

UK NONLINEAR PDE CONSULTATION MEETING

LMS, De Morgan House, 57-58 Russell Square,
London WC1B 4HS
29 May 2007

MEETING REPORT

EPSRC has recently invested significant resources in the analysis of nonlinear PDEs in the UK by making two Science and Innovation awards in this area to the University of Oxford and to the Maxwell Institute, Edinburgh (comprising the mathematics departments of Edinburgh and Heriot-Watt universities). In addition to new posts in the three institutions, there will be a large programme of activities to benefit the entire UK nonlinear PDE community. Included in this programme are international conferences, workshops, instructional conferences/summer schools, activity groups/networks, and a visitor programme. Following this round of EPSRC Science and Innovation awards the University of Bath was also asked to put together a responsive mode bid for a UK network in 'Nonlinear PDEs and related fields'.

The consultation meeting was held with the aim of obtaining genuine expressions of interest, suggestions and feedback with respect to the proposed events programme within each centre and the establishment of a nonlinear PDEs network. The meeting attracted around 40 participants from the nonlinear PDE community. The meeting was hosted by LMS, whom we would like to acknowledge for the financial and administrative support they provided.

The directors of the two centres, John Ball (University of Oxford), Tony Carbery, Sergei Kuksin (Maxwell Institute) and the Meeting Chair, John Toland (University of Bath) welcomed participants to the meeting there followed an opening presentation on the background of the Science and Innovation awards given by Caroline Batchelor from EPSRC. Each of the centre directors and the meeting chair gave presentations on their respective proposals. Several responses were received from the community in response to an open call for ideas, suggestions and expressions of interest, under the headings:

- international conferences;
- workshops;
- instructional conferences/summer schools;
- activity groups/networks;
- visitor programme.

A summary of these responses was presented at the meeting and was followed by a general discussion.

Science and Innovation awards

Caroline Batchelor from EPSRC gave a presentation on the Science and Innovation Awards explaining that the purpose of these awards is to secure strategically important

research areas that are missing or 'at risk' in the UK. This funding is outside of the existing schemes EPSRC operates. PDEs were highlighted as a real concern in the UK in the 2004 International Review (<http://www.cms.ac.uk/irm/irm.pdf>) which involved wide scale community consultation to discover which areas were in need of development. In order to achieve the strategic aim of the awards EPSRC identified key requirements to accompany the awards in mathematics. These were:

- significant commitment from the universities to build a strong framework and ensure sustainable growth;
- improving the interface between pure and applied mathematics to increase the impact of the awards.

The two centres awarded funding were in direct competition with disciplines outside of mathematics which is a huge achievement for the nonlinear PDEs community. If the awards are successful in achieving their aims the development of a nonlinear PDE community in the UK could lead to further collaboration and enhanced links with the European, and in particular the French, community.

Maxwell Institute Award

Tony Carbery, one of the directors of the Maxwell Institute Centre for Analysis and Nonlinear Partial Differential Equations, followed Caroline's presentation with an overview of the Edinburgh award. Their proposal included funding to cover various activities including:

- 2 large international conferences (100-120 participants), one of which will be held by the Maxwell Institute in June 2008;
- 5 instructional conferences/workshops at the research level, one of which will be in conjunction with the conference being held in June 2008;
- a visitor programme of approximately 20 x 1-2 week lecture tours involving 2-3 centres and 6 x one-month visits involving a lecture series in collaboration with a piece of research.

