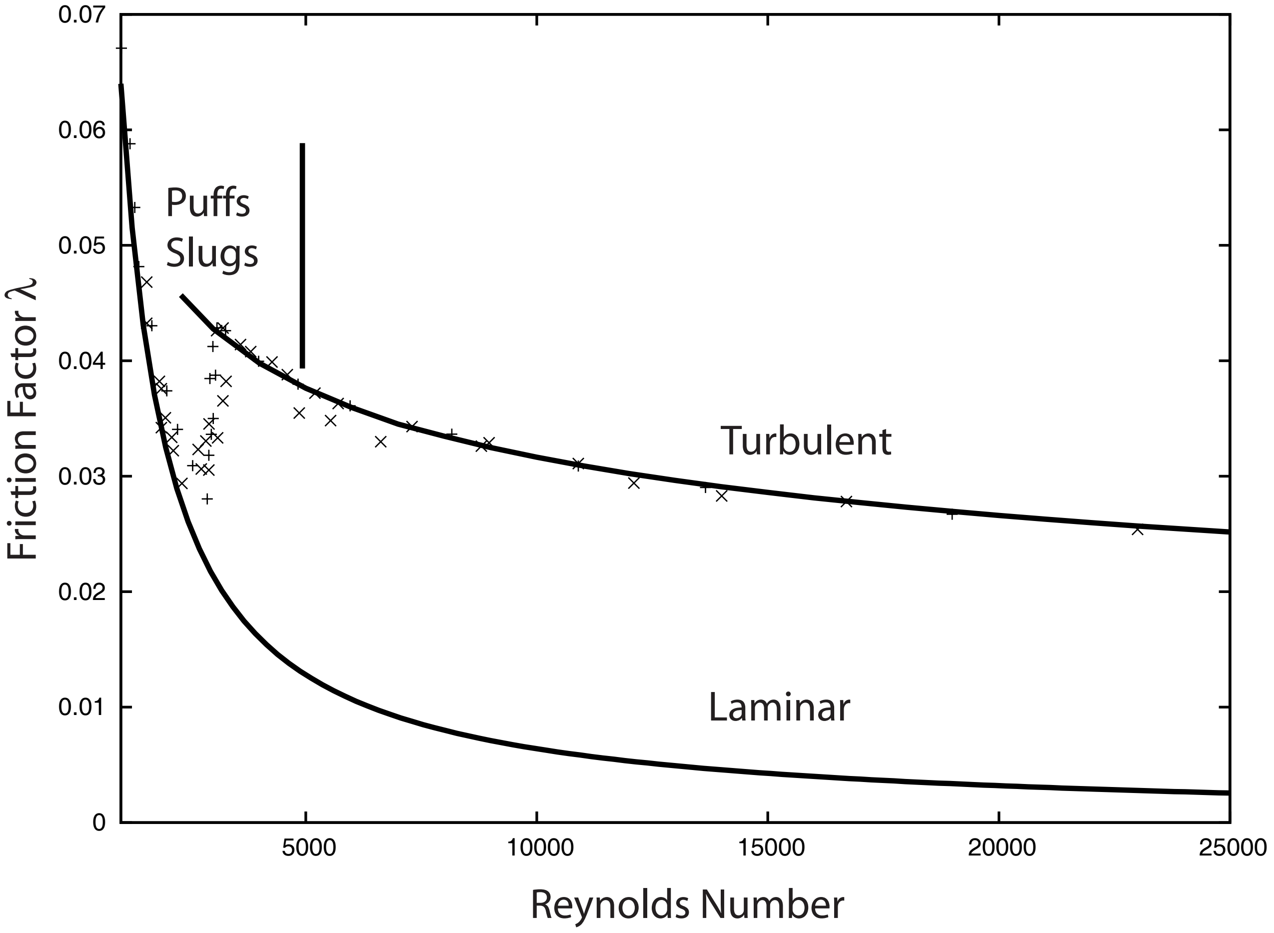
Puffs -> slugs by 'puff splitting' Nishi et al JFM (2008)

Re = 10,000

Snapshots of 'turbulent' pipe flow over a range of Re.



Friction Factor for Pipe Flow



$$\lambda = \frac{-d}{\frac{\rho}{2}\,\bar{u}^2}\,\frac{dp}{dx}$$

Laminar $\lambda = Re/64$ Turbulent λ Blasius Law

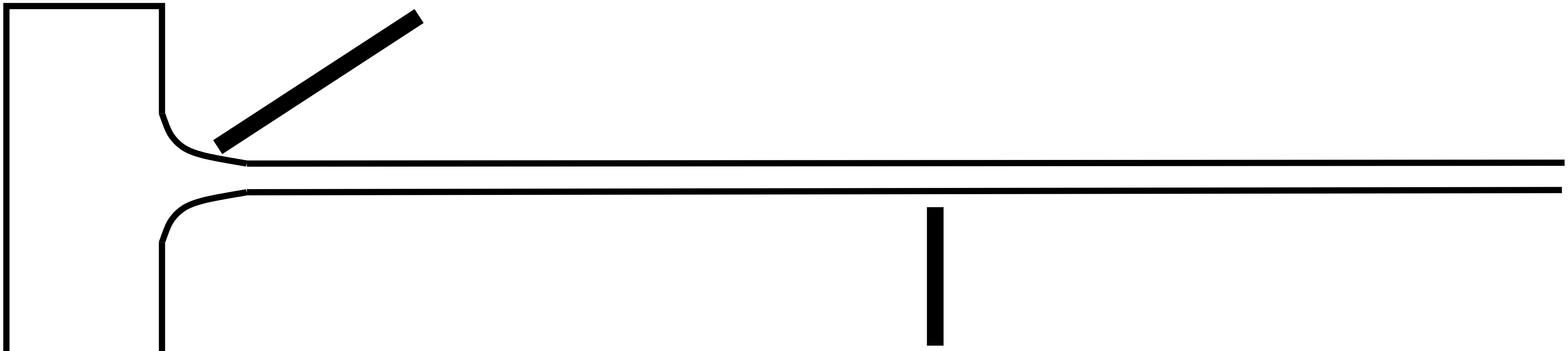
Experimental Data: Nikuradse (NASA 1966), Swanson et al (JFM 2002).

Overview of Pipe Flow Transition

$Re = \frac{u\,d}{\nu}$

Entrance flow

Practical interest, sensitivity to disturbances, focus of most experiments
linear instability Re ~10,000 (Tatsumi,da Silva & Moss,Duck),
in practice disturbed inlet produces transition at Re ~2,000.



Fully developed flow

Re/30 diameters to develop Poiseuille flow,
linear stability, --> finite amplitude transition.
(whole pipe important in an experiment)

Fully developed flow is a cleaner problem for experiment and theory.

We focus on this case.

Summary of Pipe Flow Facts

Fully developed circular pipe flow (Poiseuille flow) is linearly stable.

Single parameter: **$Re = Ud/\nu$.**

When Re >2000 most pipe flows are turbulent in practice.

BUT

Laminar Poiseuille flow can be obtained at Re~100,000 (Pfenniger 1961).

Suggests finite amplitude threshold required for transition.

Also global stability for Re < 2000.

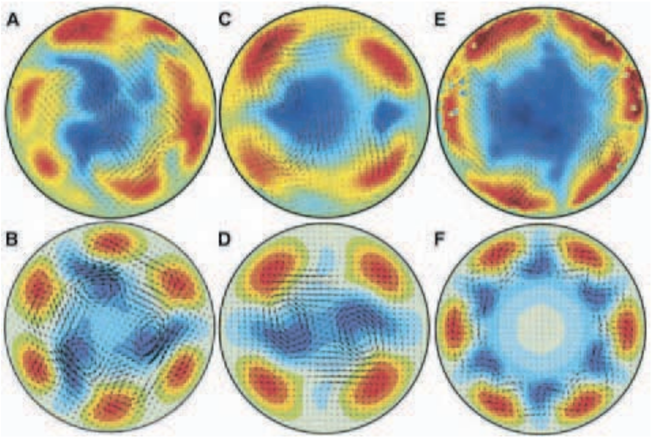
Modern Theoretical Developments

Kerswell Nonlinearity 18 (2005)

Finite Amplitude Solutions : travelling waves.

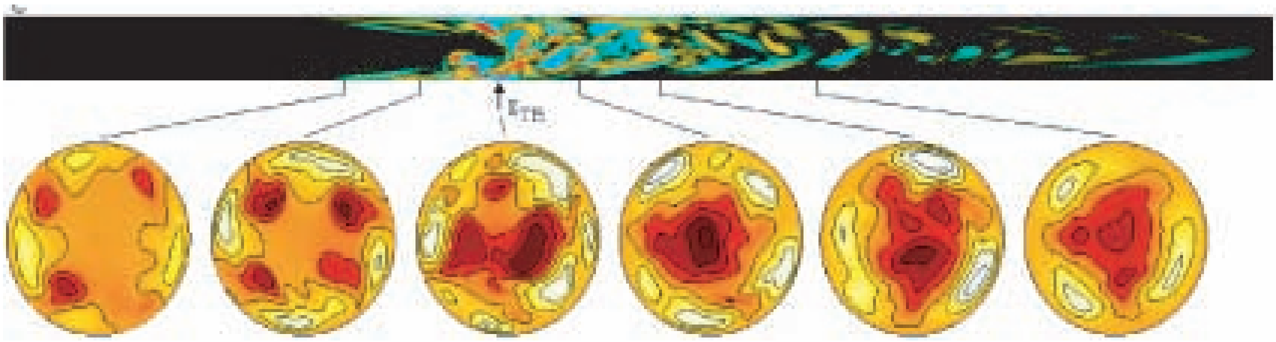
Faisst & Eckhardt PRL (2003)
Wedin & Kerswell JFM (2004)

Poiseuille flow state: other travelling wave states NOT connected to it. (All unstable)



Hof et al Science (2004)

Willis & Kerswell PRL (2008)
TW's relevant Re $\lesssim$ 2,800



Turbulent 'puff' : flow wanders between unstable travelling wave states.

