



UNIVERSITY OF  
**OXFORD**

# Oxford Centre for Nonlinear PDE

## Lectures in PDE

Mary Oglivie Lecture Theatre, St Anne's College  
Thursday 5th June 2008

**EPSRC**

Engineering and Physical Sciences  
Research Council

**OXFORD CENTRE FOR NONLINEAR PDE  
LECTURES IN PDE**

**St Anne's College, Oxford  
5 June 2008**

**PROGRAMME**

- 9:30-10:30: Alfio Quarteroni (EPFL)  
Heterogeneous Domain Decomposition  
Methods
- 10:30-11:00 Tea, Coffee & Biscuits*
- 11:00-12:00 Laure Saint-Raymond (Paris VI & ENS)  
Weak compactness methods for singular  
penalization problems with boundary layers
- 12:00-13:00 Bryce McLeod (Oxford)  
A problem in dislocation theory
- 13:00-14:00 Buffet Lunch*
- 14:00-15:00 Tom Bridges (Surrey)  
Degenerate conservation laws, bifurcation of  
solitary waves and the concept of criticality  
in fluid mechanics
- 15:00-15:30 Tea, Coffee & Biscuits*
- 15:30-16:30 Neshan Wickramasekera (Cambridge)  
Frequency functions and singular set bounds  
for branched minimal graphs

**Alfio Quarteroni (EPFL)**

**“Heterogeneous Domain Decomposition Methods”**

Classical mathematical models are used since several decades to describe complex phenomena in continuum mechanics, with applications in many diverse fields of engineering and technology. When complexity becomes a concern, recent developments in domain decomposition, particularly heterogeneous domain decomposition (HDD) and multiscale geometrical modeling (MGM), have opened the way to progress in modeling such complex systems as the human circulatory system and environmental systems. In this presentation I will illustrate the main motivation behind HDD and MGM, review the different mathematical approaches that can be set up to face these problems, comment on the associated algorithmic aspects, and show the numerical results that can be obtained using this approach.

**Laure Saint-Raymond (Paris VI & ENS)**

**“Weak compactness methods for singular penalization problems with boundary layers”**

We are interested in large scale oceanic motions forced by some wind at the surface, and braked by solid friction at the bottom. As the effect of the Coriolis force is dominant, this is a typical singular penalization problem set in a bounded domain, which has been already studied by filtering methods. We propose here an alternative “weak compactness” method, based both on compensated compactness arguments and on some 2-scale analysis. It only characterizes the limiting mean motion, but requires much less explicit computations and thus should be extended to more complex models.

**Bryce McLeod (Oxford)**

**“A problem in dislocation theory”**

A problem in dislocation theory leads to the solution of an infinite set of nonlinear algebraic equations. We discuss how to obtain a solution and look at the associated asymptotics.

Tom Bridges (Surrey)

**“Degenerate conservation laws, bifurcation of solitary waves and the concept of criticality in fluid mechanics”**

The relation between three seemingly different topics will be discussed. First a new mechanism for the bifurcation of homoclinic orbits will be discussed. It is based on degeneracy of relative equilibria (RE) in the Hamiltonian context, and uses novel geometric properties  $DU_{xx}$  of the momentum map. The more exotic the (RE) the more exotic the bifurcating homoclinic orbit (TJB, J Diff Eqns, 2008). These homoclinic orbits are of interest as solitary waves in the PDE setting.

The second observation is both an application and a new interpretation of criticality in fluid mechanics. Criticality is most familiar in the shallow water context where Froude number unity separates supercritical from subcritical flow, and is associated with the creation of the KdV solitary wave. Criticality can be re-interpreted as degeneracy of relative equilibria. The classical theory of criticality is recovered, but more importantly the concept of criticality is generalised and it shows how to find new classes of solitary waves. Two examples are the bifurcation of dark solitary waves from a critical Stokes wave, and a homoclinic bifurcation related to the breaking of large amplitude water waves. This is joint work with Neil Donaldson (TJB & NMD, J Fluid Mech, 2006). A related context for both criticality and bifurcation of waves is degenerate conservation laws. Regularised conservation laws of the following type are of interest,

