We are at a moment in the history of Oxford Mathematics that is so self-evidently special that we cannot help but marvel at it. Two years after moving into our splendid new home, it is still thrilling to enter the Andrew Wiles Building and sense the intellectual excitement all around us. Students and visiting mathematicians are inspired by the statement that it makes: mathematics is important, and you are part of this great enterprise.

A different sense of recognition came with the results of the government’s 2014 Research Excellence assessments: Mathematical Sciences in Oxford was ranked first across the UK in all categories. Oxford’s reputation as a world centre for mathematics has never been stronger.

‘What then?’ sang Plato’s ghost, ‘What then?’.

Sam Howison’s tenure as Head of Department ended in September 2015. At a small reception, Sam spoke graciously about those who had supported him, and passed me the baton. Immediately after finishing my champagne I received a sharp reminder of the constant need for renewal in mathematics. Half a dozen of our Statutory Professors are about to retire in quick succession. This loss is daunting, but it presents us with an opportunity to reflect: what are the most exciting trends in world mathematics, and where should we be providing leadership? With our emeriti remaining active, this process of renewal can lift us to new heights.

In 2001 we had 54 permanent faculty members; today we have 100, as well as many postdocs and over 1000 students. We treasure our informality and the unique sense of community that grows from our shared passion for mathematics and its propagation. But we’re a big family and we have to organise ourselves well if we are to maximise our contribution to mathematics and society.

Conscious of this, the Department is prioritising its efforts to raise philanthropic funds for DPhil studentships that are open to students of all nationalities working in all areas of mathematics. This is central to our plans for the future, and I hope that many of you will see it as an opportunity to support the life of the Institute.

It has been a pleasure to see so many alumni and friends at our recent public events. I hope to see many more of you here during my time as Curator.
10 December 2015 was the 200th anniversary of the birth of the computing pioneer Ada Lovelace. Various events took place in Oxford to mark this anniversary, including a two-day symposium at the Mathematical Institute and an exhibition at the Bodleian Library.

Famous in her day as the daughter of the Romantic poet Lord Byron, Ada is noted amongst mathematicians and computer scientists for a paper published in 1843, which translated and extended a review (by an Italian engineer, Luigi Menabrea) of Charles Babbage’s ideas for a steam-powered computer, his unbuilt ‘Analytical Engine’. Ada’s substantial appendices contain an account of the principles of the machine, and a table displaying how it might compute Bernoulli numbers, often described as ‘the first computer program’.

From an early age Ada showed both interest and ability in mathematics and science, and received tuition from a number of people, including Mary Somerville. Although her mathematical pursuits were put on hold following her marriage and the birth of three children, she later went back to mathematics and studied with the prominent British mathematician Augustus De Morgan. In a two-year correspondence course, preserved in the Bodleian, we see her working through much of the same material as De Morgan’s students at UCL – limits, series, functions, differential equations, and the like.

This background stood Ada in good stead when she came to translate and extend Menabrea’s article. Babbage’s Analytical Engine was, in modern terms, a general-purpose computer, programmed with punched cards, similar to those used at the time to control Jacquard looms. In principle, it could calculate any function. Ada’s paper presents it, not in terms of ironmongery, but as what we would now call an ‘abstract machine’, describing the functions of memory, CPU, registers, loops, and so on. What is truly remarkable to the modern computer scientist reading the paper is Ada’s high-level view, and how her speculation on the capabilities and potential of the machine mirrors present-day concerns.

Later in life Ada continued to pursue her mathematical interests. Although she did no further work on Babbage’s engines, she and Babbage remained friends and appear to have pursued recreational mathematics together: on one charming document, for instance, we find an exploration of the problem concerning the Bridges of Königsberg.

The Oxford Ada Lovelace Symposium, which took place on 9–10 December, saw a great variety of talks on subjects connected with Ada, ranging from her contributions to computing, to the literary influence on her scientific thought, to a new (mixed) assessment of her mathematical abilities.

