John Wallis (1616–1703), Oxford's Savilian Professor of Geometry from 1649 to 1703, was the most influential English mathematician before the rise of Isaac Newton. His most important works were his *Arithmetic of Infinitesimals* and his treatise on *Conic Sections*, both published in the 1650s. It was through studying the former that Newton came to discover his version of the binomial theorem. Wallis's last great mathematical work *A Treatise of Algebra* was published in his seventieth year.

UNIVERSITY OF OXFORD
Mathematical Institute

John Wallis

"In the year 1649 I removed to Oxford, being then Publick Professor of Geometry, of the Foundation of Sr. Henry Savile. And Mathematicks which had before been a pleasing diversion, was now to be my serious Study."
John Wallis

Wallis' time-line

1616	Born in Ashford, Kent
1632–40	Studied at Emmanuel College, Cambridge
1640	Ordained a priest in the Church of England
1642	Started deciphering secret codes for **Oliver Cromwell**'s intelligence service during the English Civil Wars
1647	Inspired by **William Oughtred**'s *Clavis Mathematicae* (Key to Mathematics) which considerably extended his mathematical knowledge
1648	Initiated mathematical correspondence with continental scholars (**Hevelius**, **van Schooten**, etc.)
1649	Appointed Savilian Professor of Geometry at Oxford
	31 October: Inaugural lecture
1655–56	*Arithmetica Infinitorum* (The Arithmetic of Infinitesimals) and *De Sectionibus Conicis* (On Conic Sections)
1658	Elected Oxford University Archivist
1663	11 July: Lecture on Euclid's parallel postulate
1655–79	Disputes with **Thomas Hobbes**
1685	*Treatise of Algebra*
1693–99	*Opera Mathematica*
1701	Appointed England's first official decipherer (alongside his grandson **William Blencowe**)
1703	Died in Oxford

Oxford Mathematics

UNIVERSITY OF OXFORD

Mathematical Institute

John Wallis (1616–1703)
Savilian Professor

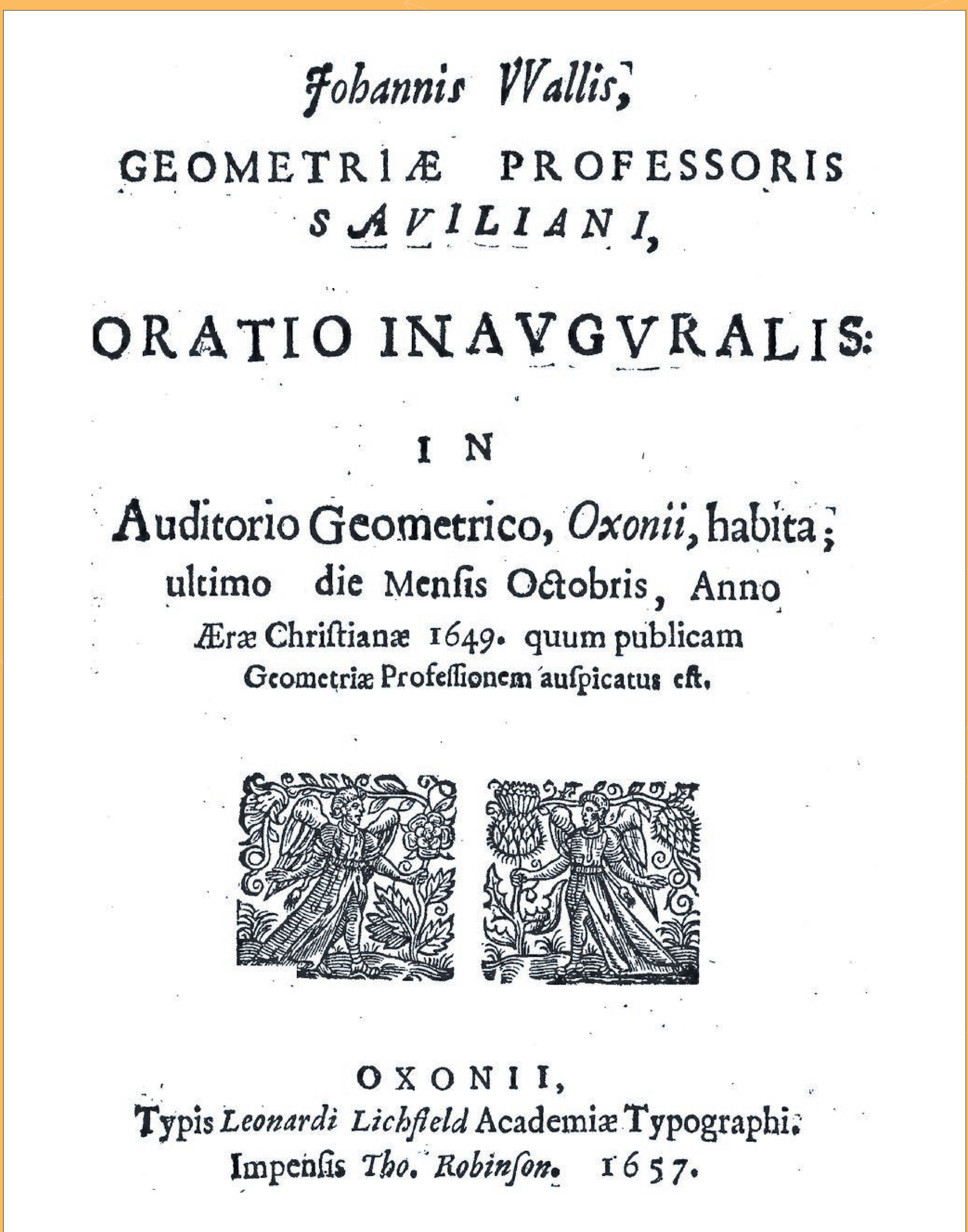
During the English Civil Wars John Wallis was appointed Savilian Professor of Geometry in 1649.

Wallis is appointed

The introduction of John Wallis into the University of Oxford was caused by politics. During the early part of the Civil Wars the University had been the Royalist headquarters, and in the subsequent reckoning most college heads and fellows were deposed.

The Savilian professors were expelled in 1648 for Royalist sympathies, and the Parliamentary Commissioners replaced them by John Wallis as Professor of Geometry and Seth Ward as Professor of Astronomy. Wallis had been a moderate supporter of the revolutionary cause during the Civil Wars.

Prior to taking up the Savilian Chair John Wallis had little mathematical experience and enjoyed no public reputation as a mathematician. However, a more far-sighted mathematical appointment on flimsier evidence is difficult to imagine. Wallis's appointment to the Savilian Chair, which he held until his death fifty-four years later, marked the beginning of an intense period of activity which established Oxford as the mathematical powerhouse of the nation, for a time at least, and promoted John Wallis as the most influential of English mathematicians before Isaac Newton.



Johannis Wallis,
GEOMETRIÆ PROFESSORIS
SAVILIANI,
ORATIO INAUGURALIS:
IN
Auditorio Geometrico, Oxonii, habita;
ultimo die Menſis Octobris, Anno
Æræ Chriſtianæ 1649. quum publicam
Geometriæ Profeſſionem auſpicatus eſt.

OXONII,
Typis Leonardi Lichfield Academiæ Typographi.
Impenſis Tho. Robinſon. 1657.