Transient growth of perturbations (linear).

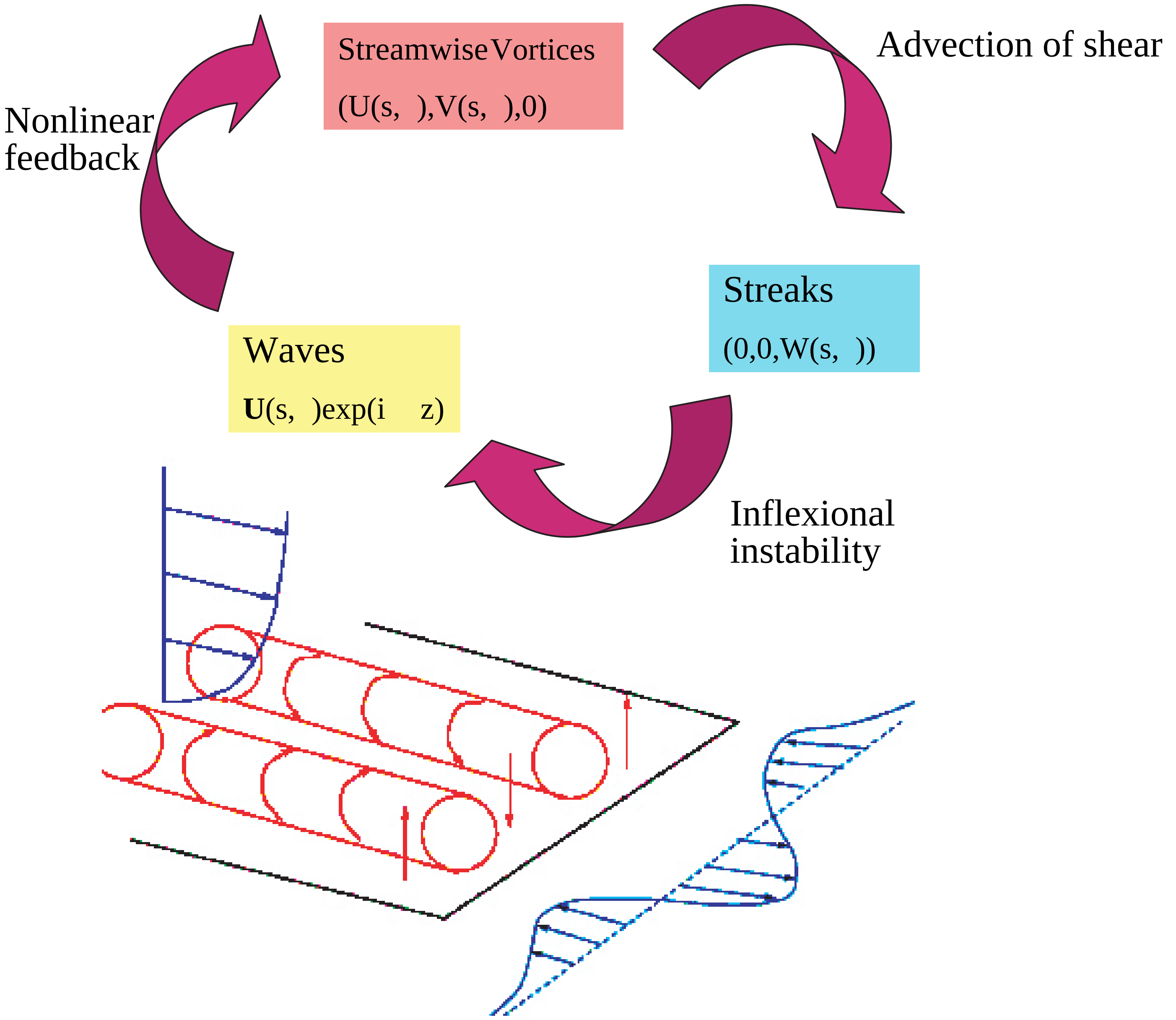
Benney, Stuart (1960's)

Brosa, Butler, Farrell, Trefethen, Schmidt,
Henningson, Chapman (1980-2002)

Infinitessimal perturbation grows algebraically ->
finite amplitude -> nonlinear effects take over.

Self Sustaining Process

(Waleffe 1997,1998,2001,2003) Hall (2010)



Streamwise Vortices

(U(s,),V(s,),0)

Advection of shear

Streaks

(0,0,W(s,))

Inflexional instability

Waves

U(s,)exp(i   z)

Nonlinear feedback

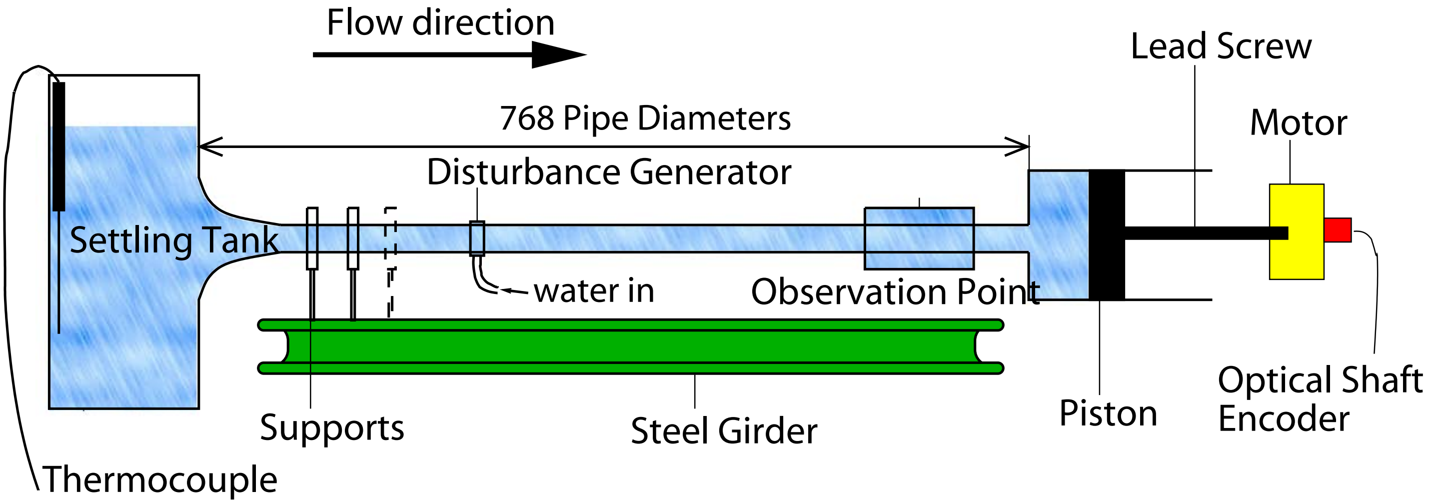
Equilibrium “puff”
Comparison Experiment Simulations
Re = 1800

A. P. Willis and R. R. Kerswell, Bristol

J. Peixinho and T. Mullin, Manchester

Our Experiment

Constant Mass Flux Pipe i.e. Re fixed.



In most other experiments, pressure gradient drives the flow. On transition, flow rate will drop, hence Re will vary.

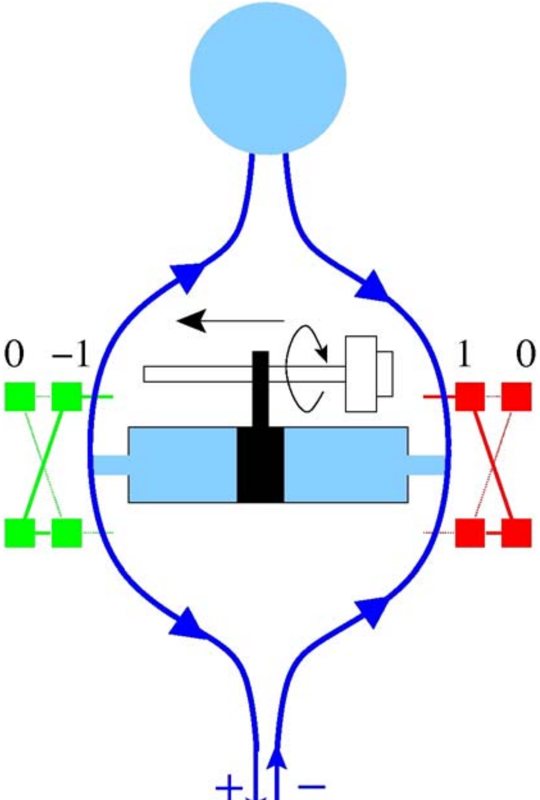
The long pipe

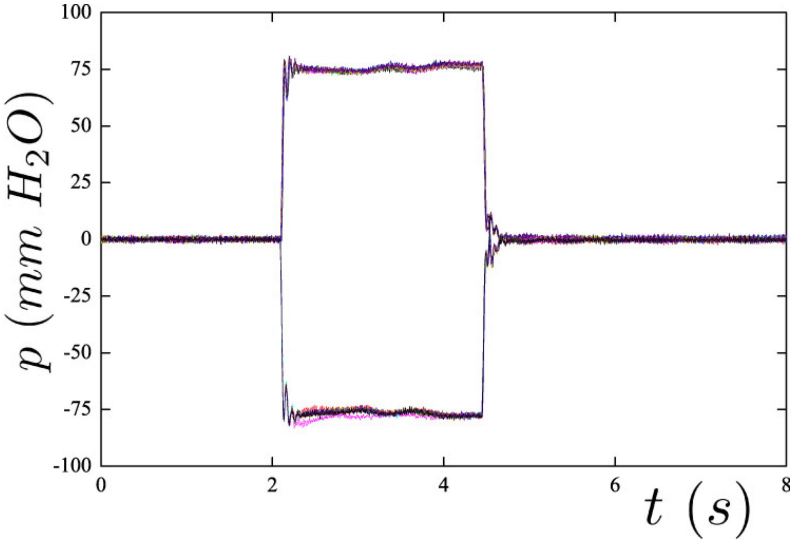
New large scale experimental facility:

- 15.5 m or 768 pipe diameters long,
- temperature control,
- new perturbation, where a spread and amplitude of perturbation are decoupled.

