Unspeakable cryptography

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Speakable information

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But there is a hidden assumption: parties share a reference frame.

More generally, you cannot transmit *resources*, like charge.

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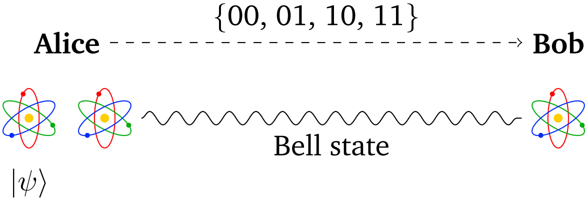
Given a classical channel to an alien species, how do you *define* 'left' and 'right'?

Unspeakable quantum teleportation



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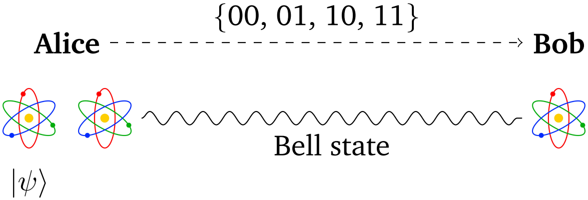

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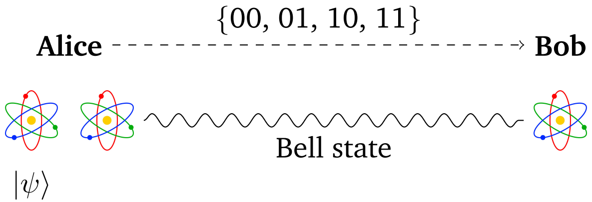

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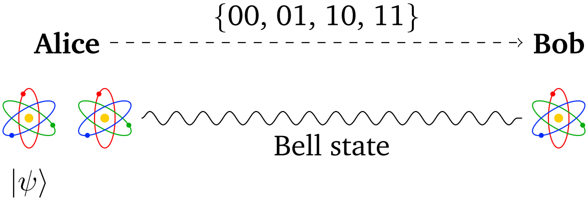

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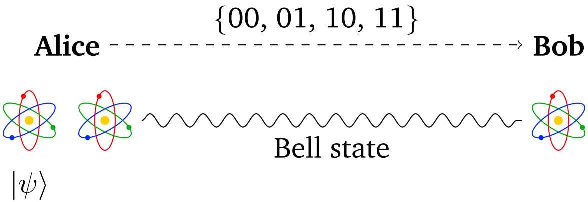

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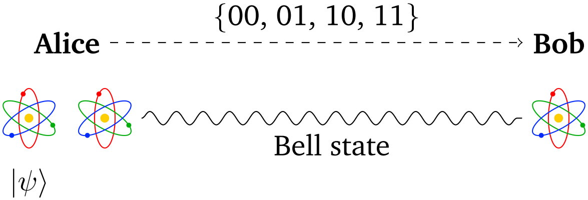

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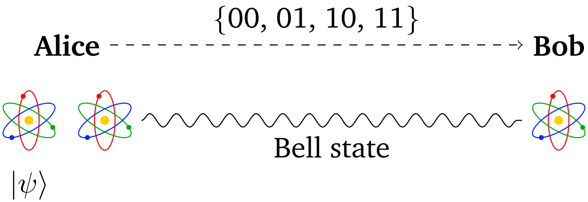

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- Success means that Bob's system is now in state $|\psi\rangle$.

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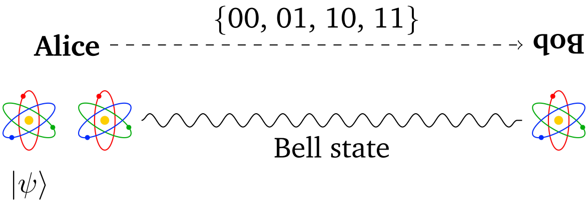

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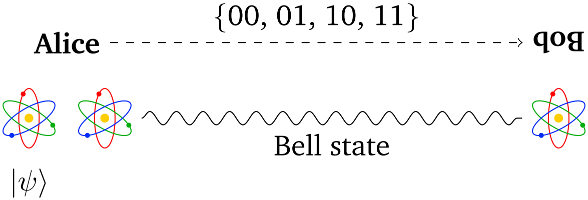

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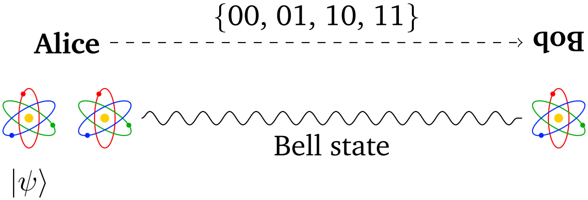

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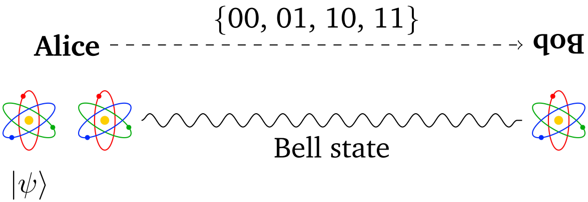

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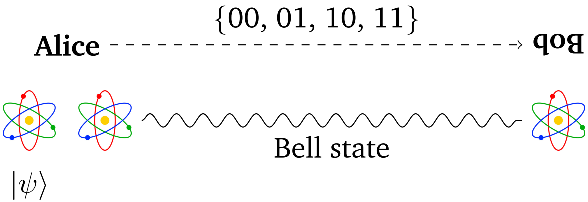

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This *breaks* quantum teleportation: "unspeakable information cannot be teleported".