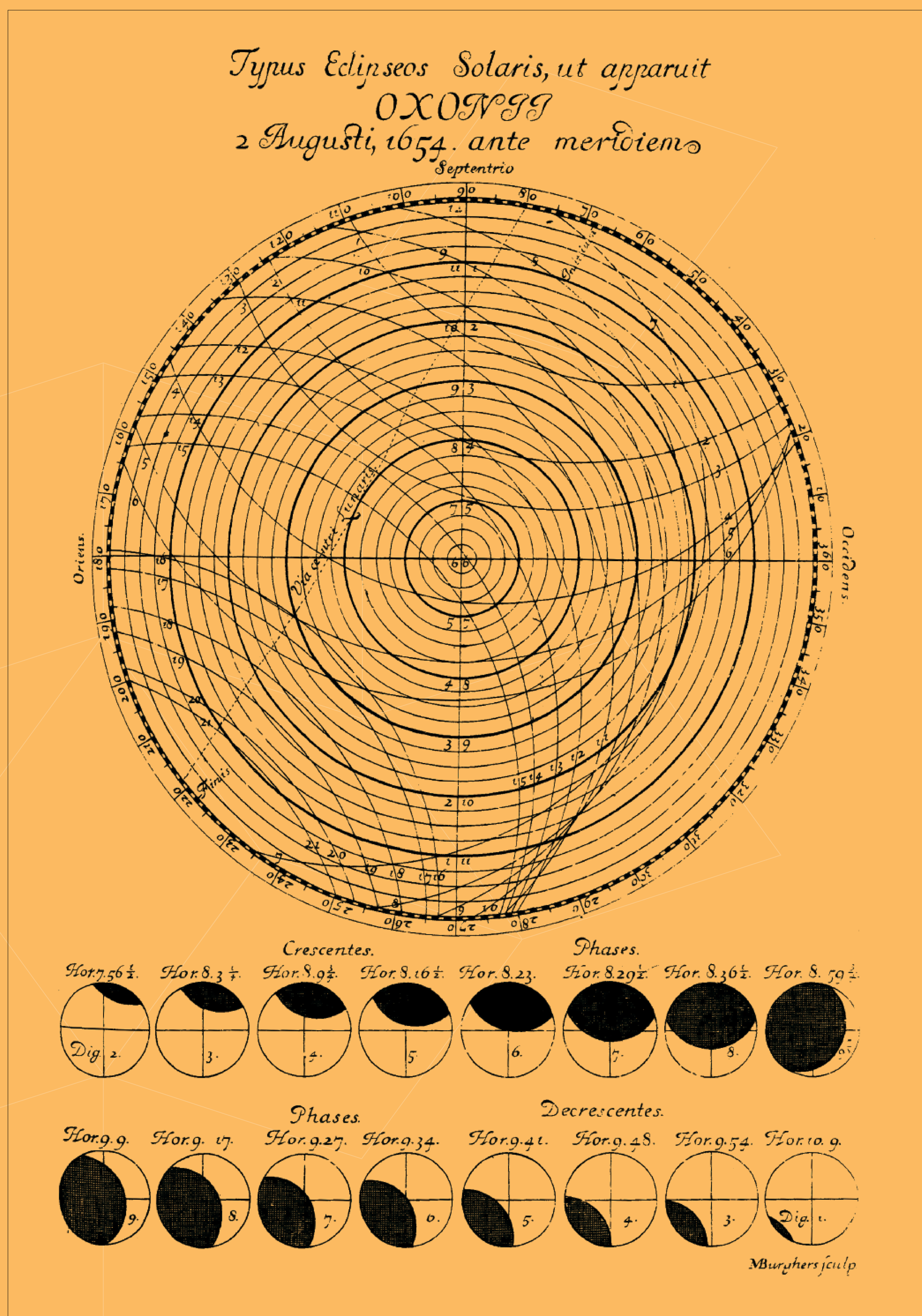
Wallis's inaugural lecture as Savilian Professor of Geometry was given on 31 October 1649 in the Geometry Lecture Room at the east end of the Schools quadrangle, now part of the Bodleian Library.

Oxford in the 1650s

The Savilian statutes obliged John Wallis to give introductory courses in practical and theoretical arithmetic. In his courses Wallis also lectured on Euclid's *Elements*, the *Conics* of Apollonius, and the works of Archimedes.

Several Oxford mathematicians of the time learned their trade from the writings of William Oughtred, whose influential algebra text *Clavis Mathematicae* (Key to Mathematics) had been published in London in 1631.

The pedagogical concerns of the Savilian professors, Wallis and Ward, were exemplified in their enthusiastic promotion of Oughtred's *Clavis* from which both had learned their algebra. With their support a new Latin edition was published in Oxford in 1652. It contained as an appendix a work of Oughtred's on sundials, translated by the 20-year-old Christopher Wren, a student at Wadham College 'from whom we may expect great things', in the prescient words of Oughtred's preface.



A solar eclipse was observed at Oxford on 2 August 1654 by John Wallis, Christopher Wren and Richard Rawlinson.

UNIVERSITY OF OXFORD
Mathematical Institute

John Wallis (1616–1703)

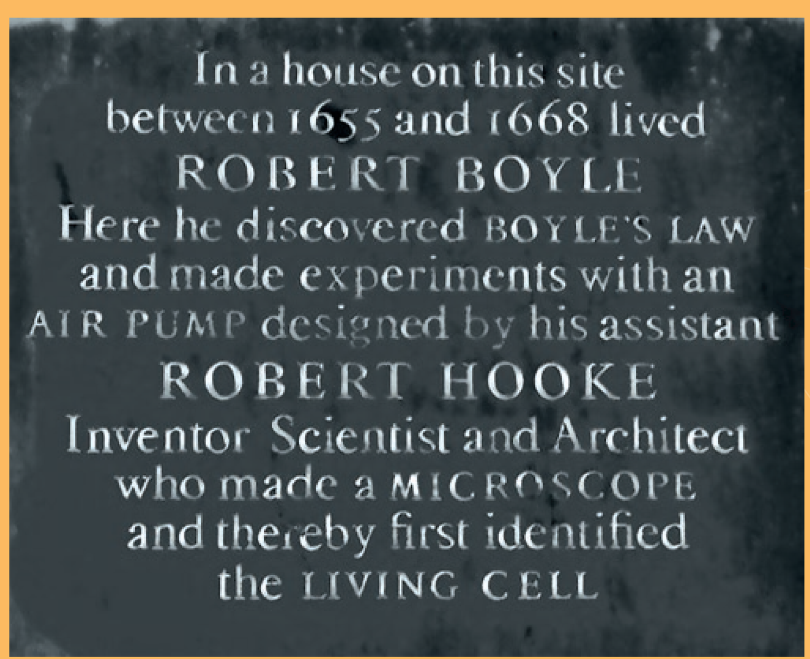
Intellectual and scientific life

Throughout his career Wallis was closely involved with England's intellectual and scientific activities.

Oxford contemporaries

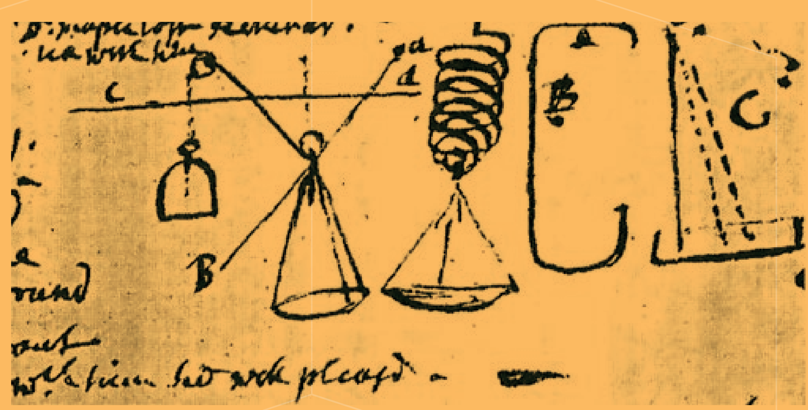
"Oxford's achievements in the 17th century were founded upon a rich culture of experimental science, and between 1648 and 1660 the city housed probably the most dynamic scientific community in Europe."
Allan Chapman

At the centre of this community was the Warden of Wadham College, John Wilkins, another Civil War appointment. Wilkins attracted Robert Boyle to Oxford, had Christopher Wren as a student, and Robert Hooke as an illustrious protégé.