* Study of perturbed Hagen-Poiseuille flow ($> 95\%$) for up to $Re \simeq 20000$.

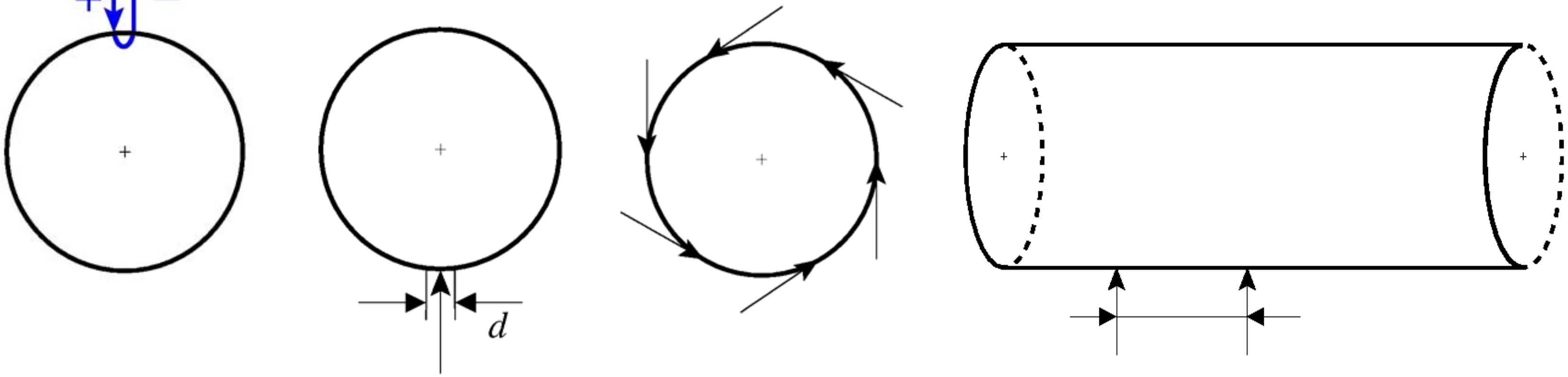
Perturbation Mechanism



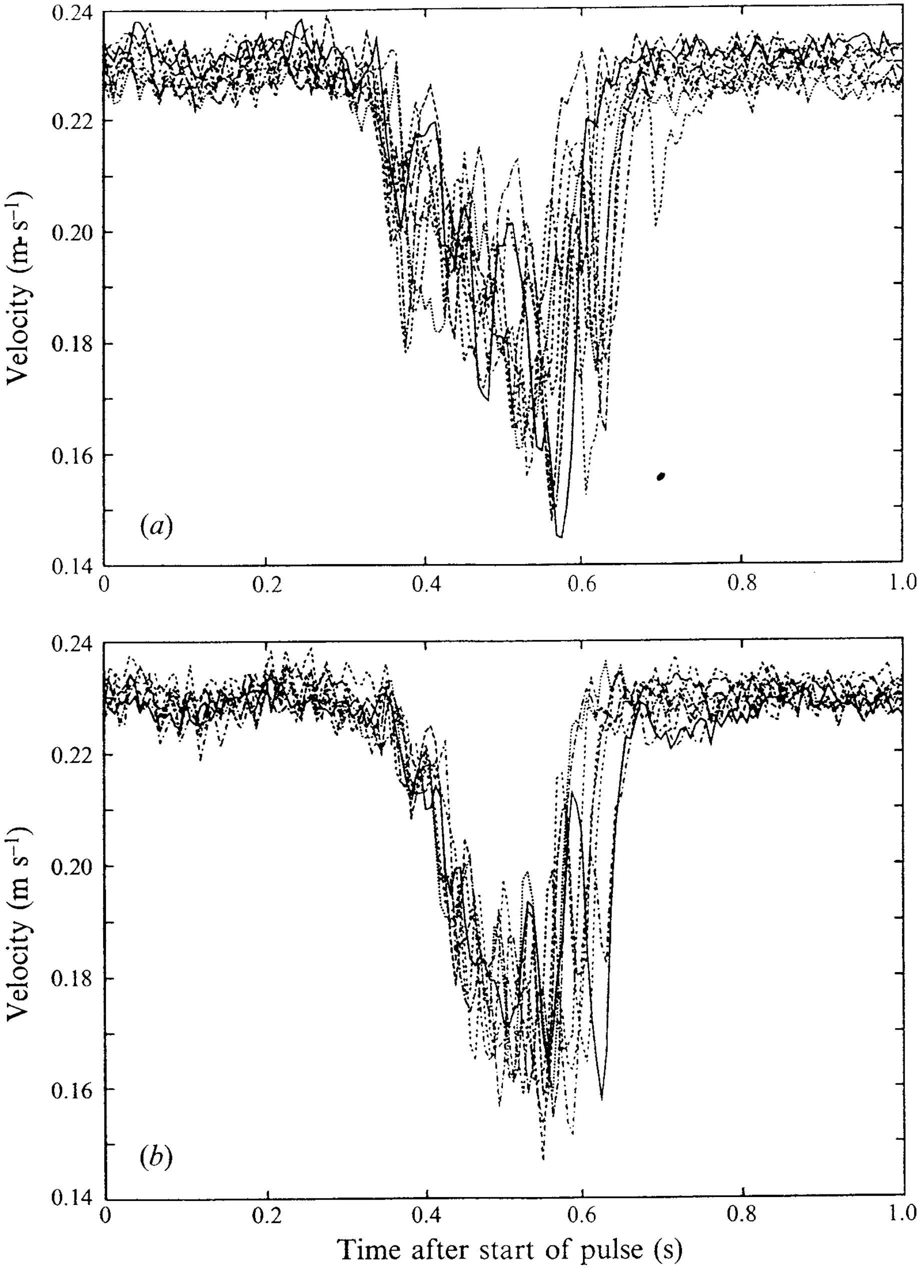


$$A = \frac{\Phi_{inj}}{\Phi_{pipe}}$$

$$L^* = \frac{Ut}{D}$$



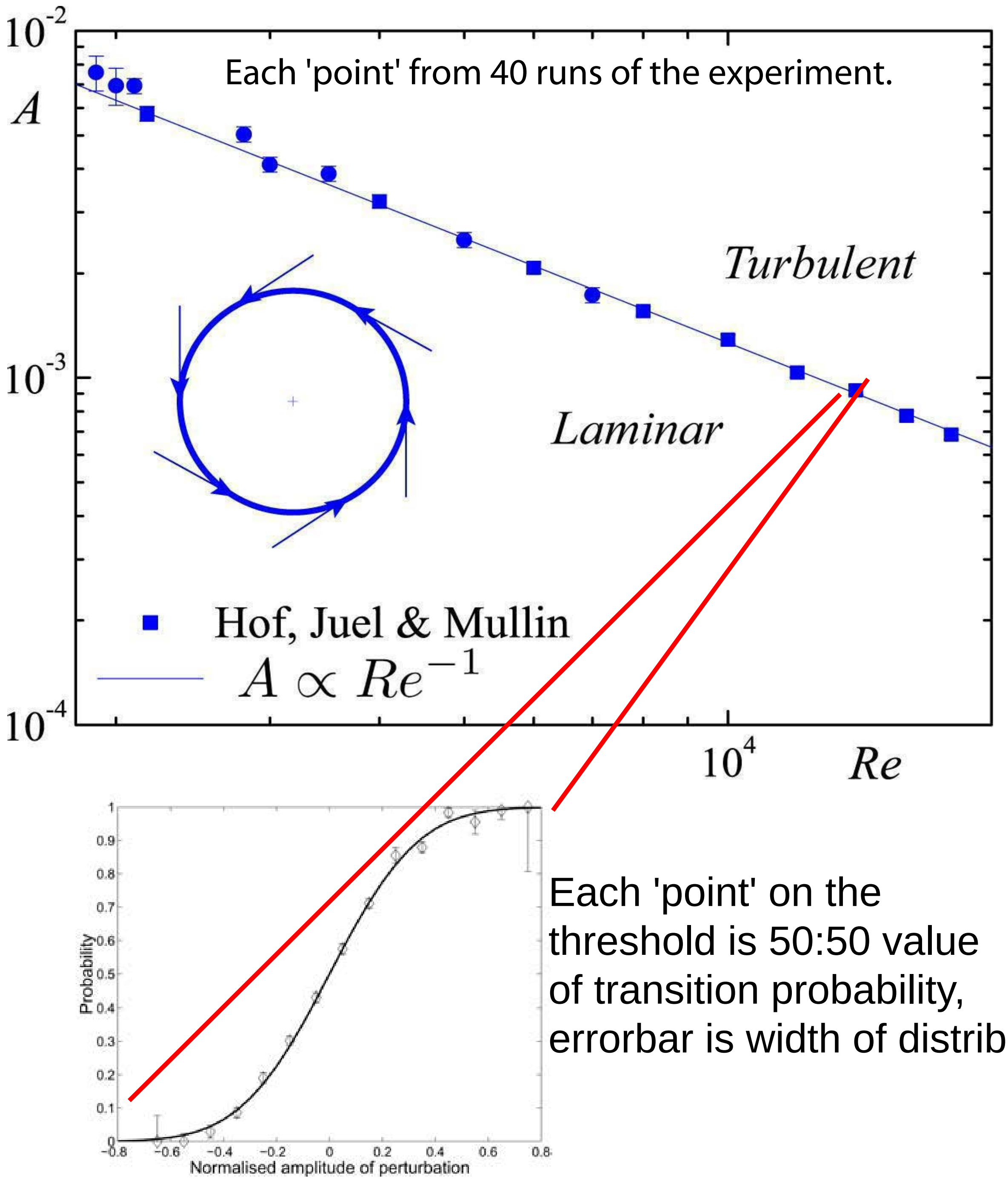
Impulsive Disturbance Applied



. Velocity time traces measured $2D_P$ from the single-jet inlet. (*a*) Disturbances that cause transition. (*b*) Disturbances that decay downstream.