ICMS has good resources to assist with the infrastructure and support of the activities described and will take on the organisation of some of these events. An open call was given before the meeting for suggestions and expressions of interest on activities and events to spread the benefit of the awards to the whole of the UK. There is some flexibility in the design of the events and the Maxwell Institute are happy to receive further expressions of interest from the community, as well as suggestions for the event themes. The Edinburgh and Oxford centres will work closely in reviewing and taking forward proposals to avoid duplication and increase coverage. The Maxwell Institute's proposal also covered training for postgraduates and undergraduates, the intention being to use the EPSRC taught course centres as well as local PhD courses and EPSRC/LMS conferences. In addition, studentships have been earmarked for nonlinear PDEs. An area of concern highlighted in the Edinburgh proposal was undergraduate training; some headway needs to be made in the undergraduate curriculum and events to entice new students to the research area should also be held. The Edinburgh proposal did not include funding for networking items. Management of the Edinburgh Centre will be an evolutionary process; initially a management committee will be setup, consisting of 2 co-

directors, 2 heads of department and 2 academics. In addition to this a steering committee will be formed including 4-5 international experts; the steering committee will review the scientific activities as well as provide Quality Assurance on the events and activities. The administration of the Centre will be handled by Laura Darling and others at the Maxwell Institute.

University of Oxford award

John Ball, director of the Centre for Nonlinear Partial Differential Equations presented the Oxford proposal which resonates strongly with the Edinburgh one. Oxford has been awarded £2.78m over 6 years from the EPSRC and HEFCE, which has been matched with £2.5m from the University of Oxford. The grant will cover lecturer and postdoctoral posts as well as studentships in nonlinear PDEs, a visitor programme and a programme of nonlinear PDE events in the UK. Oxford identified several UK events for funding including:

- a major conference to be held in Oxford during year four of the award;
- annual two-day general PDE conferences followed by one-day techniques sessions;
- annual two-day thematic workshops followed by one-day techniques sessions;
- programmes at the INI and ICMS;
- mini-symposia at BMC and BAMC;
- graduate summer schools for PhD students;
- events for postgraduate students and final or penultimate year undergraduates.

In collaboration with Edinburgh, Oxford gave an open call for suggestions and proposals before this meeting and is open to receiving further proposals and suggestions in relation to events and visitors. Oxford is making six new appointments due to commence in October 2007. Bryce McLeod will join the centre in an emeritus post. In addition to these posts a chair in the Analysis of nonlinear PDE, a five-year Titchmarsh research fellowship and 2 three-year research fellowships will be appointed. In their proposal Oxford also suggested the establishment of a national steering committee for nonlinear PDE.

UK Network Proposal - 'Nonlinear PDEs and related fields'

After the third round of Science and Innovation awards in December 2006 during which Oxford and Edinburgh were awarded funding, EPSRC asked the University of Bath to work on a Network proposal for the UK in 'Nonlinear PDEs and related fields' for a responsive mode bid. The meeting chair, John Toland asked for feedback on this idea. It is proposed that the network would support PDE activity in the UK and provide a mechanism for bridging the gap between analysis and applied research. Caroline Batchelor from EPSRC supported the creation of such a network adding that the goals of the network should be to bring the community together and add momentum to the work of the centres. The importance of keeping the applications of the network in clear sight and providing outreach from pure analysts to applied areas was raised by Brian Davies, who is in strong support of a broad and encompassing network.

Expressions of Interest

In response to the open call for suggestions and expressions of interests for events in the area of nonlinear PDEs in the UK eleven proposals were received from the community prior to this meeting. A summary of these proposals was presented by Endre Suli (University of Oxford).

Proposals:

1. Two International Workshops & Visitor Programmes

a. Functional & Geometric Inequalities and Nonlinear PDE:

Connections to optimal transportation methods, and applications to nonlinear PDE, incl. nonlinear diffusion eq's, Monge-Kantorovich minimization problems, the Boltzmann equation, and other PDE of mathematical physics.

b. Nonlinear Spectral Problems:

Stability of Stationary States & Applications. "A recent change in modern nonlinear analysis is that the most interesting problems in nonlinear PDE, esp. those arising in engineering applications, often lead to entirely new or difficult unsolved linear problems."

Ari Laptev (ICL), E. Brian Davies, Eugene Shargorodsky (KCL), Marco Marletta, Michael Levitin (Cardiff)

2. Network on Integrable Nonlinear PDE

Aim: Link main UK centres [Cambridge, Glasgow, Edinburgh, Kent, Leeds, Loughborough, Manchester, Oxford, Reading]; co-ordinate initiatives, establish common visitor programme.