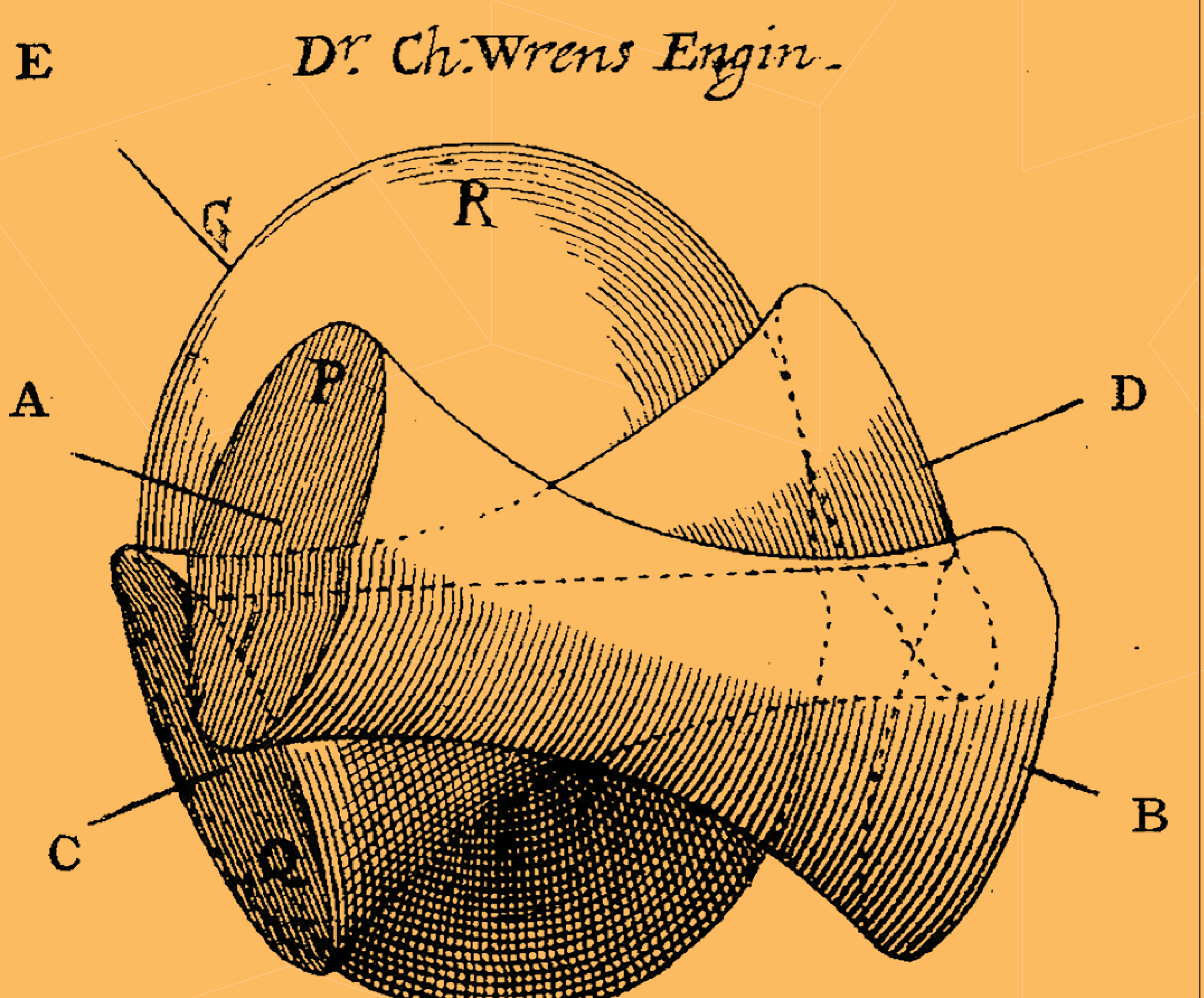
A plaque commemorates the High Street site of Boyle and Hooke's laboratory.

Robert Hooke (1635–1703) was interested in the mathematical principles underlying many of his experiments. In this extract from his diary for 21 August 1678 he records a visit to a coffee house with Christopher Wren, where they exchanged information on their recent inventions, including Hooke's *'philosophicall spring scales'*.



Although Sir Christopher Wren (1632–1723) is mainly remembered as an architect, his early career was as an astronomer, and he was one of the outstanding geometers of the age.

Wren's blend of mathematical and practical insights is seen in his design of an engine for grinding hyperbolic lenses, shown to the Royal Society in July 1669.



The Royal Society

John Wallis was an active member of the Oxford experimental philosophy group which met frequently at Wadham College and Oxford coffee houses and led to the formation of the Royal Society. The Royal Society was founded on 28 November 1660 at Gresham College in London following a lecture given by Christopher Wren, Gresham Professor of Astronomy. Wren was later appointed Savilian Professor of Astronomy in Oxford.

William Brouncker, the first President of the Royal Society, is the left-hand figure in the frontispiece of Thomas Sprat's 1667 history of the Society. Wallis encouraged Brouncker's mathematical researches and published some of Brouncker's results in his books.

Wallis was an early and active Fellow of the Royal Society. Along with many of his contemporaries in the Royal Society, he had remarkably broad interests.

The pages of the Society's *Philosophical Transactions* reflect the range of things he had views on and wanted to communicate, as well as topics reflecting his interest in the history of mathematics.

In 1685 Wallis, in the *Philosophical Transactions*, supported his argument that one's memory is better at night by reporting that he calculated the square root of 3 in his head to twenty decimal places, arriving at the correct answer 1.73205 08075 68877 29353, and retained it in his mind before writing it down the next day.

Oxford
Mathematics

UNIVERSITY OF OXFORD
Mathematical Institute

John Wallis (1616–1703)
Publications

Wallis's main impact was through his publications.
Two of his most important ones are featured here.

Arithmetica Infinitorum

John Wallis's *Arithmetica Infinitorum* (Arithmetic of Infinitesimals, or a New Method of Inquiring into the Quadrature of Curves, and Other More Difficult Mathematical Problems) appeared at a critical time for the development of mathematics.

Approaches using geometrical indivisibles had previously been used to find areas under curves, and had been successful for any curve of the form $y = x^n$, where n is a positive whole number, but Wallis associated numerical values to the indivisibles, allowing him to extend the result to the case when n is fractional. The word 'interpolation' (in its mathematical sense) was introduced by Wallis in this work.