Thanks are due for the generosity of the Clay Mathematics Institute and the descendants of Ada Lovelace. The presentations from the symposium may be watched online at http://podcasts.ox.ac.uk/series/ada-lovelace-symposium-celebrating-200-years-computer-visionary. Images and transcriptions of Ada’s mathematical writings will be released online at www.claymath.org in 2016.
Celebrating John Wallis

Philip Beeley and Raymond Flood

2016 marks the 400th anniversary of the birth of John Wallis, who held the Savilian Chair of Geometry for 54 years. Oxford’s Wallis Chair of Mathematics is named after him.

The introduction of John Wallis into the University of Oxford was caused by politics. During the early part of the Civil Wars the University had been the Royalist headquarters, and in the subsequent reckoning most college heads and fellows were deposed. The Savilian professors were expelled in 1648 for Royalist sympathies, and the Parliamentary Commissioners replaced them by John Wallis as Professor of Geometry and Seth Ward as Professor of Astronomy. Wallis had been a moderate supporter of the revolutionary cause during the Civil Wars.

Prior to taking up the Savilian Chair John Wallis had little mathematical experience and enjoyed no public reputation as a mathematician. What counted in his favour was the esteem in which he was held in government circles for his codebreaking abilities. A more far-sighted mathematical appointment on false grounds is difficult to imagine. Wallis’s appointment to the Savilian Chair, which he held until his death fifty-four years later, marked the beginning of an intense period of activity which established Oxford as the mathematical powerhouse of the nation.

John Wallis’s *Arithmetica Infinitorum* (‘Arithmetick of Infinites’) appeared at a critical time for the development of mathematics. Approaches using geometrical indivisibles had previously been used to find areas under curves, and were successful for any curve of the form $y = x^n$, where $n$ is a positive integer, but Wallis associated numerical values to the indivisibles, allowing him to extend the result to the case when $n$ is fractional.

The word ‘interpolation’ (in its mathematical sense) was introduced by Wallis in this work, and he would always be associated with supplanting geometrical techniques to ‘quadratures’ with more rigorous arithmetical ones. It was also here that Wallis produced his celebrated formula for what we would call $4/\pi$ (and which he presented as pictured at the top of this page).

Wallis’s fundamental engine of discovery was the exploration and recognition of pattern. His career had been set in motion by his cryptographical skills during the Civil Wars, and they seem to have characterised his mathematical style as well. Wallis deciphered throughout his professional career, right up to the final days before his death; he was justly regarded as Europe’s greatest codebreaker, working mainly on complex French numerical substitution ciphers. It is humbling to think how the course of events in Europe was in part being decided by an Oxford mathematician labouring in his scarce free time in his lodgings in New College Lane.

Although Wallis was often conservative in his use of mathematical notation he introduced two new symbols that we still use today, for ‘infinity’ and for ‘greater than or equal to’.

Wallis’s last great mathematical work, *A Treatise of Algebra, Both Historical and Practical*, was published in 1685, in his seventieth year. Of his vast output, it was this work that was most widely read over the next hundred years, and was the first substantial history of mathematics in the English language.

Although Wallis’s main contribution to learning was undoubtedly through his innovative and important work in the broad sphere of mathematics, he excelled in other fields, too, including classical scholarship and studies on language. He also served for nearly half a century as keeper of the archives, one of the highest ranked officers in the University at that time, assiduously defending the institution’s ancient rights and privileges.

Some measure of the impression that Wallis left on contemporaries may be seen in the remarkable full-length portrait of him at the age of 86, by the court painter Sir Godfrey Kneller. This painting – in which the aged Wallis, swathed in scarlet like some Renaissance prince-prelate, stares out at the viewer with unmistakable pride – was commissioned by his friend Samuel Pepys for presentation to the University of Oxford.

Wallis’s life and work will be celebrated at a one-day meeting in the Mathematical Institute on 9 June 2016.

![John Wallis, by Sir Godfrey Kneller, Bt](image-url)
Transcendental numbers

Jonathan Pila FRS

Transcendental numbers, like π (= 3.14159… and e (= 2.71828…), have long fascinated mathematicians. There are obstinate problems (is π + e transcendental, or even irrational?), fruitful connections, and an overarching conjectural picture.

A number that satisfies a polynomial equation with integer coefficients is called **algebraic** – for example, √2, 3√2, and √2 + √3 are all algebraic, satisfying the equations \(x^2 = 2\), \(x^3 = 2\), and \(x^4 - 10x^2 + 1 = 0\), respectively. If there’s no such non-zero polynomial, the number is **transcendental**.