Network would also include:

- i. short graduate courses that any international visitor under the programme would be required to contribute to;
- ii. short meetings (e.g., 1-2 days) + workshops (1 week).

Peter Clarkson (Kent), Alexander Mikhailov (Leeds), Alexander Veselov (Loughborough), Beatrice Pelloni (Reading)

3. Conference, Workshops, Network & Visitor Programme on Nonlinear PDE

- i. Medium-scale international conference (Possibly: Free-boundary-value problems, in 2011);
- ii. Workshops (approx. 1/year): on nonlinear PDE, planned for 2008;
- iii. Two-week instructional conferences
- iv. Activity networks:
 - Oxford-Cambridge network on nonlinear PDE and relativity;
 - Bath-Cambridge-Edinburgh network on geometric integration;
- v. Visitor Programme:

Topics: Bose-Einstein condensation; New trends in Hamilton-Jacobi eqs.; Homogenisation of deterministic and stochastic transport eqs.; Spectral & pseudospectral methods for highly oscillatory PDE.
Mihalis Dafermos, Thanassis Fokas, Arieh Iserles, Peter Markowich, Benjamin Schlein, David Stuart (Cambridge)

4. Nonlinear PDE and General Relativity

Objective: Programme on Nonlinear PDEs in General Relativity within S&I award:

- i. Workshop series: annual 1-week workshops (10-30 participants), held alternately in Oxford and Cambridge.
- ii. Visitor programme: on the scale of three 1-month visitors/year in Oxford in nonlinear PDE in General Relativity (~ 1/term).

Piotr Chrusciel and Paul Tod (Oxford)

5. Geometric Nonlinear PDEs & Minicourses on Nonlinear PDE:

Objective: build up UK nonlinear PDE in general, and broaden the reach of the local activity on geometric nonlinear PDE via the S&I award, by involving UK participants in particular.

Minicourses:

Objective: Small number of minicourses (approx. 3-4 lectures each), with international speakers, in areas which are of general interest and are not covered currently in the UK.

Audience: 50% local and 50% other UK, from Ph.D. student to (non-expert) faculty level.

Prerequisites: basics of modern PDE theory.

Peter Topping (Warwick)

6. Nonlinear PDE in Mathematical Biology & Nonlinear Kinetic PDE with Dispersive Effects

- i. **Nonlinear PDE in Mathematical Biology**
3-day instructional conference followed by 3-day conference.
Emphasis: *New* challenges in the analysis of nonlinear PDE in mathematical biology, incl. interaction with modelling.
- ii. **Nonlinear kinetic PDE with dispersive effects**
3-day conference.
Emphasis: Nonlinear kinetic PDE arise in a wide variety of applications: relativity, cosmology, plasma physics, Boltzmann's kinetic theory of gases (physics), chemotaxis, biomotors, adaptive evolution (biology). Relevant mathematical techniques: PDE theory, harmonic analysis, nonlinear analysis and geometry

Nikolaos Bournaveas (Edinburgh)

7. Numerical Approximation of Stochastic PDE

- i. One-week instructional conference + one-week research conference.

- (a) Aim of the instructional conference: Provide intro. to SPDE and numerical algorithms for PDE.
- (b) Aim of the research conference: Disseminate analytical results on numerical approx. of SPDE.

ii. Subsequent years: 3 short workshops with different emphases.

István Gyöngy (Edinburgh)

8. One-day Workshop on Pattern Formation in Reaction-Diffusion Systems with Biological Applications

Time: 2008; mainly UK speakers plus some from overseas.

Matthias Winter (Brunel)

9. Workshop on Nonlinear PDE in Applications

Objectives:

- Bring together pure and applied areas of mathematics, as well as modeling concerned with nonlinear PDE.
- Attract analysts interested in seeing their work being applied & the applied-end researchers seeking new insights from analysis.

Analysis joining with numerical computations is included, but computation without analysis is not.