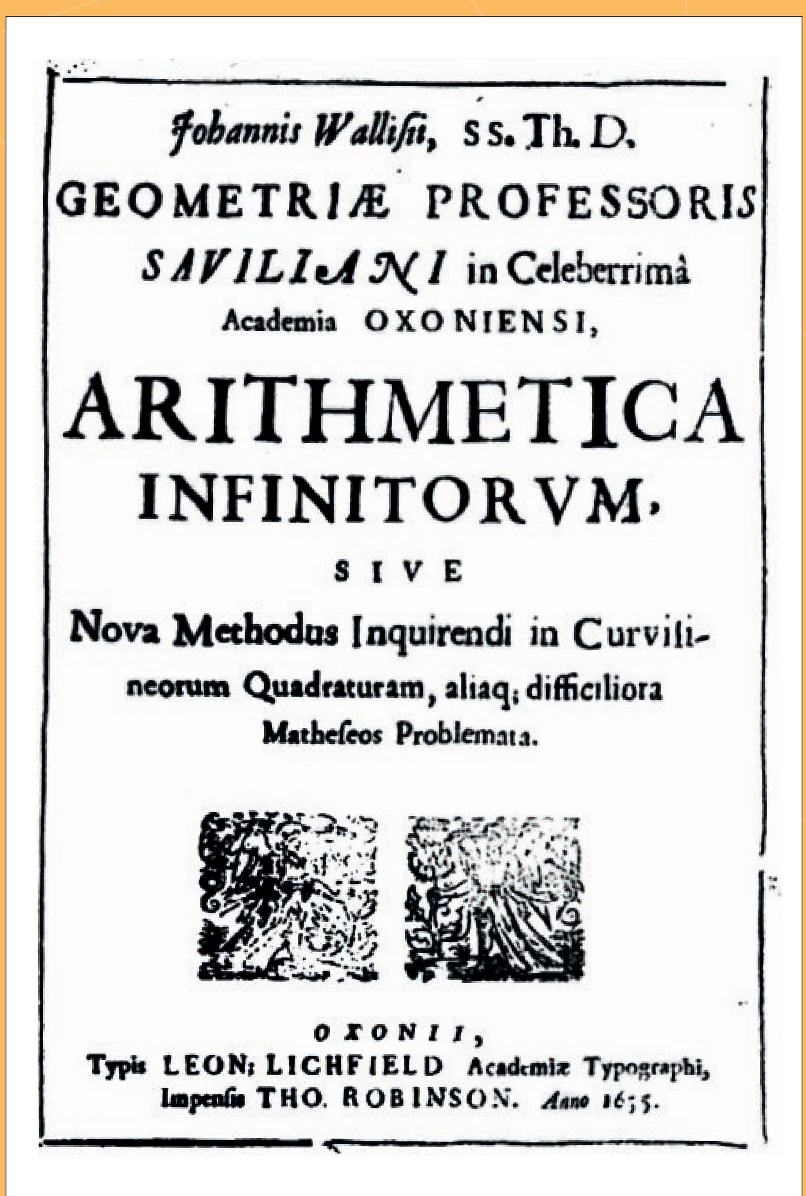
It was here that Wallis produced his celebrated exact formula for $4/\pi$ – namely,

$$\frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times \dots}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times \dots}$$

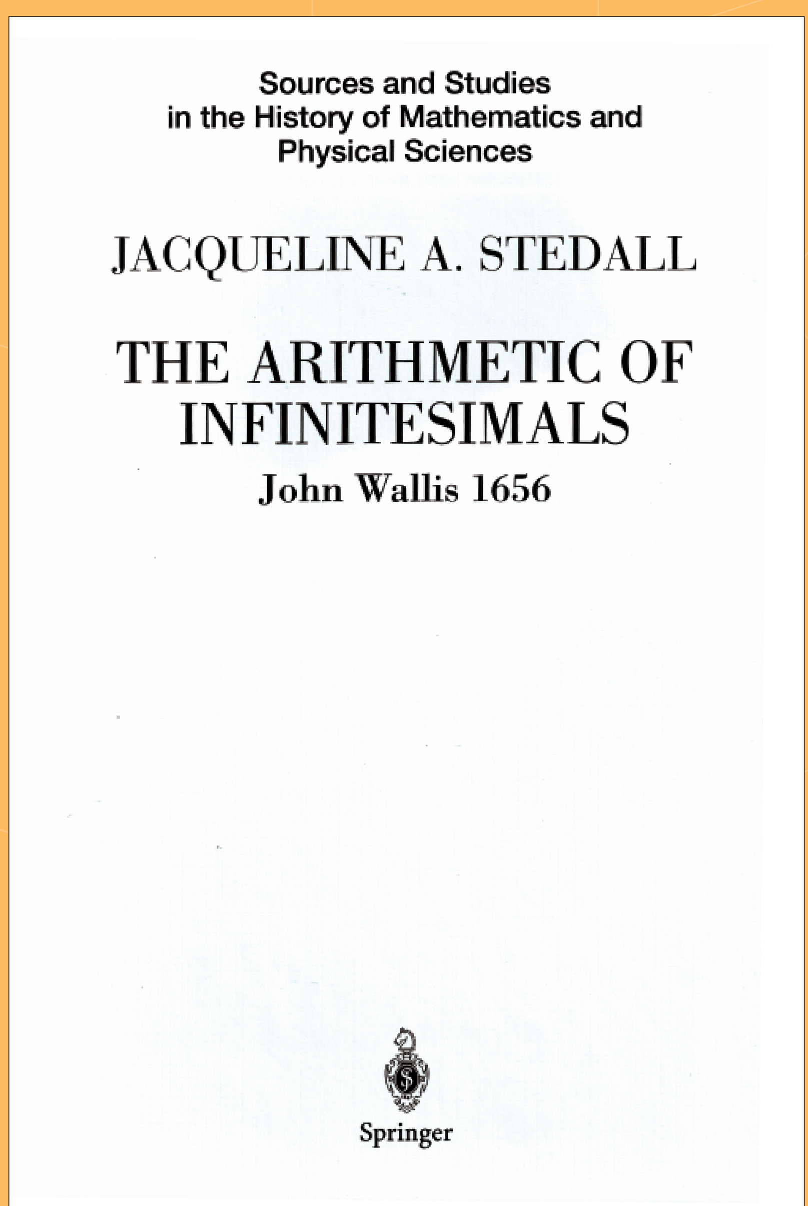
"… Wallis drew on ideas originally developed in France, Italy and the Netherlands: analytic geometry and the method of indivisibles. He handled them in his own way, and the resulting method of quadrature, based on the summation of indivisible or infinitesimal quantities, was a crucial step towards the development of a fully fledged integral calculus some ten years later."

Jacqueline Stedall
Introduction to her translation of Arithmetica Infinitorum

Among the mathematicians influenced by *Arithmetica Infinitorum* was Isaac Newton: it was through his study of this work in the mid-1660s that he came to discover the general binomial theorem. Newton was attracted to Wallis's fundamental engine of discovery, the exploration and recognition of pattern. Wallis's career had been set in motion by his cryptological skills, and they seem to have characterised his mathematical style as well. Wallis deciphered throughout his professional career, right up to the final days before his death; he was justly regarded as Europe's greatest code-breaker, working mainly on complex French numerical substitution ciphers.


Johannis Wallisii, SS. Th. D.
GEOMETRIÆ PROFESSORIS
SAVILIANI in Celeberrimâ
Academia OXONIENSI,
ARITHMETICA
INFINITORVM,
SIVE
Nova Methodus Inquirendi in Curvilineorum Quadraturam, aliaq; difficiliora
Matheseos Problemata.
OXONII,
Typis LEON: LICHFIELD Academiæ Typographi,
Impensis THO. ROBINSON. Anno 1655.

Wallis's *Arithmetica Infinitorum* was published in 1656.