$$U_t + F(U)_x = DU_{xx} \text{ or } U_t + F(U)_x = DU_{xxx}, \quad (1)$$

where  $U \in \mathbb{R}^n$ ,  $F: \mathbb{R}^n \rightarrow \mathbb{R}^n$  and  $D$  is an  $n \times n$  matrix. A conservation law is *degenerate* if  $DF(U_0)$ , the Jacobian, is singular for some  $U_0 \in \mathbb{R}^n$ . In applications the flux vector is often of the type  $F = M \nabla E(U)$  for an *indefinite* symmetric matrix  $M$  and function  $E: \mathbb{R}^n \rightarrow \mathbb{R}$ , with  $M^{-1}D$  symmetric. In this case, the steady systems of (1) are gradient and Hamiltonian respectively, and in the latter case, the flux vector can be characterised as a momentum map.

What can one say about the *dynamics* near degeneracy of (1)? It is very similar to criticality, but here one wants to retain both space and time. As a first step, in the case of simple degeneracy, one can derive a Burger's equation or a KdV equation that locally describes the dynamics near the direction of the eigenvector of  $DF(U_0)$ . In the case of dissipative regularisation, one can prove that the Burger's dynamics

is valid. The proof of validity in the parabolic case is joint work with Sergey Zelik (TJB & SZ, in preparation). Higher order degeneracies, which occur in models for stratified flow, lead to coupled Burger's equations or coupled KdV equations.

**Neshan Wickramasekera (Cambridge)**

**"Frequency functions and singular set bounds for branched minimal graphs."**

We will discuss bounds on the size of the singular sets for various classes of minimal submanifolds, focusing on some recent joint work with Leon Simon on a PDE method for estimating the branch sets of minimal immersions.

# OXFORD CENTRE FOR NONLINEAR PDE

## NEWS

### VACANCIES

#### **Professorship in the Analysis of Partial Differential Equations**

Applications are invited for the above post, tenable from 01 October 2008, or as soon as possible thereafter. The professorship will be held in the Mathematical Institute, in association with a non-stipendiary professorial fellowship at Keble College.

The university is seeking a world-class mathematician who, through leadership and distinction of his or her contribution to the field, will ensure the continuing pursuit of excellence in research and teaching in partial differential equations. The person appointed will be expected to play a leading role in the development of the newly-formed Oxford Centre for Nonlinear PDE (OxPDE).

Further particulars, including details of how to apply, are available from <http://www.admin.ox.ac.uk/fp/> or from the Registrar, University Offices, Wellington Square, Oxford OX1 2JD, UK (tel: +44(0)1865 270200). The closing date for applications is Monday 23 June 2008.

Further information may be obtained on a strictly confidential basis from Professor Sir John Ball ([ball@maths.ox.ac.uk](mailto:ball@maths.ox.ac.uk)).

#### **4-year Postgraduate Studentship in nonlinear PDE — University of Oxford**

Applications are invited for a 4-year postgraduate studentship based at the new Oxford Centre for Nonlinear Partial Differential Equations, which opened on 1 October 2007. Major funding for new faculty appointments, research fellows, research students and visitors is provided through an EPSRC Science and Innovation award. The Centre, which is directed by Professor Sir John Ball FRS, aims to create a stimulating research environment with the critical mass to make major research progress and provide leadership in the area of non-linear PDEs within the UK. The person appointed will have a unique opportunity to benefit from this initiative. Applicants may apply for one or more of the following projects:

- Micromagnetics and domain wall dynamics
- Polarisation tensor for composite materials and applications to inverse problems
- Viscous incompressible Newtonian and Non-Newtonian fluids
- Uniqueness of stationary black hole space-times

The studentship is available from 1 October 2008 for four years. The studentship is available to applicants of any nationality, but only includes

the University and College fees charged to EU citizens. It does not cover the higher level of fees charged to students who are not EU citizens (the current difference being approximately £7,500pa). If appointed, a non-EU citizen may apply to outside sources of funds to cover the higher fees. Success in such applications cannot be guaranteed. The maintenance stipend is £12,600 this academic year.

The closing date for applications is Monday 16th June 2008; late applications may be considered at the discretion of the selection committee.