The ancient Greek geometers tried unsuccessfully to ‘square the circle’ – that is, to use a ruler and compasses to construct a square whose area equals that of a given circle. They also tried to ‘double the cube’. It turned out that only a rather special kind of algebraic number is **constructible** using ruler and compasses (one using the usual rules of arithmetic, together with the taking of square roots). Doubling the cube is impossible because \(3\sqrt{2}\) is algebraic, but is not constructible. This was proved in the 1830s with the development of algebra – but algebraic methods cannot deal with π.

The transcendence of π was proved in 1882 by Ferdinand Lindemann, who had discussed the matter with Oxford’s Henry Smith in 1876. His proof generalised the methods of Charles Hermite, who proved the transcendence of e a few years earlier: in fact Lindemann proved that if \(x\) is algebraic but non-zero then \(\exp(x)\) is transcendental. It follows from this that π is transcendental, because \(\exp(2\pi i) = 1\) is algebraic. Likewise, the logarithm of a non-zero algebraic number is transcendental.

Leonhard Euler speculated that, if \(a\) and \(b\) are algebraic with \(a \neq 0\) or 1, and \(b\) irrational, then \(a^b\) should be transcendental: this includes \(2^{\sqrt{2}}\), \(\sqrt[3]{2}\), and \(\exp(i\pi) = (-1)^i\): proving this was number 7 on Hilbert’s famous 1900 list. It was affirmed by Gelfond and Schneider in the 1930s and generalised by Alan Baker in the 1960s.

How might one prove such results? Proofs depend on the interaction between the analytic and arithmetic properties of the exponential function – in particular, the fact that it has a power series with rational coefficients. We assume, for a contradiction, that there is an algebraic relationship, and then construct a quantity that (for analytic reasons) lies between 0 and 1, but is also an integer. This contradiction proves the result.

We might expect numbers and their exponentials to be as ‘algebraically independent’ as they can be, subject to the constraints imposed by the identity \(\exp(x + y) = \exp(x) \exp(y)\).

A comprehensive conjecture was formulated by Stephen Schanuel in the 1960s:

**Schanuel’s conjecture**: if \(x_1, \ldots, x_n\) are complex numbers that are linearly independent over the rationals, then at least \(n\) of the numbers \(x_1, \ldots, x_n, \exp(x_1), \ldots, \exp(x_n)\) are algebraically independent over the rationals.

Schanuel’s conjecture implies all known transcendence results and conjectures about the exponential function. For example, its truth would imply the unproved algebraic independence of \(e, \exp(e), \exp(\exp(e)), \ldots\) (and similarly for π). For \(n = 1\), Schanuel’s conjecture reduces to the Hermite–Lindemann theorem, but it is not known in general for \(n = 2\). On the positive side, Nesterenko proved in 1986 that \(\pi\) and \(\exp(\pi)\) are algebraically independent, using the analytic and arithmetic properties of certain modular functions studied by Ramanujan.

While progress towards Schanuel’s conjecture has been slow, it has been fruitful. First, the conjecture has been generalised by Grothendieck and others to a bigger picture including zeta function values, such as

\[
\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3}
\]

which was proved irrational by Apéry (1979) and is thought to be transcendental (but this is unknown): work of Oxford’s Francis Brown is connected with these ideas. Second, various analogues of the conjecture can be proved: in particular, it was proved for functions by Ax (1971). Third, the conjecture serves as a paradigm in model theory, guiding and inspiring much work, including new diophantine conjectures of Oxford’s Boris Zilber (2001) which give a far-reaching extension of a famous 1922 conjecture of Mordell (proved by Gerd Faltings in 1983) about rational points on curves.

Thus, Schanuel’s conjecture guides much current work, including much of my own in Oxford, which revolves around generalisations of Ax’s functional versions of Schanuel’s conjecture and applications to the diophantine conjectures of Zilber and others.

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Art in the Andrew Wiles building

Sam Howison

The relationship between mathematics and art has been debated for centuries. Some of its aspects are straightforward, such as proportion, perspective and projective geometry, but others are more allusive. The Andrew Wiles Building has provided our Department with a venue in which to make our own contribution.