Sources and Studies
in the History of Mathematics and
Physical Sciences
JACQUELINE A. STEDALL
THE ARITHMETIC OF
INFINITESIMALS
John Wallis 1656
Springer

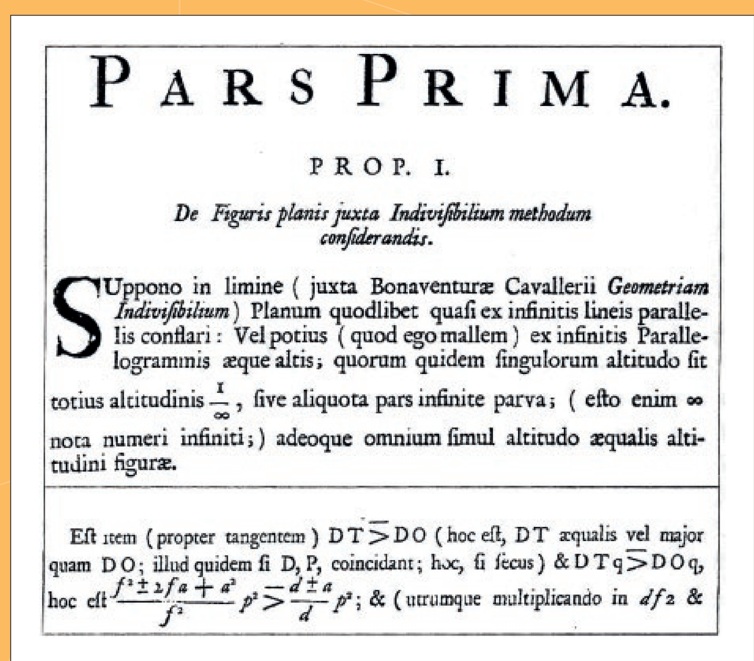
Jacqueline Stedall's translation and commentary.

De Sectionibus Conicis

Another important contribution that Wallis made in the 1650s was his investigation of conic sections. The conics and their properties had been known from antiquity, but the curves had been viewed as sections of a cone, arising from three-dimensional geometry.

In 1656 Wallis published his investigations of conic sections, called *De Sectionibus Conicis* (On Conic Sections). He regarded them as plane curves, with no reference to the cone after the initial derivation, and obtained their properties through the use of the techniques of algebraic analysis introduced by René Descartes.

Although Wallis was often conservative in his use of mathematical notation he did introduce two new symbols that are still in current use: the 'infinity' sign and the symbol for 'greater than or equal to'.


PARS PRIMA.
PROP. I.
De Figuris planis juxta Indivisibilium methodum considerandis.
SUppono in limine (juxta Bonaventuræ Cavallerii *Geometriam Indivisibilium*) Planum quodlibet quasi ex infinitis lineis parallelis conflari : Vel potius (quod ego mallem) ex infinitis Parallelogrammis æque altis; quorum quidem singulorum altitudo sit totius altitudinis $\frac{1}{\infty}$, sive aliquota pars infinite parva; (esto enim ∞ nota numeri infiniti;) adeoque omnium simul altitudo æqualis altitudini figuræ.
Est item (propter tangentem) DT $\geqq$ DO (hoc est, DT æqualis vel major quam DO; illud quidem si D, P, coincidant; hoc, si secus) & DTq $\geqq$ DOq, hoc est $\frac{f^2 \pm 2fa + a^2}{f^2} p^2 \geqq \frac{d \pm a}{d} p^2$; & (utrumque multiplicando in $df2$ &

Wallis introduced the 'infinity' sign and the symbol for 'greater than or equal to' in his 1656 book on conic sections.

Even where useful notations were yet to be introduced, such as those for fractional and negative indices, Wallis went far towards laying the groundwork, writing in his *Arithmetica Infinitorum*: *"1/x whose index is –1"* and *"√x whose index is ½"*.

Wallis's concern for mathematical symbolism is but one facet of a lifelong exploration of issues of language and communication. One of his first and most successful books was not a mathematical treatise but an *'English grammar'*.


Johannis Wallis,
Geometriæ Professoris
SAVILIANI
in celeberrima Academia
OXONIENSI,
GRAMMATICA
Linguæ Anglicanæ.
Cui præfigitur,
DE LOQUELA
sive Sonorum Formatione,
TRACTATUS
Grammatico-Physicus.
OXONIÆ,
Excudebat LEON. LICHFIELD
Acad. Typographus. Veneunt
apud Tho. Robinson. 1653.

"At the beginning of my mathematical studies, when I had met with the works of our celebrated Wallis, on considering the series by the intercalation of which he himself exhibits the area of the circle and the hyperbola …"

Isaac Newton
on how he came to discover the general binomial theorem

John Wallis (1616–1703)

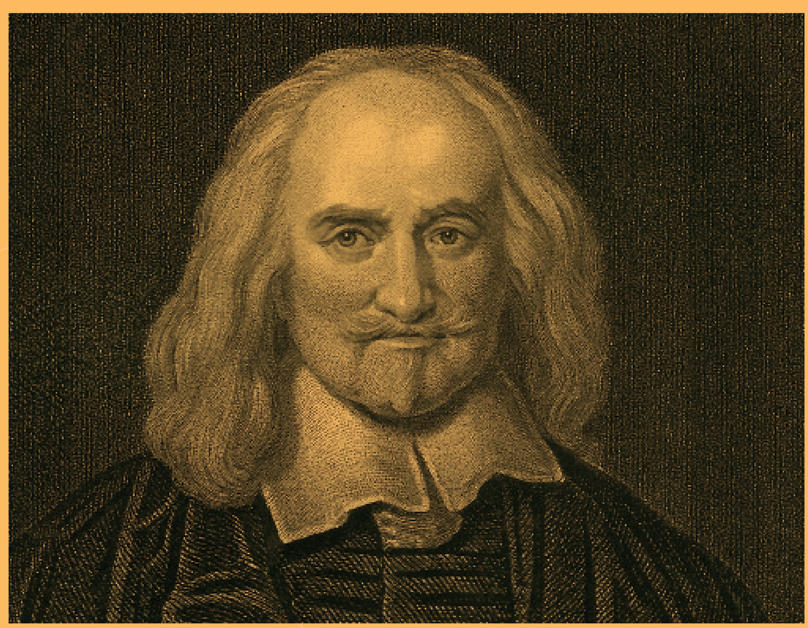
Controversy and collaboration

University of Oxford
Mathematical Institute

John Wallis was involved with many fierce disputes, as well as productive collaborations.

Disputes with Thomas Hobbes

John Wallis



Thomas Hobbes

Wallis was blessed with a formidable intellect, a prodigious memory, and a robust constitution. A man of short temper and robust dialogue he also possessed a highly contentious nature and created many enemies.