Stability Threshold

6 jet disturbance



Each 'point' from 40 runs of the experiment.

Each 'point' on the threshold is 50:50 value of transition probability, errorbar is width of distribution.

Details of Transition Difficult to Discern

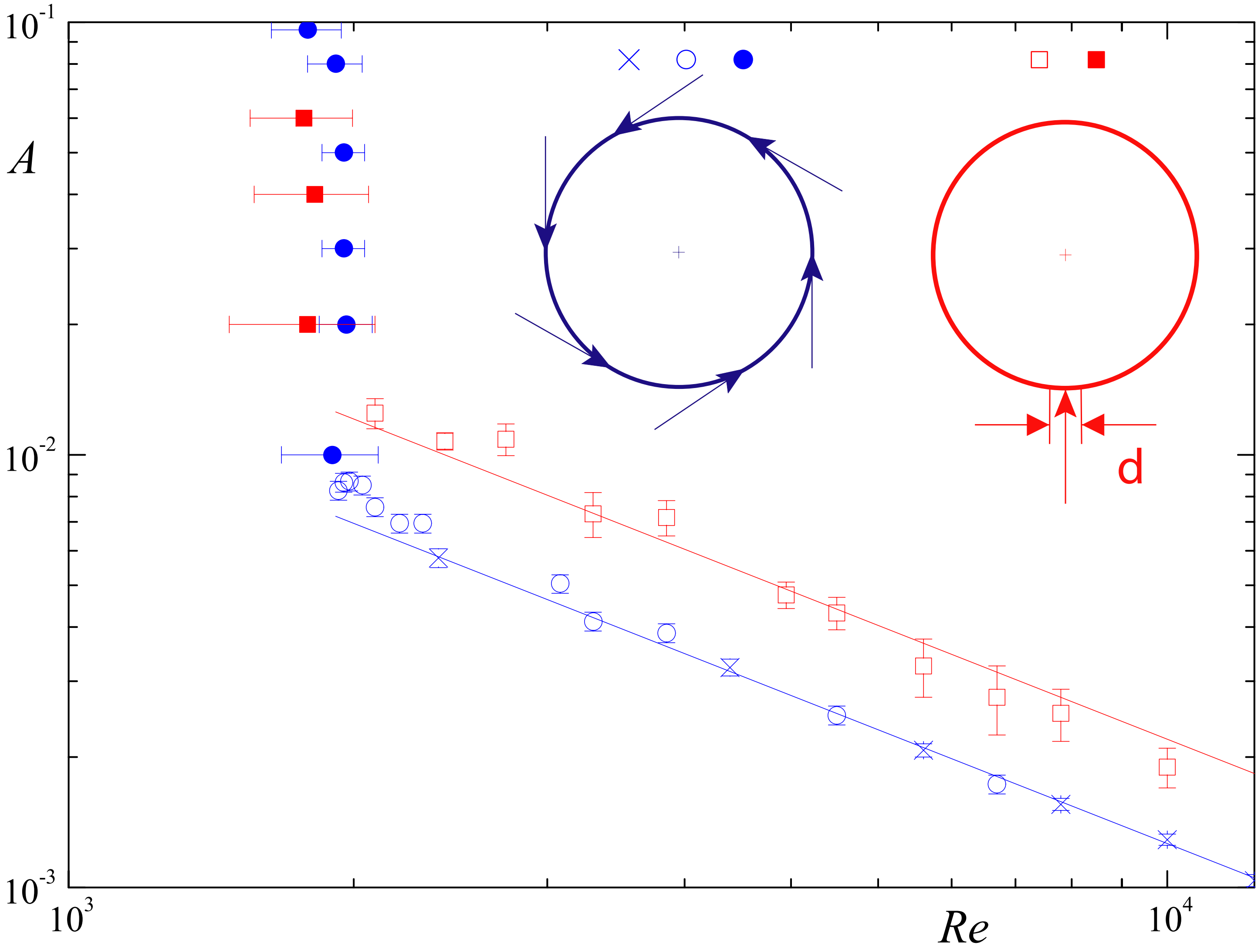
Re = 2,000

Disturbance decays

Transition downstream

Finite Amplitude Stability Curves

Single and six jets

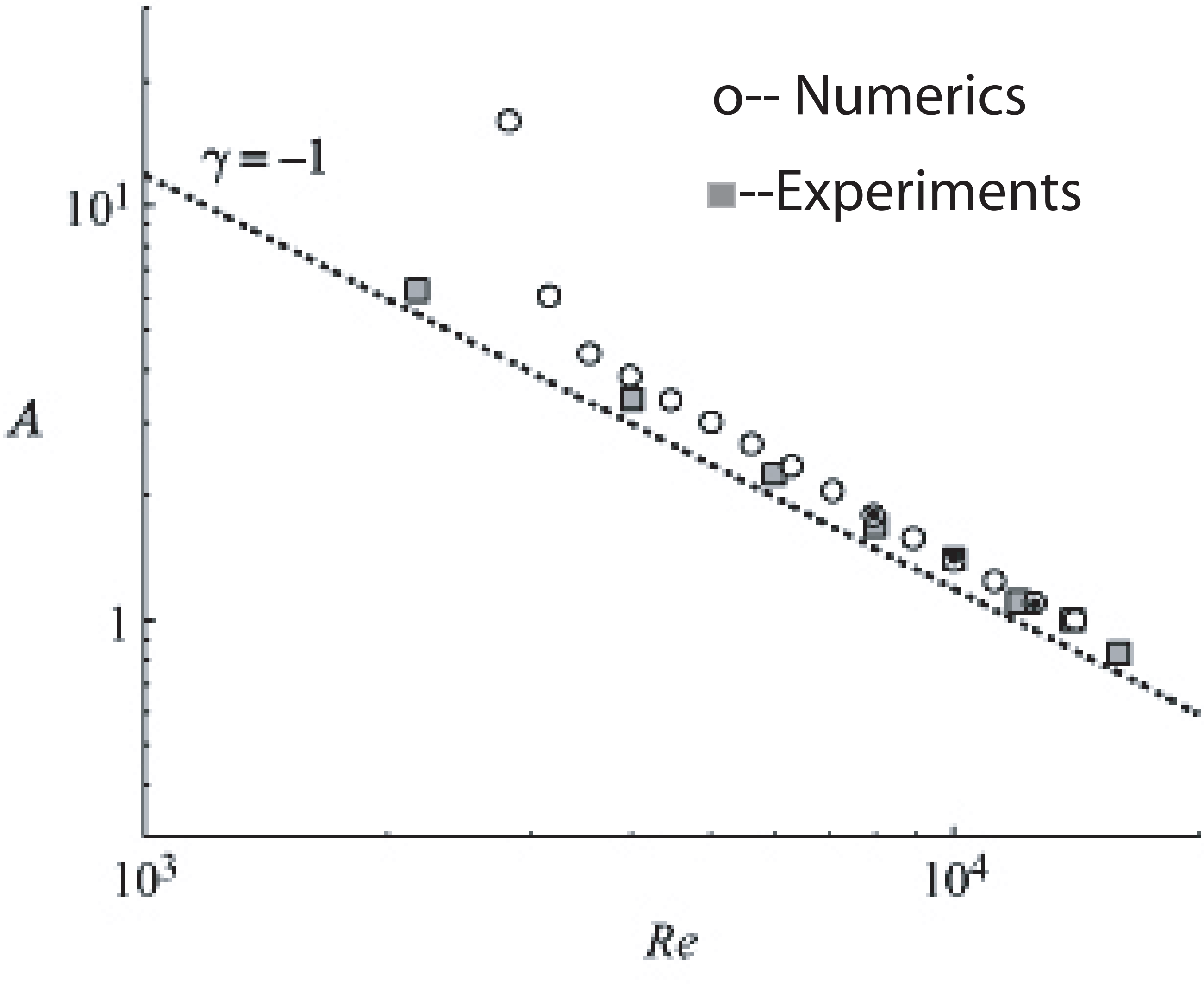


Both give: $A \ \alpha \ Re^{-1}$