The building itself does this. Many visitors have commented that the interior brings to mind M. C. Escher’s work, especially his impossible staircases: Roger Penrose’s recent BBC programme on this artist and his mathematical connections was partly filmed there. We followed it up with an Escher symposium featuring a showing of the film and talks about the underlying mathematics by Jon Chapman and Roger himself; you can watch it on our website.

As well as the staircases, we have the two crystals and the wonderfully decorative Penrose tiling; these all reinforce the message that this is both a building for mathematics and a work of art in its own right.

We have commissioned two major pieces for the building. Matt Chivers’ sculpture Axiom, which greets visitors in the entrance lobby, shown in Newsletter 14, is based on a triangulation of a shape generated by expanding bubbles; it explores a number of mathematical ideas, including the interesting interface between mathematical perfection and the imperfections of the real world.

On either side of the entrance bridge is Antoni Malinowski’s wall-based commission Spectral Flip, which probes the interplay of colour, surface textures and illumination.

The ‘permanent collection’ has been complemented with temporary exhibitions in the Mezzanine public space. The most recent of these was Illegitimate Objects, a group show in which artists made new work in response to the Department’s collection of 19th-century models of geometric surfaces. These models, most likely bought by J. J. Sylvester, Oxford’s Savilian Professor of Geometry at the time, were once at the cutting edge of research and teaching. Changing tastes in mathematics relegated most of them to the store cupboard, but they turned up in the move to the new building, and we now have a complete catalogue, with explanations of their purpose, which you can see on our website’s ‘History’ section.

Models of this kind have inspired many artists, including Man Ray, Henry Moore and Barbara Hepworth, and Illegitimate Objects was a fascinating addition to this tradition. The display cabinets were filled with models and sculptures, alongside a number of paintings and drawings, and there was also a poetry event featuring new work by a number of poets. It was a thought-provoking contribution to the art–mathematics debate, and we hope that it will be the first of many.

For more details of all our art, see www.maths.ox.ac.uk/about-us/art-and-oxford-mathematics

Sir Christopher Zeeman

Sir Christopher Zeeman FRS died in February, aged 91. Although most of his distinguished mathematical career was as Founding Professor of Mathematics at Warwick University, working in catastrophe theory and dynamical systems, he spent the years 1988 to 1996 in Oxford as Principal of Hertford College. He was also President of the London Mathematical Society and Vice-President of the Royal Society, but is best remembered in the wider world for his popularisation of mathematics, giving the Royal Institution Christmas lectures in 1978 and founding the Royal Institution Masterclasses for schoolchildren.
Appointments...

We are delighted to welcome the following new Faculty members.

**Alvaro Cartea (UCL)**: Lecturer in Mathematical Finance. Research interests: algorithmic and high-frequency trading, commodities and energy finance, mathematical finance and financial economics.

**Christopher Hollings** (Oxford): Lecturer in Mathematics and its History, and Senior Research Fellow at the Queen’s College. Research interests: development of modern algebra, Soviet mathematics, scientific communication during the Cold War.

**Harald Oberhauser** (UCL): Associate Professor in Probability and Tutorial Fellow at St Hugh’s College. Research interests: systems that evolve under the influence of randomness, and their applications.

**Damian Rössler** (Toulouse): Professor of Mathematics and Tutorial Fellow at Pembroke College. Research interests: algebraic geometry in positive characteristic, number theory, Arakelov theory.


**Melanie Rupflin** (Leipzig): Associate Professor in the Analysis of Nonlinear PDEs, and Tutorial Fellow at Trinity College. Research interests: geometric analysis, in particular the study of geometric flows, harmonic analysis and minimal surfaces.

Recognition...

This has been another good year for awards.