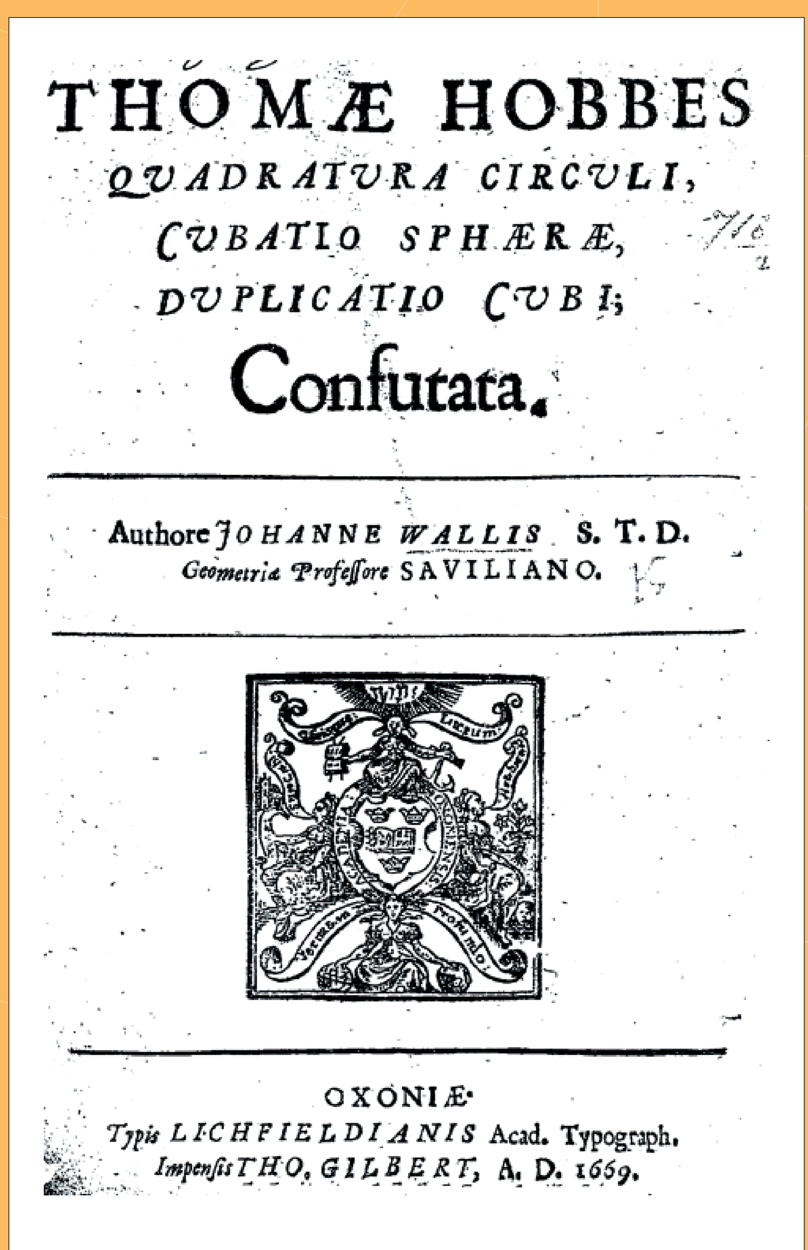
Wallis quarrelled with the French mathematicians Pierre de Fermat and Blaise Pascal, but his most virulent dispute, lasting nearly a quarter of a century from the mid-1650s, was with the philosopher, Thomas Hobbes (1588–1679). A friend of Francis Bacon, Galileo, Descartes, and Mersenne, Hobbes was one of the outstanding intellectual figures of the age.

The following account from Aubrey's *Brief Lives* tells how Hobbes became interested in mathematics.

"He was 40 years old before he looked on Geometry; which happened accidentally. Being in a Gentleman's Library, Euclid's Elements lay open, and 'twas the 47 El. libri I [the Pythagorean theorem in Book I]. He read the Proposition. By G—, sayd he (he would now and then sweare an emphaticall Oath by way of emphasis) this is impossible! So he reads the Demonstration of it, which referred him back to such a Proposition; which proposition he read. That referred him back to another, which he also read. Et sic deinceps [and thus, in succession] that at last he was demonstratively convinced of that trueth. This made him in love with Geometry."

Hobbes's new-found enthusiasm for mathematics subsequently pervaded his philosophical approach and style of writing. His materialist and anti-clerical Leviathan (1651) created widespread controversy, even before he aroused the ire of both Ward and Wallis through his attacks on the post-revolutionary state and performance of the universities, which he saw as riddled with priestcraft and outmoded learning.

Thomas Hobbes's claim in 1655 that he had solved the ancient Greek problem of 'squaring the circle' (constructing a square equal in area to a given circle) drew a fierce reaction and rebuttal from John Wallis.



THOMÆ HOBBES
QUADRATURA CIRCULI,
CUBATIO SPHÆRÆ,
DUPLICATIO CUBI;
Confutata.

Authore JOHANNE WALLIS S. T. D.
Geometriæ Professore SAVILIANO.

OXONIÆ
Typis LICHFIELDIANIS Acad. Typograph.
Impensis THO. GILBERT, A. D. 1669.

The title page from one of Wallis's tracts against Hobbes

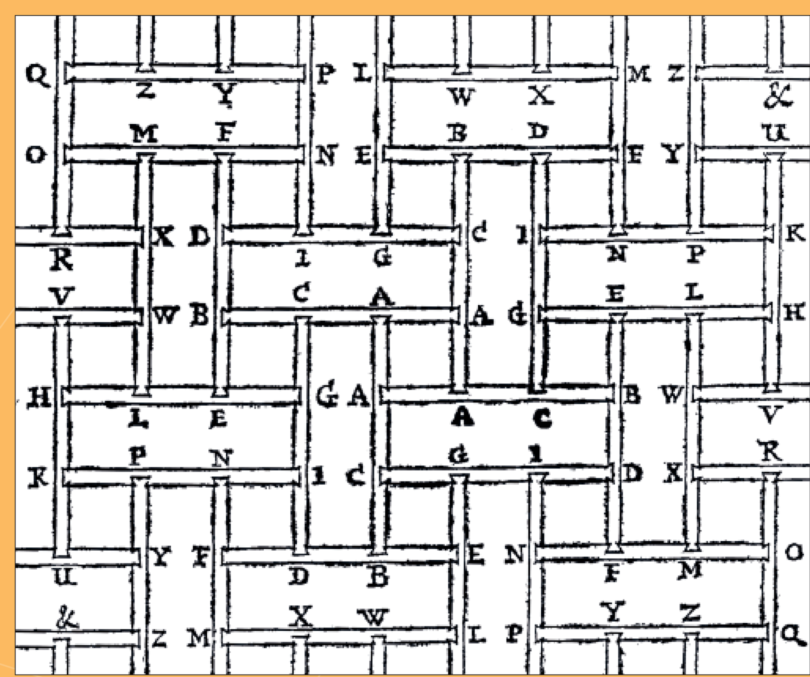
The Sheldonian Theatre

Oxford's Sheldonian Theatre, designed on an ancient Roman model by the Savilian Professor of Astronomy, Christopher Wren, exemplifies the creative tension between antiquity and innovation that characterised the Wallis era. In particular, the expanse of the Sheldonian's flat ceiling, supported by trusses, caused a sensation.

Oxford's Sheldonian Theatre ceiling

An alternative approach to supporting a ceiling by beams that were much shorter than the length or width of the ceiling had been worked out earlier by John Wallis in the 1650s.

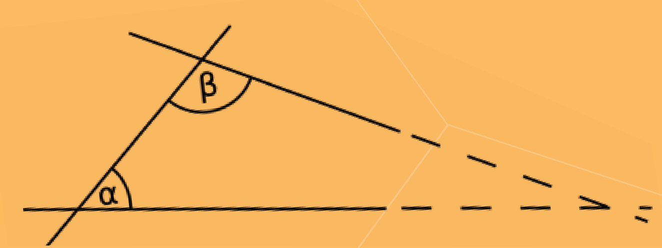
His interlocking beam structure needed support only where its edges rested on the walls. Wallis had worked out the mathematics of these interlocking beams in an innovative calculation involving his solution of no fewer than twenty-five simultaneous equations.



A diagram of Wallis's interlocking flat ceiling structure.

The Parallel Postulate

The Parallel Postulate states that given any straight line and a point not on it, there exists one and only one straight line which passes through that point and never intersects the first line, no matter how far they are extended. It is equivalent to Euclid's Fifth Postulate.



Euclid's Fifth Postulate states that if the sum of the interior angles α and β is less than 180°, then the two straight lines, produced indefinitely, meet on that side.

On the evening of 11 July 1663 Wallis lectured in Oxford on the parallel postulate, and presented a seductive argument purporting to derive it from Euclid's other axioms.

This lecture by Wallis was the first mature Western attempt to derive the parallel postulate as a theorem.

As Wallis observed, his argument assumes that similar figures can take different sizes. Wallis found this assumption very plausible, and if it were true then the Parallel Postulate would be a consequence of the other axioms of Euclid.