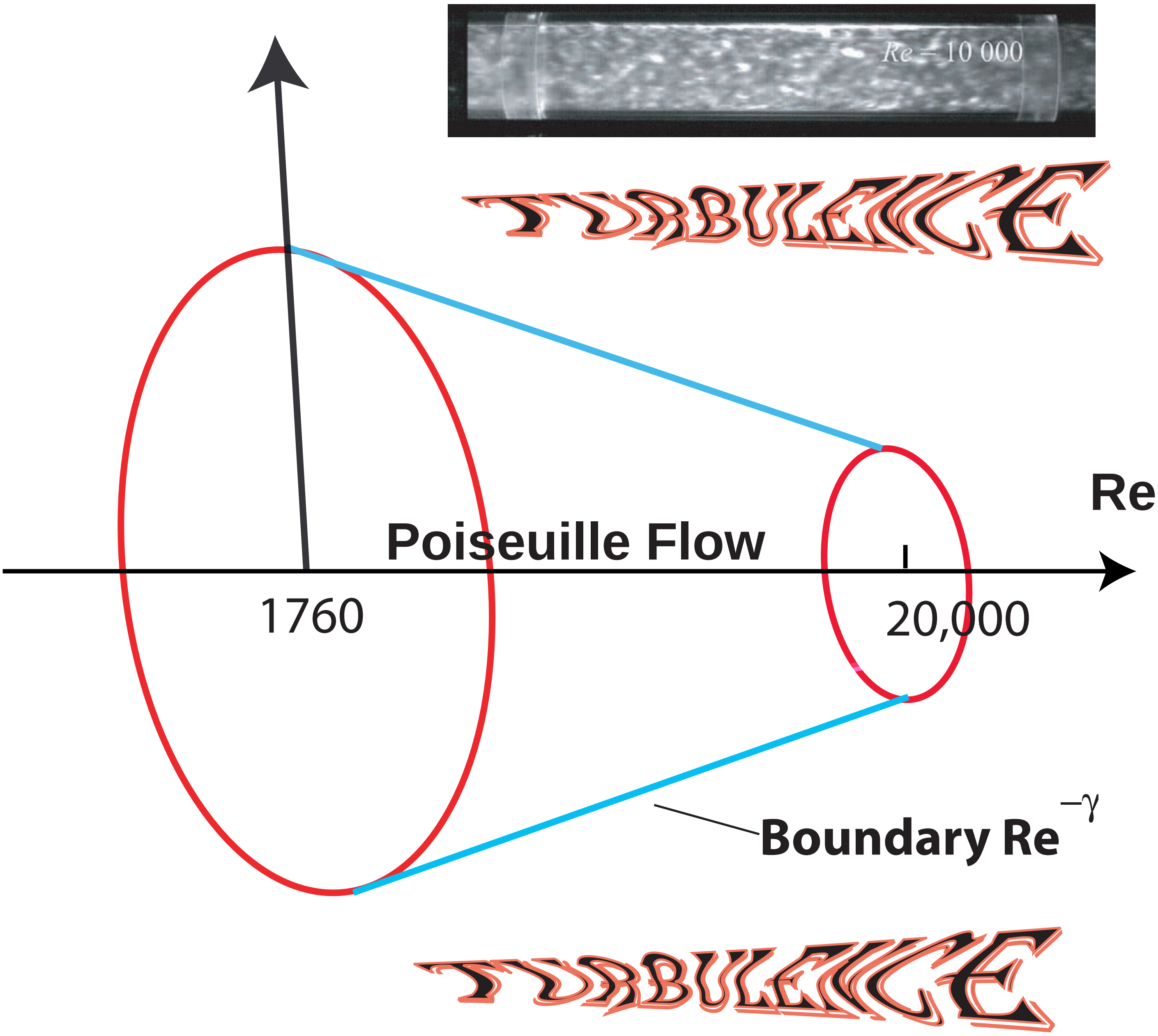
Robust simple scaling

Transition is abrupt in both cases

Numerical Calculation of Threshold by Mellibovsky & Meseguer PRSA (2009)



NB: 'Amplitudes' normalised



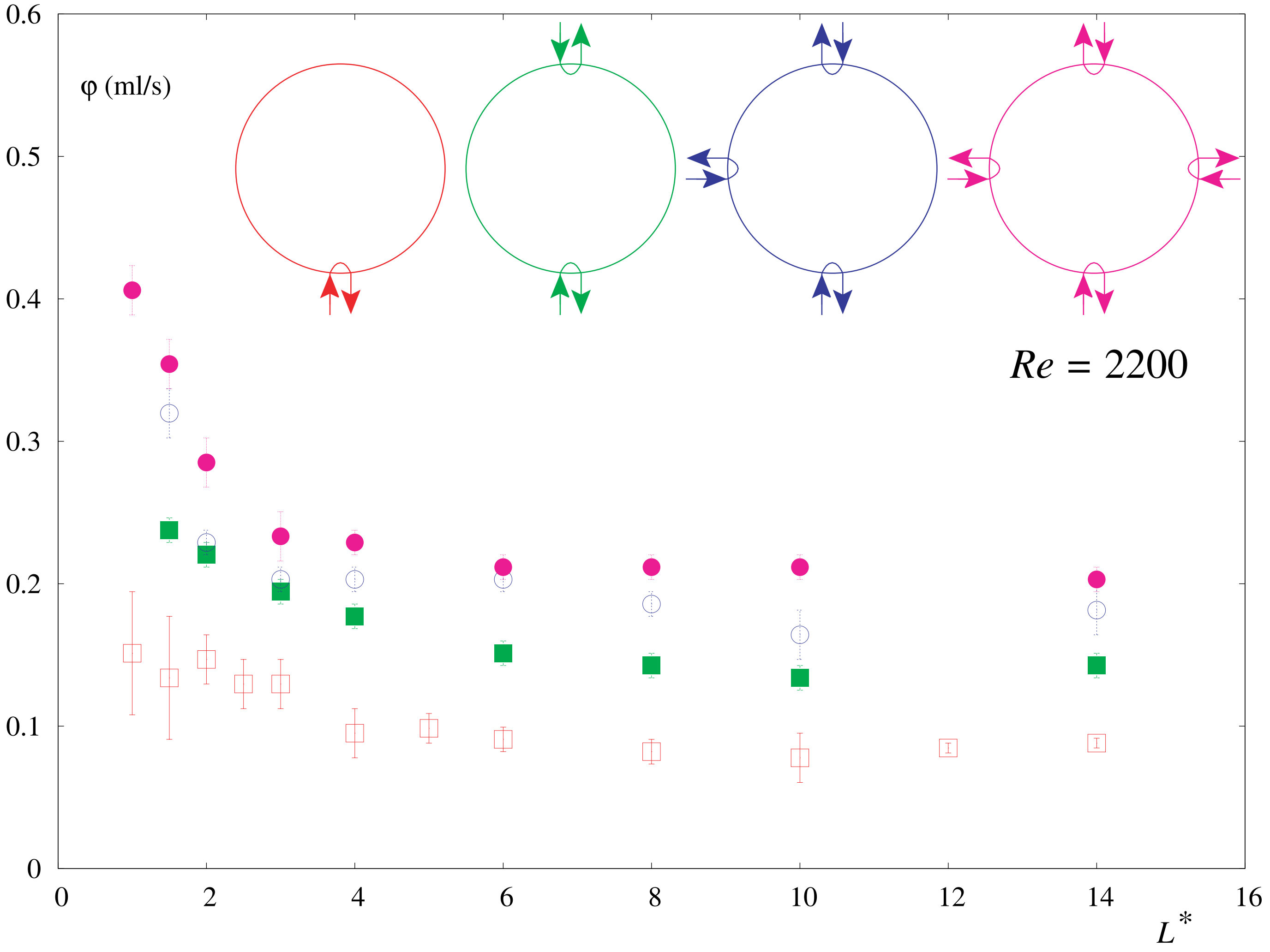
Schematic based on experimental estimates of boundary between laminar & turbulent flow.

γ (1 τo 1.4)

Boundary is **Probabilistic**

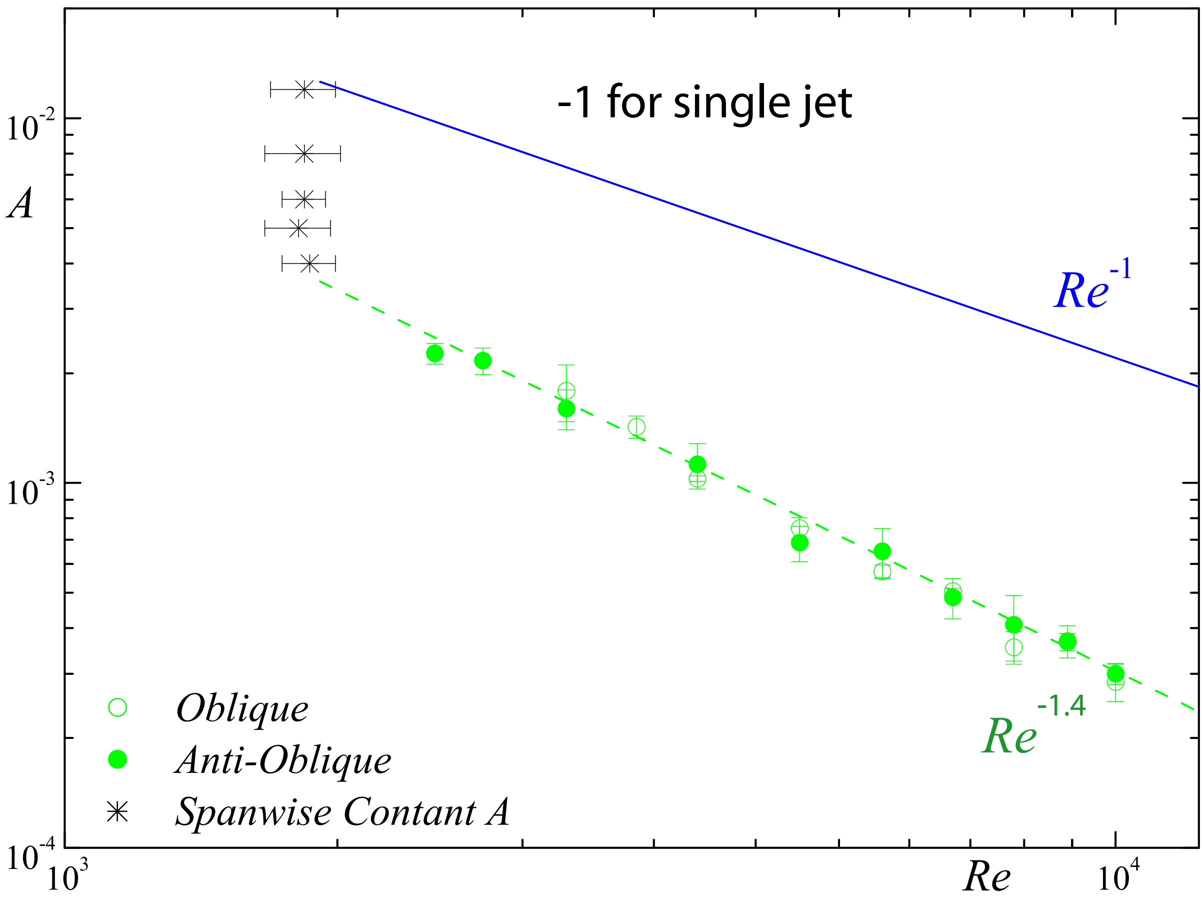
Can we see the details of the edge of turbulence?