**London Mathematical Society prizes**

Six prizes were awarded to Oxford mathematicians by the London Mathematical Society:

- **Naylor prize and Lectureship in Applied Mathematics** to Jon Chapman. Professor of Mathematics and its Applications, for his contributions to modelling and methods development in applied mathematics.
- **Pólya Prize** to Boris Zilber. Professor of Mathematical Logic, for his contributions to model theory and its applications.
- **Whitehead prizes** to Peter Keevash. Professor of Mathematics, for his work in combinatorics, in particular his proof of the existence of combinatorial designs for all appropriate parameters; to James Maynard, Research Fellow, for his results on gaps between prime numbers, in particular answering a major unsolved conjecture of Paul Erdős; to Mason Porter, Professor of Nonlinear and Complex Systems, for his interdisciplinary contributions, and in particular to the emerging field of network science; and to Dominic Vella, Associate Professor of Applied Mathematics, for his contributions to the modelling of instability and interfacial phenomena in fluids and solids.

**Oxford awards**

Vicky Neale, Whitehead Lecturer, has won an Oxford University Student Union Teaching Award in the Most Acclaimed Lecturer category.

Jan Obloj has been appointed the title of Professor in the University’s annual Recognition of Distinction exercise.

Catherine Wilkins has won a Mathematical, Physical and Life Sciences Division award for teaching excellence.

Thomas Woolley and William Binzi were awarded runner-up prizes in the OxTALENT competition for their work in creating a video series of A Mathematician’s Holiday.

**Fellows of the Royal Society**

The following have been elected Fellows of the Royal Society:

- **Alison Etheridge**, Professor of Probability, has made significant contributions in the theory and applications of probability and in the links between them. Her particular areas of research have been in measure-valued processes (especially super processes and their generalisations), in theoretical population genetics and in mathematical ecology. Recently she has created a flexible framework for modelling biological populations which, for the first time, combines ecology and genetics in a tractable way, while introducing a novel and mathematically interesting class of stochastic processes.

- **Philip Maini**, Professor of Mathematical Biology, is renowned for his mathematical and computational modelling of spatiotemporal processes in biology and medicine, which has led to significant scientific advances in both. He has, for instance, developed multiscale models for wound healing and for vascular tumour growth, and thereby elucidated the underlying mechanisms by which particular growth factors reduce scar formation and provided detailed insight into the design of combination cancer therapy.

- **Jonathan Pila**, Reader in Mathematical Logic, has pioneered the study of the distribution of rational points on sets defined by real analytic conditions, leading (with collaborators) to a powerful theorem in the setting of o-minimal structures, a part of model theory. This theorem has found significant applications to central problems in diophantine geometry and has opened up a rich new connection between diophantine geometry and mathematical logic.

**Iain Smears** has shared the 2015 Leslie Fox Prize in Numerical Analysis (named after Oxford’s first professor of Numerical Analysis and founder of the Oxford University Computing Laboratory) by the Institute of Mathematics and its Applications; a second prize was awarded to **Patrick Farrell**.
External relations

Dyrol Lumbard

External relations may seem a one-way street – we tell the world what we do to promote Oxford Mathematics – but we’d rather characterise it as a mutual exchange: we tell you what we do, and you in turn tell us what you’re doing and what you’d like to hear about.

With that in mind we’ve developed our Alumni Stories feature where alumni give us a Q & A version of their lives since leaving Oxford. Please have a look at the stories so far and let us know yours (www.maths.ox.ac.uk/about-us/engagement/alumni-stories).

Equally, several of you have told us that you want more on our latest research, so that is both a feature of this annual Newsletter and of the electronic newsletters that we send throughout the year. If you’re not currently receiving them, please fill in the form on www.alumniweb.ox.ac.uk/secure/maths_email.

We’ve also launched a new Oxford Mathematics Alphabet on our Life in Oxford Mathematics webpage, which features an area of our research for each letter of the alphabet – beginning with Aperiodic tiles (by Sir Roger Penrose) and Bayesian inference (Robin Evans).

Public lectures

Our Oxford Mathematics Public Lectures series, aimed at sixth-formers and above, is now well established and plays to packed audiences. Amongst others over the past year we’ve had Martin Bridson talking on symmetry and undecidability, Roger Penrose on Escher, Étienne Ghys on the dancing skills of vortices, and Marcus du Sautoy on the logistical and mathematical challenges that Santa Claus faces every Christmas Eve. If you want an enjoyable late afternoon out, please come along – and if you can’t make it, these lectures are all online on our events pages.