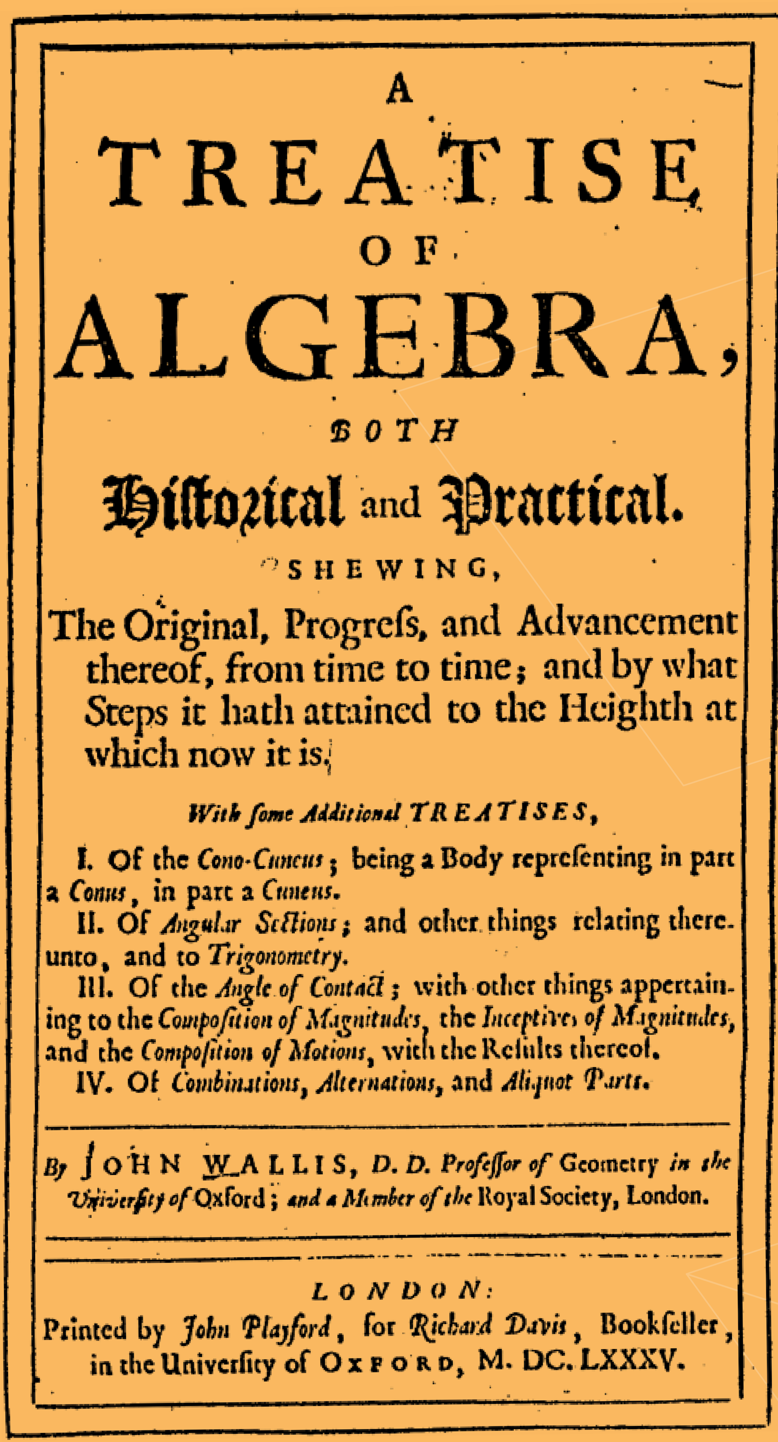
It does, however, imply a remarkable result: *in any geometry in which the parallel postulate does not hold, similar figures must be identical in size as well as in shape,* and so scale copies can never be made. It would also have the consequence, as Johann Heinrich Lambert observed a century later, that there would have to be an absolute measure of length.

John Wallis (1616–1703)

Final years

In his final years John Wallis published his influential *A Treatise of Algebra*.
In 1969 Oxford University established the *Wallis Chair of Mathematics* in his memory.

A Treatise of Algebra



A
TREATISE
OF
ALGEBRA,
BOTH
Historical and Practical.
SHEWING,
The Original, Progreſs, and Advancement thereof, from time to time; and by what Steps it hath attained to the Heighth at which now it is.

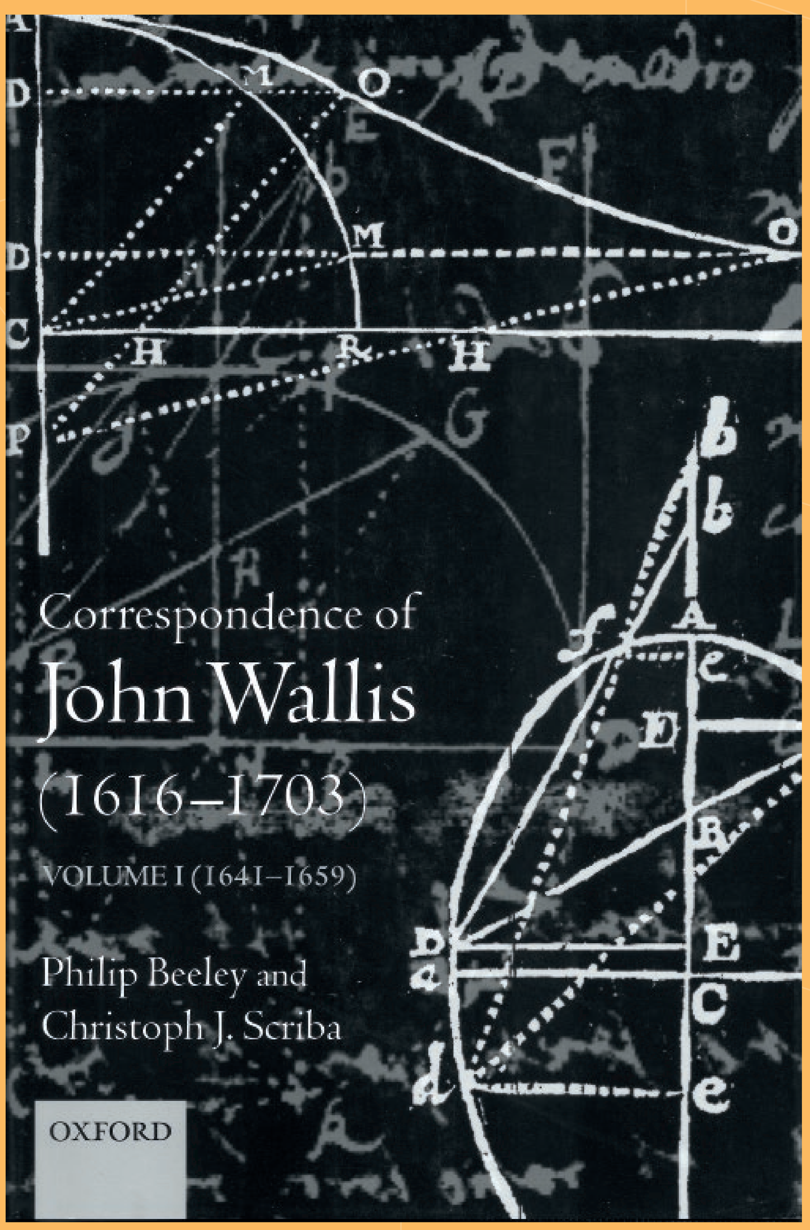
With ſome Additional TREATISES,

I. Of the *Cono-Cuneus*; being a Body repreſenting in part a *Conus*, in part a *Cuneus*.
II. Of *Angular Sections*; and other things relating thereunto, and to *Trigonometry*.
III. Of the *Angle of Contact*; with other things appertaining to the *Compoſition of Magnitudes*, the *Inceptives of Magnitudes*, and the *Compoſition of Motions*, with the Reſults thereof.
IV. Of *Combinations*, *Alternations*, and *Aliquot Parts*.