Four disturbances with different symmetries.



Note:simplest is most efficient in giving transition.

Threshold Curves with Two Different Perturbations.



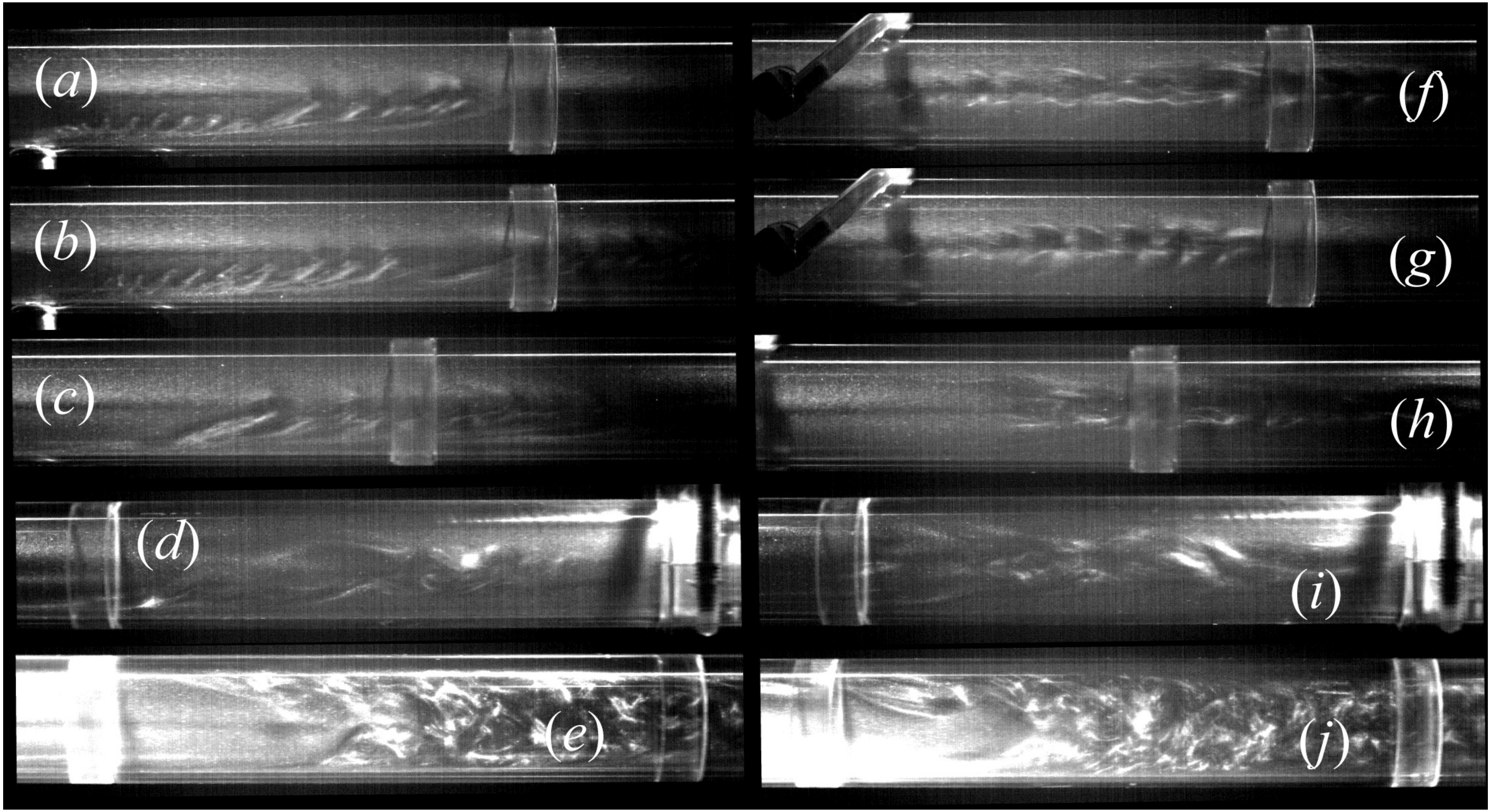
Localised push-pull slope -1.4

Chapman(2002) predicts -1.5 for plane Poiseuille flow

Note order of magnitude reduction in amplitude.