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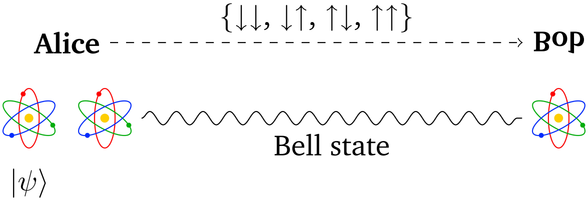

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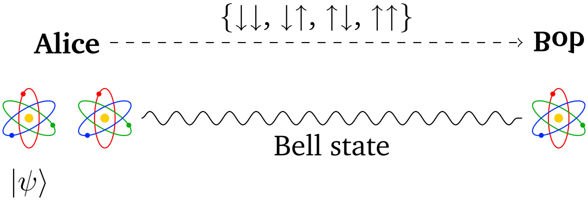

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Alice' result	**Bob's action**	**Bop's action**
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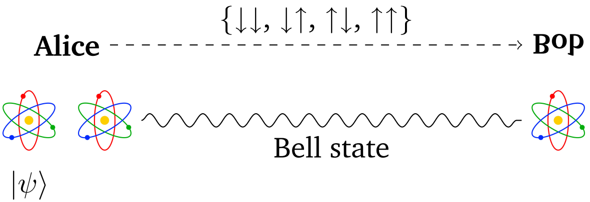

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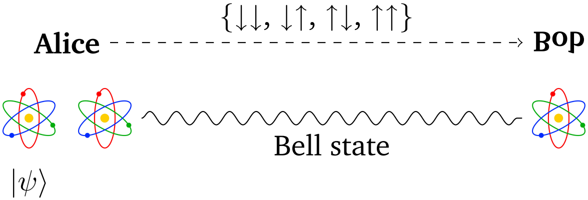

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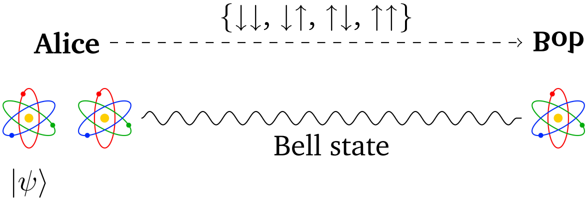

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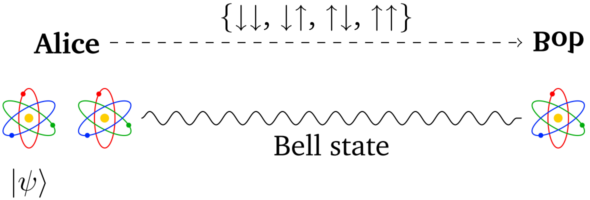

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If columns 2 and 3 are identical, teleportation always succeeds.

Unspeakable quantum teleportation

Here is a solution:

$$U_{\downarrow\downarrow} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \qquad U_{\downarrow\uparrow} = \frac{1}{4}\begin{pmatrix} -\sqrt{2}-\sqrt{6} & -\sqrt{2}+\sqrt{6} \\ -\sqrt{2}+\sqrt{6} & \sqrt{2}+\sqrt{6} \end{pmatrix}$$

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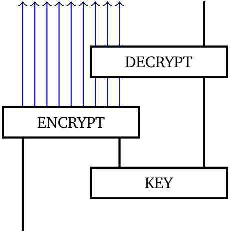

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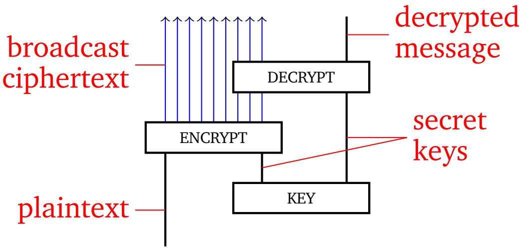

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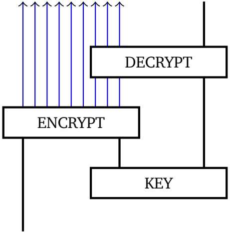

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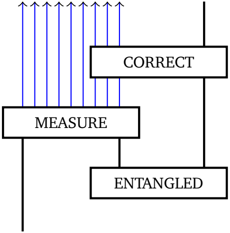

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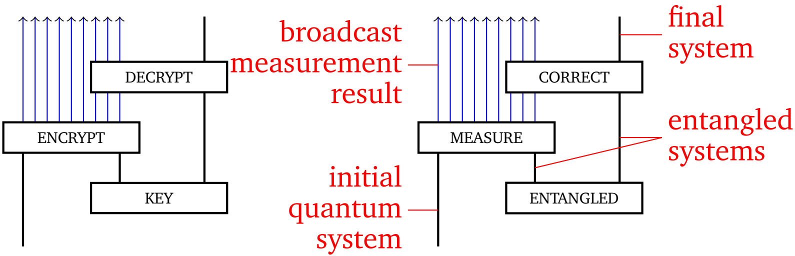

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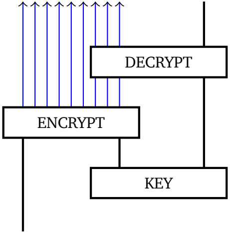

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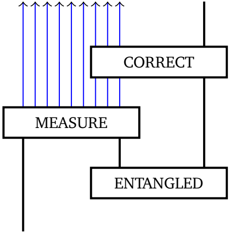

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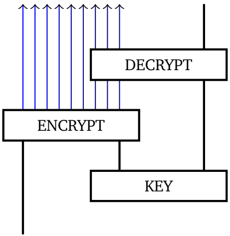

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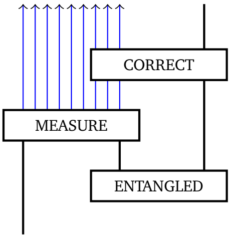

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This lets us translate ideas between the two settings.

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Key and ciphertext can’t *both* be speakable or unspeakable.

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