By JOHN WALLIS, *D.D. Profeſſor of* Geometry *in the Univerſity of* Oxford; *and a Member of the* Royal Society, London.

LONDON:
Printed by *John Playford*, for *Richard Davis*, Bookſeller, in the Univerſity of OXFORD, M. DC. LXXXV.

Wallis's last great mathematical work, *A Treatise of Algebra, Both Historical and Practical*, was published in 1685, in his seventieth year. Of all his vast output, it was this work that was most widely read over the next hundred years, and it remains an extraordinary work.

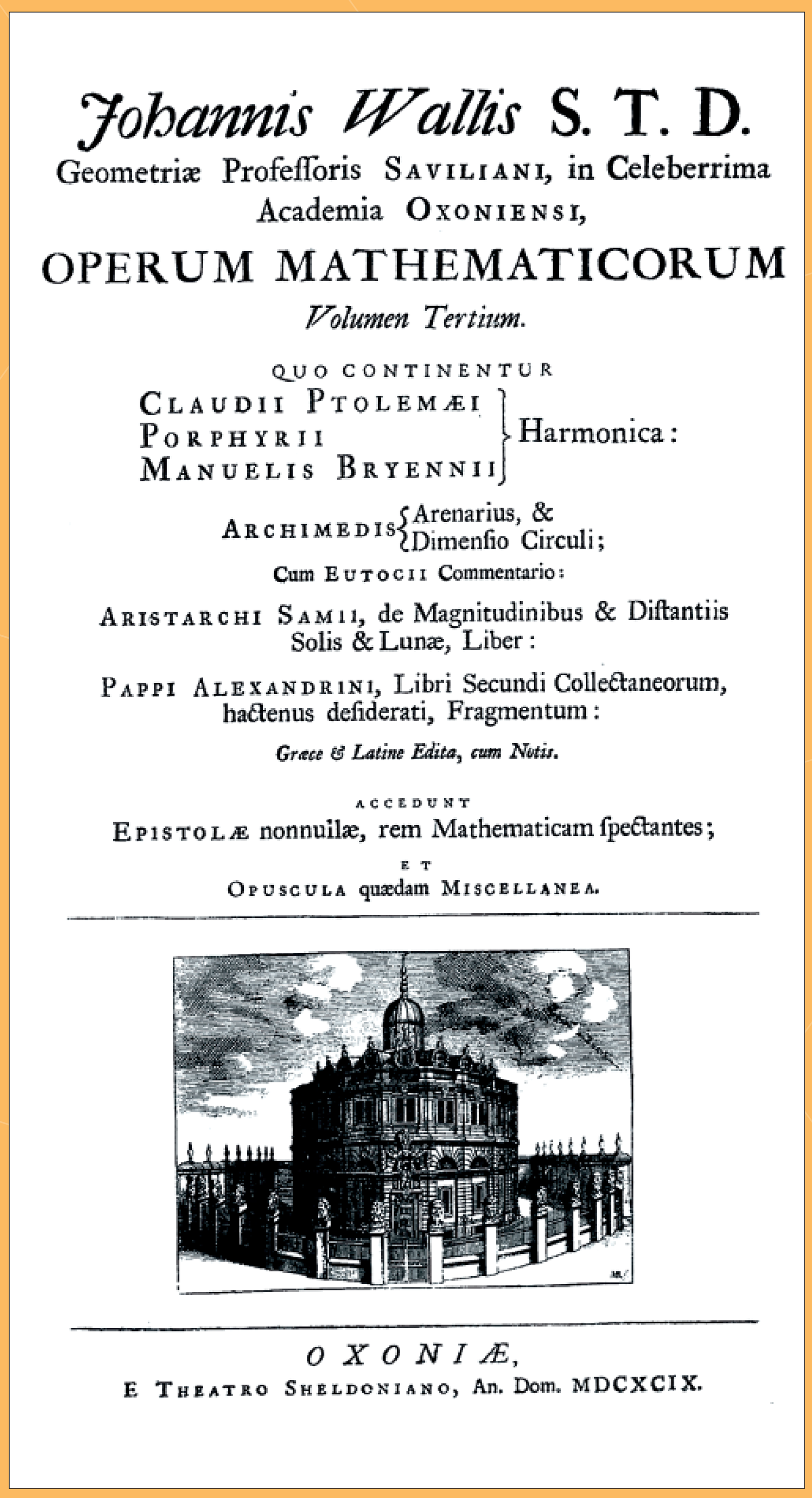


The first volume of an eight-volume collection of the correspondence of John Wallis.

The idea of presenting modern mathematics together with an account of its development was itself a novel one – this was the first substantial history of mathematics in the English language.

The *Treatise* provides a full and judicious survey of the achievements of past centuries. But when he came to his own century his past difficulties with French mathematicians influenced his judgement in rather a startling way as he left the reader in no doubt that recent French achievements had a solidly English basis.

Not only had Thomas Harriot already made Descartes's discoveries, Wallis asserted, but Descartes had actually plagiarized Harriot's results.



Johannis Wallis S. T. D.
Geometriæ Profeſſoris SAVILIANI, in Celeberrima Academia OXONIENSI,
OPERUM MATHEMATICORUM
Volumen Tertium.
QUO CONTINENTUR
CLAUDII PTOLEMÆI, PORPHYRII, MANUELIS BRYENNII Harmonica:
ARCHIMEDIS Arenarius, & Dimenſio Circuli;
Cum EUTOCII Commentario:
ARISTARCHI SAMII, de Magnitudinibus & Diſtantiis Solis & Lunæ, Liber:
PAPPI ALEXANDRINI, Libri Secundi Collectaneorum, hactenus deſiderati, Fragmentum:
Græce & Latine Edita, cum Notis.
ACCEDUNT
EPISTOLÆ nonnullæ, rem Mathematicam ſpectantes;
ET
OPUSCULA quædam MISCELLANEA.
OXONIÆ,
E THEATRO SHELDONIANO, An. Dom. MDCXCIX.

John Wallis's *Collected Works* were published in the 1690s. The third volume contained Wallis's editions of works by Ptolemy, Archimedes, Aristarchus, Pappus and others.

John Wallis's portrait

Wallis retained his vigour to the end. In 1699 an Oxford colleague wrote to Samuel Pepys, a friend of Wallis for over thirty years:

"He says 83 is an incurable distemper. I believe Death will no more surprise him than a Proposition in Mathematicks."

Some measure of the impression that Wallis left on contemporaries may be seen in the remarkable full-length portrait of him at the age of 86, by the court painter Sir Godfrey Kneller. This painting – in which the aged Wallis, swathed in scarlet like some Renaissance prince-prelate, stares out at the viewer in cold disdain – was commissioned by Pepys for presentation to the University of Oxford.

John Wallis, by Sir Godfrey Kneller (1702). The portrait hangs in the Examination Schools in Oxford.

Wallis Professors of Mathematics

The Wallis Professorship of Mathematics was established in 1969 in honour of John Wallis. John Kingman held the professorship from 1969 to 1985, followed by Simon Donaldson. Terry Lyons succeeded Simon Donaldson in 1999 and has held the professorship since then.

John Kingman

Simon Donaldson

Terry Lyons

These posters were conceived by Raymond Flood and Robin Wilson with help from Philip Beeley, Adrian Rice and Dyrol Lumbard

University of Oxford Mathematical Institute