OBLIQUE PUSH-PULL DISTURBANCE

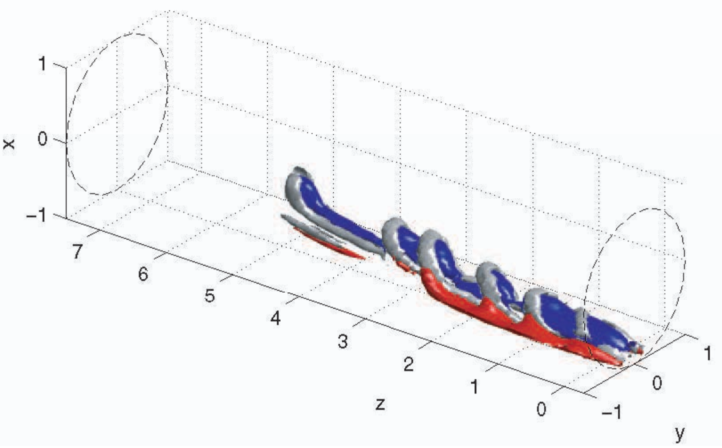
Side View Bottom View

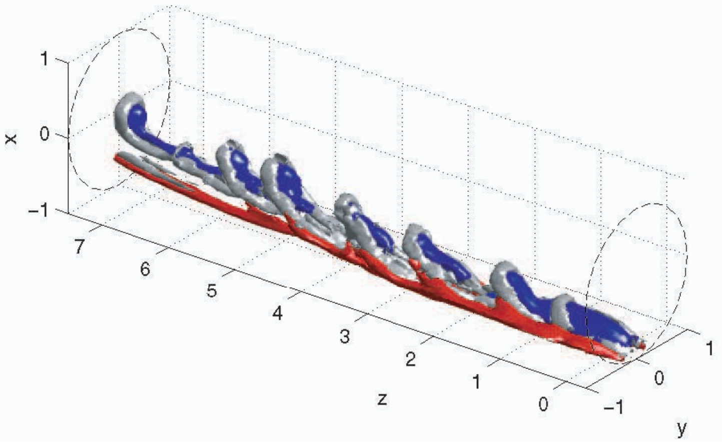


$Re = 3000$

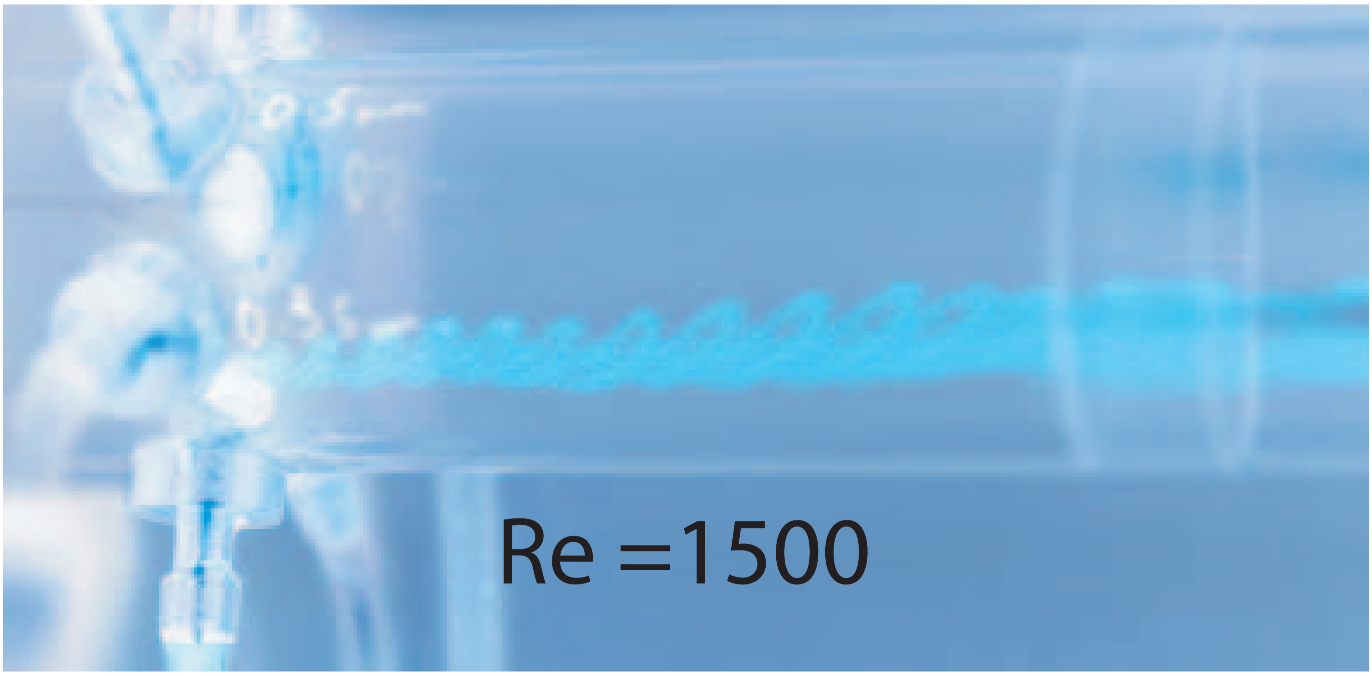
Calculations by Per-Olov Asen

Comp. & Fluids (2010) Asen, Kreiss & Rempfer

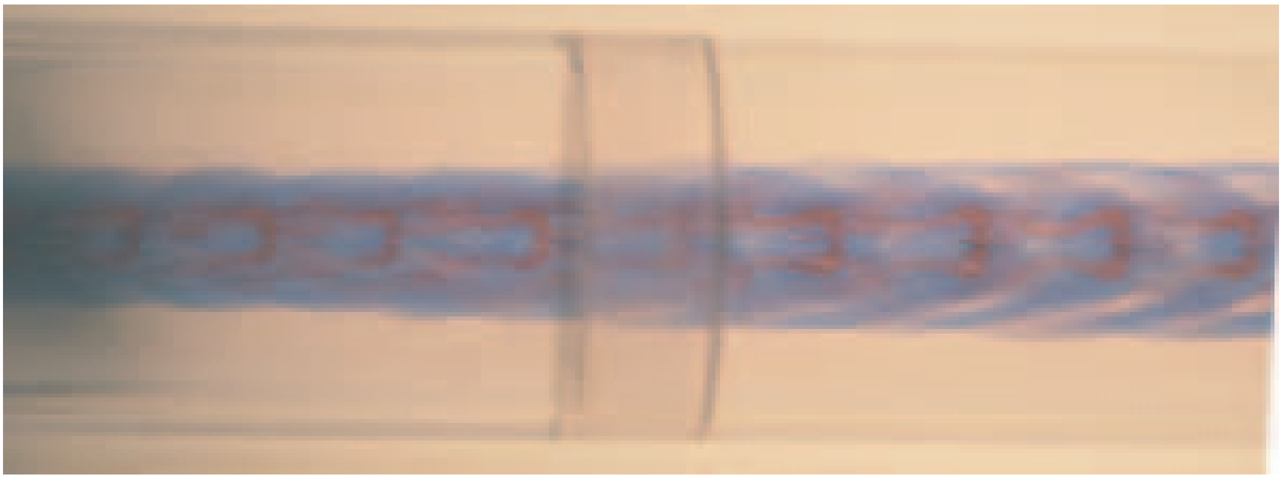




Jet Through Small Hole


Re =1500

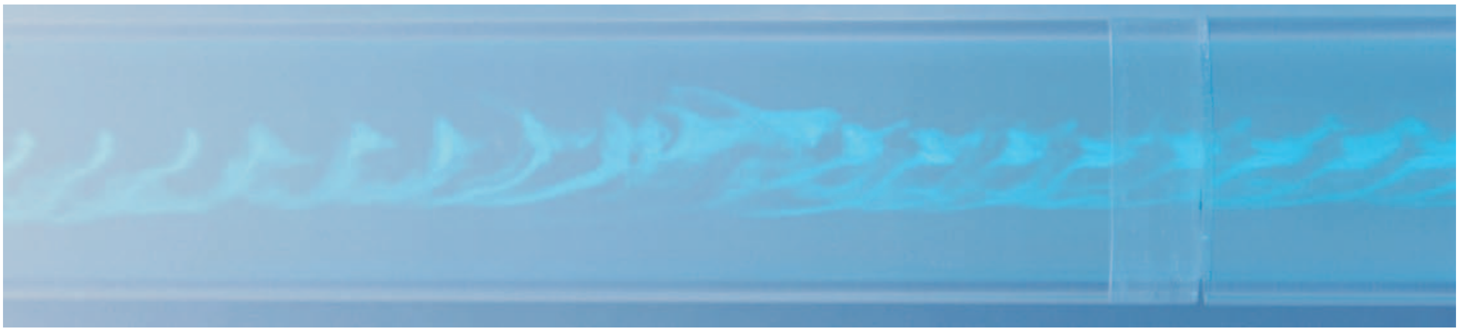
0.3mm diameter hole: Amp. 0.1%



0.3mm diameter hole: Amp. 0.1%

Re =1900

0.3mm diameter hole: Amp. 0.18%



Reynolds Number 2000

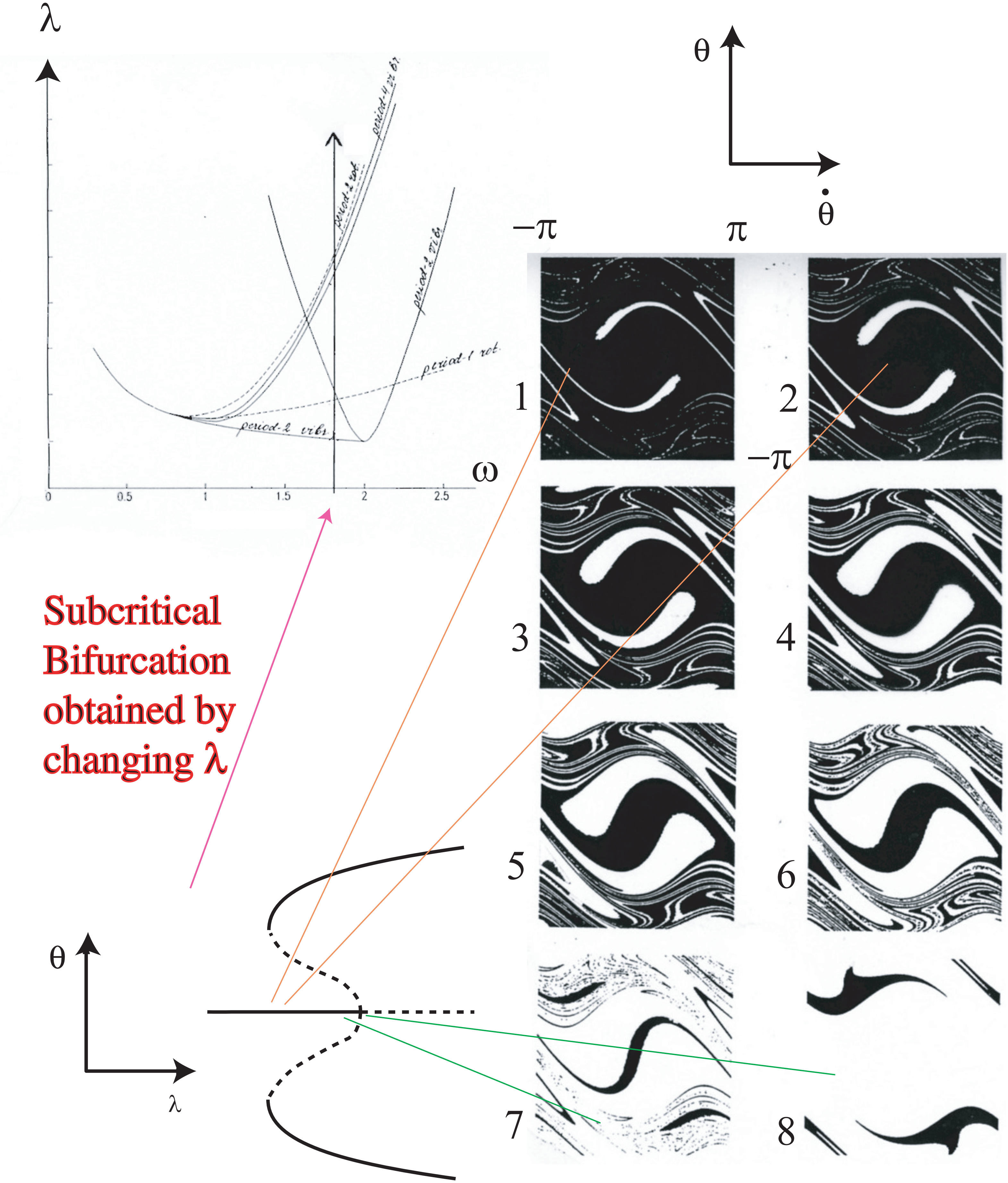
Video Playback 1/10 real time

Pert. Amplitude 0.1%

Pert. Amplitude 0.15%

Single pulse injected through 0.3mm hole 6D from LH edge of image. Pulse length ~10 diameters long.