Further particulars can be found at:

<http://www2.maths.ox.ac.uk/oxpde/news/vacancies.php>

## **EVENTS**

### **Workshop on the mathematics of weather and climate prediction**

Met Office, Exeter, 30 March – 3 April 2009

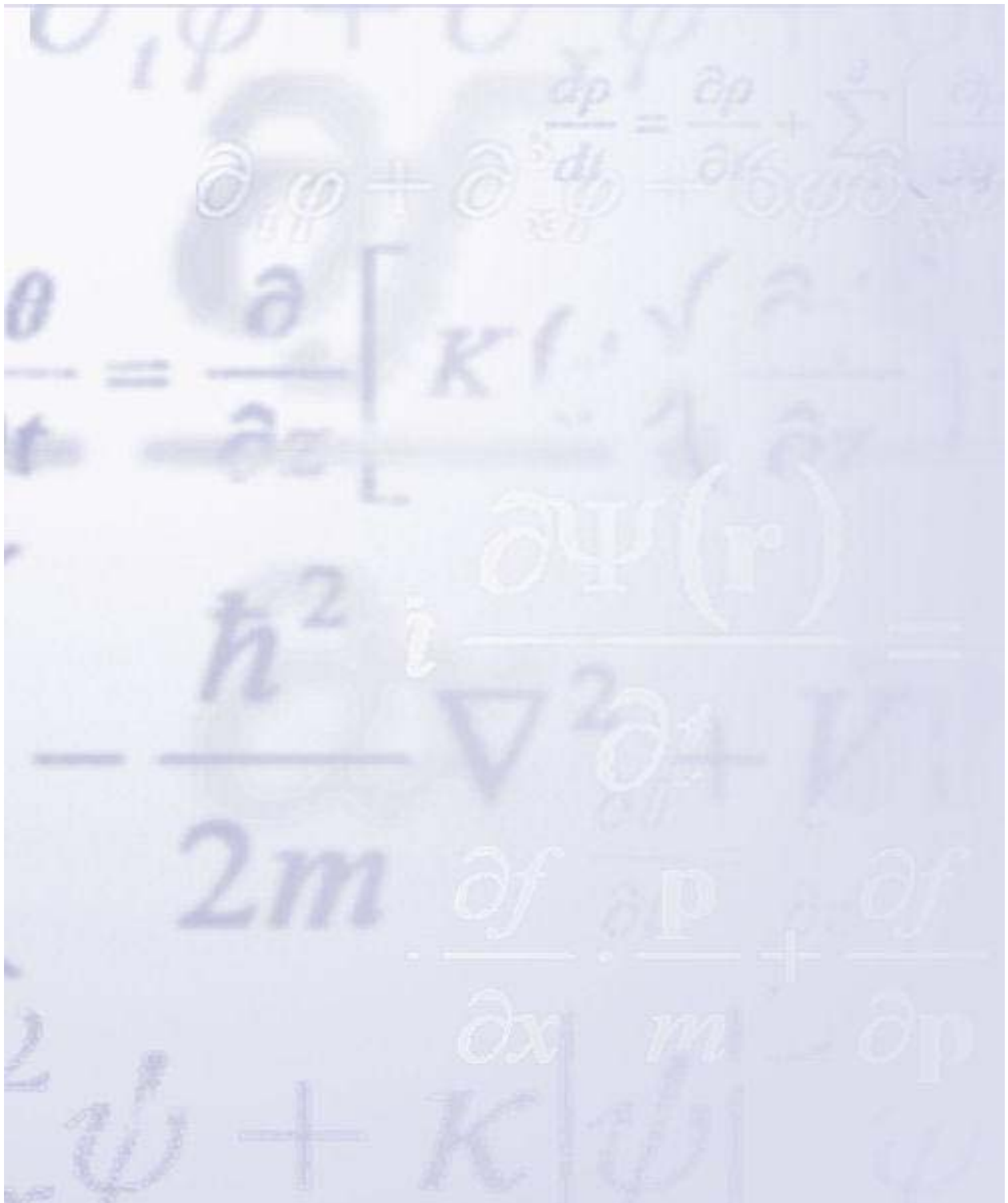
*Organisers: John Ball (Oxford), Mike Cullen (Met Office), Sergei Kuksin (Edinburgh)*

Registration and further details coming soon

## **WEBSITE**

visit our website for further information:

[www2.maths.ox.ac.uk/oxpde/](http://www2.maths.ox.ac.uk/oxpde/)



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