Main applications: Biomedical, industrial and environmental areas.

Frank Smith & Dima Vassiliev (UCL)

10. Meteorology, Oceanography & Nonlinear PDE

Objectives:

- Bridge gap between nonlinear PDE theory and Meteorology;
- Develop understanding of benefits mathematics can bring;
- Develop effective communication of the problems between mathematically literate meteorologists & mathematicians.

Means to achieve these:

- Targeted research workshops: communication of relevant results already achieved + statement of problems where a rigorous solution would be helpful;
- Collaborative research: on physically relevant problems.

Mike Cullen (Met. Office, Exeter)

11. Weekend Meeting for Final-year Undergraduates:

Introduction to Nonlinear PDE

Justification: Low density of researchers in analysis of nonlinear PDE in the UK is largely due to small number of UK undergrads who choose to embark on a Ph.D. in analysis.

Aim: Review modern PDE theory + UK research opportunities

Structure: 4 or 5 talks on varied aspects of the analysis of nonlinear PDEs + extra talk on the realities of life as a Ph.D. student and mechanisms for funding.

James Robinson (Warwick)

Open discussion on the S&I awards, the Centres and the proposed network

Various questions were raised by the meeting participants on a number of issues relating to the awards, the scope of the awards and raising the profile of research in nonlinear PDE within the UK. Below is a short summary of some of the discussion.

There is little leverage for changing undergraduate curricula across the UK to include material necessary for progression into the area of nonlinear PDE; however in institutions where curricula can be influenced this could lead to an increase in the number of students from those institutions going on to PhD positions in the area of nonlinear PDE throughout the UK. One way to encourage students with an aptitude or interest in nonlinear PDE research to stay within the UK is to develop a strong infrastructure and expand the opportunities available in nonlinear PDE. The studentships funded by the two centres will go some way to achieving the overall aims of establishing an increased interest in nonlinear PDE research, and although the centres will not be able to use their funding for studentships in other institutions, there could be potential in funding graduate attendance at events and conferences. As well as teaching new skills to graduates, events will raise the profile of this research area. Identifying the ‘Grand Challenges’ in PDE may be a consideration in attracting new graduates.

The Science and Innovation awards aim to improve research capacity in the study of nonlinear PDE, and bridging the gap between pure and applied mathematics could be a key to achieving this aim. There is much discussion within the community around identifying problems that are central to the analysis of nonlinear PDEs which also couple to the needs and interests of the applied community. The establishment of a UK network in nonlinear PDE, including a national steering committee, either as an integrable part of, or in addition to, the network could address some of these issues. It could also provide support to smaller communities that would benefit from the establishment of an inclusive network in nonlinear PDE, such as those within pure integrable systems. Other solutions could include industry days, conferences either driven by theory or application, with involvement from both communities, other activities which highlight the link between PDE and other disciplines. Clearer collaboration between the two communities could result in increased sponsorship and funding for students. Further forums for discussions of this type could also be the Royal Society, the British Mathematics Colloquium, and the British Applied Mathematics Colloquium.

The events and visitor programmes included in the awards will need close liaison between the centres to be successful. It will be important to ensure that there is even coverage of themes as well as regions. In order to tackle this issue a calendar of UK nonlinear PDE events and a calendar of nonlinear PDE visitors to the UK will be developed and maintained by Oxford. Oxford envisages that departments and groups

throughout the UK will be able to log in to these calendars and enter information as they wish. The calendars will be publicly viewable. In identifying appropriate themes the gaps within UK nonlinear PDE need to be considered; there are arguments to the degree of specialization of the content of courses and workshops, and the likely outcome will be a mixture of both broad and specialized events. The centres aim to build capacity in the whole UK nonlinear PDE community and not just within the institutions hosting the centres. The responses for expressions of interest on the events and visitor programme were considered valuable and the call was deliberately vague to allow the community to make suggestions freely. All of those who submitted proposals will receive feedback from Edinburgh or Oxford and the centres continue to invite suggestions, ideas and proposals from the community.