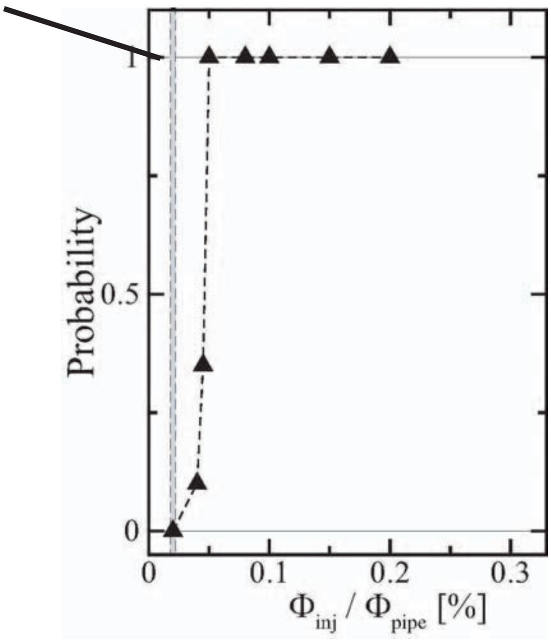
Basin of Attraction: Parametric Pendulum



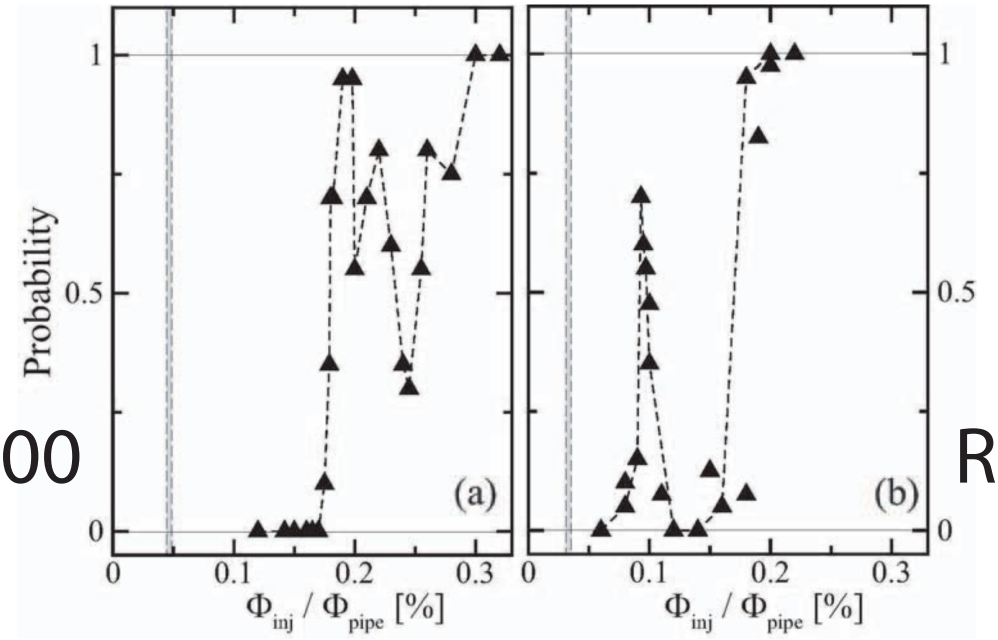
Subcritical Bifurcation obtained by changing λ

Transition Probabilities:

Onset of waves

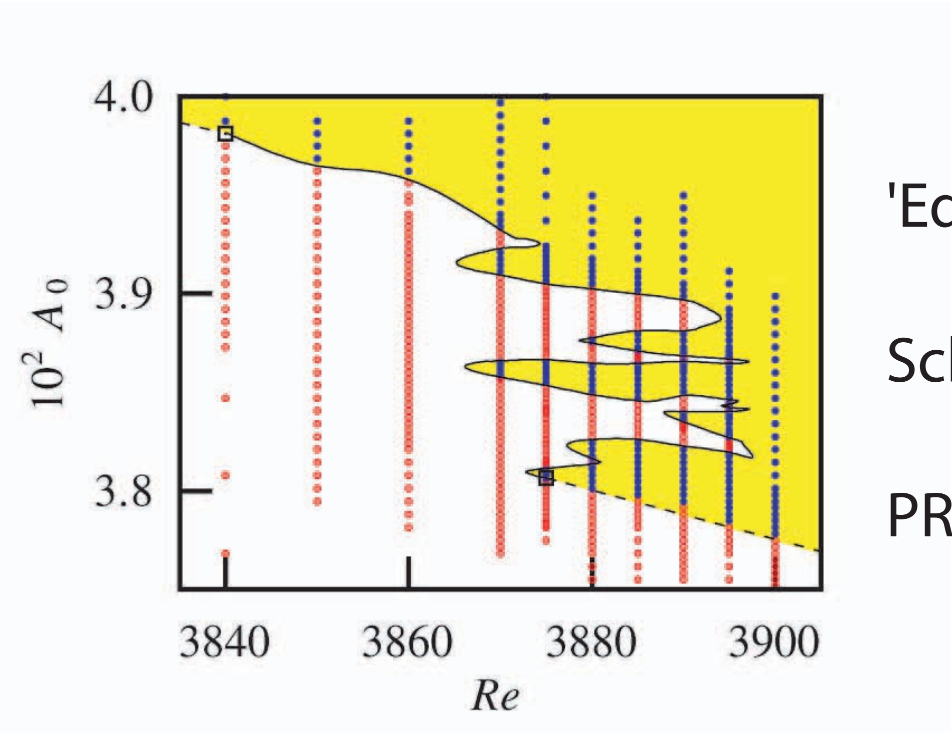


Re=3,000



Re=1900

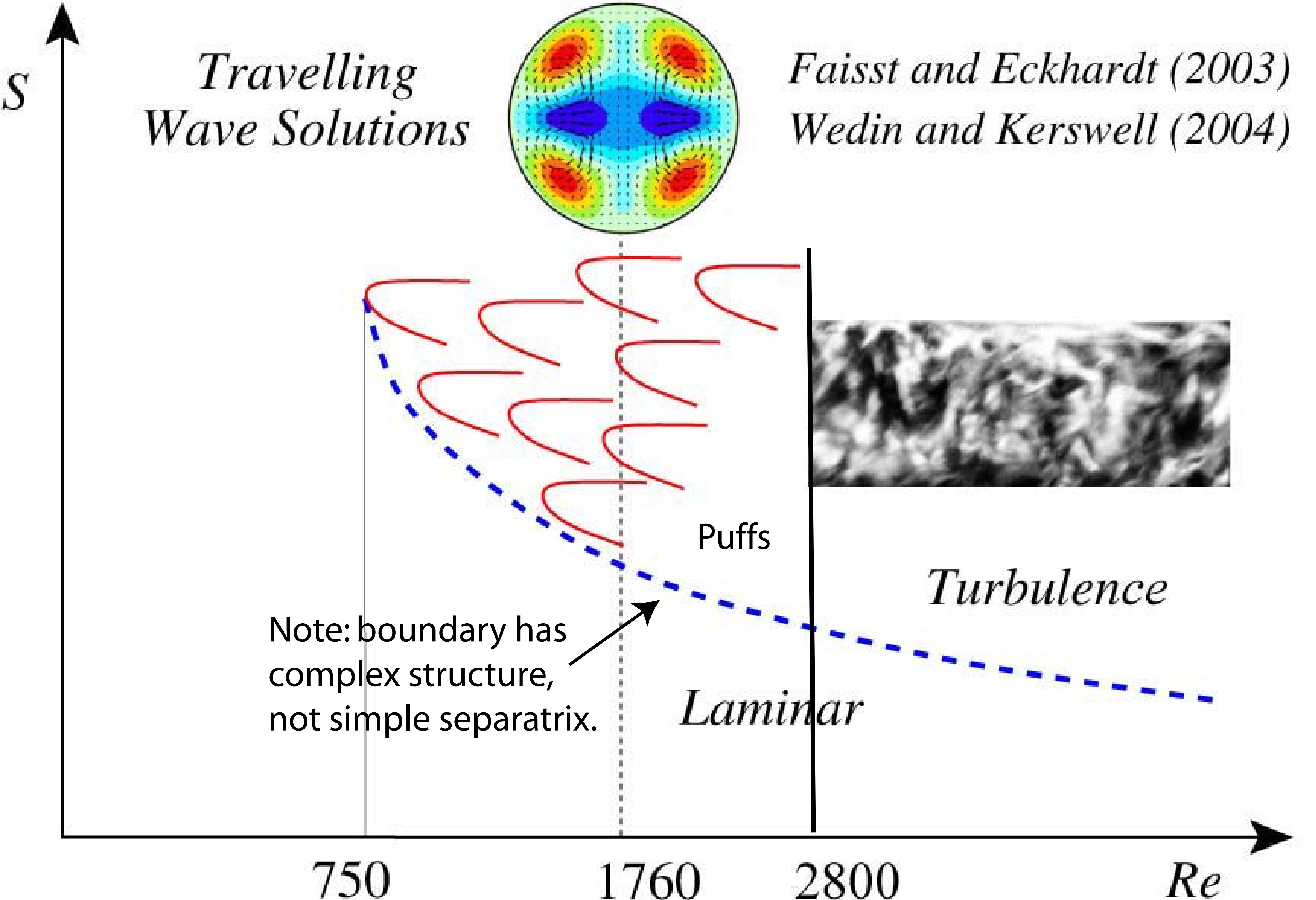
Re=2100



'Edge of chaos'

Schneider et al.

PRL 99 (2007)



Note: boundary has complex structure, not simple separatrix.

Conclusions

- Scaling laws:

  -1 exponent --> balance of
  viscous and inertial terms.

  -1.4 --> possibility of
  transient growth.

- Hairpin vortices suggest definite transition step.

- Threshold is complex for $Re < 3{,}000$

PRL 91(2003) 244052, PRL 96 (2006)
JFM 582, 169 (2007)
Phys. Today (2004) Feb.