Working with schoolchildren

Rebecca Cotton-Barratt

The Andrew Wiles Building is proving to be an inspiring place for mathematical events for schoolchildren. We’ve hosted Royal Institution Masterclasses on Saturdays for local teenagers who are so keen that they keep coming back year after year, events for the Further Mathematics Support Programme giving more students access to Further Maths at A Level, and events for the UK Mathematics Trust ranging from team maths challenges for 13-year-olds from Oxfordshire and nearby to summer schools for 16-year-olds invited from across the country. In addition we hold the Maths and Maths & Stats UNIQ summer schools as part of Oxford’s work on widening participation. Our days to promote mathematics to schoolgirls last April (see following article) proved so popular that we ran them again in January for several hundred more students.

We now look forward to the return of PROMYS Europe next summer, with a select group of older teenagers from across Europe immersing themselves in number theory for six weeks. We’re delighted that we can share our building with over 3000 school students a year, as we spread the word about the wonder of mathematics and Oxford’s place in its study.

Women in Maths meeting

Frances Kirwan

In 2015, the London Mathematical Society’s annual ‘Women in Maths day’ was expanded to four days and hosted in Oxford as part of the LMS 150th anniversary celebrations, with two days for school students and two days for university mathematicians and mathematical scientists.

The meeting was a great success, with about 430 participants for the first two days, and 250 for the last two. As the LMS President wrote, ‘the event, which is being talked about far and wide, has been one of the highlights of the Society’s 150th Anniversary year so far, and as a result of its success the Women in Mathematics Committee is hoping to be able to further develop its work with school students and undergraduates’.

A short video of the meeting, produced by Mareli Augustyn, can be viewed on www.maths.ox.ac.uk/events/conferences/past-events/women-maths, together with photographs by Jennifer Balakrishnan, a photo collage created by participants to show the present and future of UK women in mathematics, and a link to the programme.
Alumni events

Last year’s Mathematical Institute Garden Party was held in the Andrew Wiles Building during the University’s Alumni weekend. The speaker was Ben Green FRS, Waynflete Professor of Pure Mathematics, on ‘Problems about points and lines’.

This year’s Garden Party will take place on 17 September in the Andrew Wiles Building. You can book online at www.maths.ox.ac.uk/about-us/alumni/alumni-garden-party. Professor Jon Chapman, Director of OCIAM, will give a lecture on ‘The mathematics of M. C. Escher’.

Ida Busbridge

A biography of Dr Ida Busbridge, who taught mathematics in Oxford from 1935 to 1970, has been written by her former student E. Clare Friedman. A popular tutor and lecturer, ‘Buz’ was appointed Fellow of St Hugh’s College in 1946, the first woman to be appointed to a college fellowship in mathematics. In 1962 she was awarded a Doctor of Science degree for her work in integral equations and radiative transfer. Her booklet on Oxford Mathematics and Mathematicians can be accessed online at www.maths.ox.ac.uk/about-us/history/busbridge-lecture.

Fellows of the Royal Society

Alison Etheridge’s election to a Fellowship of the Royal Society (see page 6) brings to six the total number of women mathematicians who have ever been elected FRS. Three of the six are currently working in Oxford, the other two being Frances Kirwan (left) and Ulrike Tillmann (right). Two of the others, Mary Cartwright and Mary Rees, were formerly students at Oxford.

Andrew Wiles portrait

A recently commissioned portrait by Rupert Alexander of Sir Andrew Wiles FRS, has been unveiled at the National Portrait Gallery. It can be viewed at www.maths.ox.ac.uk/node/14675

Undergraduate summer research projects

In the summer vacation of 2015, over 25 undergraduates took part in summer research projects supervised by members of the Oxford Mathematical Institute. Some of the students described their work in posters currently on display in the Mezzanine of the Andrew Wiles Building. At the moment, we are helping students who want to do projects in 2016 to match with willing supervisors, and are seeking funding to try to enable all those who want to do a summer project to be able to get a taste of mathematical research, with the aim of inspiring the next generation of research students.

Cheers...

At a meeting in Edinburgh on The Science of Beauty at which all three were guest speakers, Sir Michael Atiyah (left) and Sir Roger Penrose (right) enjoyed a relaxing drink with Sir David Attenborough.