MAT Syllabus Practice

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The Maths Admissions Test (MAT) has a published syllabus[1], which contains maths that you should know by the time you sit the test. People often ask me how they can check that they know the maths on the syllabus, so I've put together these practice questions; if you can do these, you've learnt enough maths to start doing MAT questions. If some of them don't make sense to you, ask a teacher for help.

Polynomials

- Solve $x^2 - x - 1 = 0$
- Solve $x^4 - x^2 - 1 = 0$
- Write $x^2 + 4x + 3$ in the form $(x + a)^2 + b$
- How many real solutions does $x^2 + bx + 1 = 0$ have? Find the different cases in terms of b.
- Factorise $x^2 + 4x + 3$
- Let $p(x) = x^3 - 13x^2 - 65x - 51$. Check that $p(17) = 0$. Factorise $p(x)$.

Algebra

- Solve the simultaneous equations $x + y = 1$ and $x - y = 3$.
- For which values of x is it true that $x^2 + 4x + 3 > 0$?
- Expand $(2x + 3)^3$
- I've got four playing cards; the ace and king of clubs, and the ace and king of hearts. I shuffle the cards together and deal them out left to right. What's the probability that the kings and aces alternate? (they alternate if they are either arranged as $AKAK$ or $KAKA$)

Differentiation

- Differentiate x^{17} with respect to x.
- Differentiate $\sqrt{x}$ with respect to x.
- Differentiate e^{3x} with respect to x.
- Differentiate $2e^{-x} - x^2$ with respect to x.
- Find the tangent to the curve $y = e^x + 1$ at $x = 2$.
- Find the normal to the parabola $y = x^2$ at $x = 3$.
- Find the turning points of the curve $y = x^4 - 2x^3 + x^2$. Identify whether the turning points are maxima or minima.
- For which values of x is $y = x^4 - 2x^3 + x^2$ increasing? For which values of x is it decreasing?
- Two points A and B are on the curve $y = x^3 + x^2 + x + 1$. A is held fixed at $(1, 4)$. The point B is moved along the curve towards A. What happens to the line through A and B?

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[1] See www.maths.ox.ac.uk/r/mat for the syllabus, as well as past papers and solutions

Integration

- Suppose that the derivative of a polynomial $p(x)$ with respect to x is $q(x)$. Find $\int q(x)\,\mathrm{d}x$.
- Find the area enclosed by the polynomial $x^2 + 4x + 3 = 0$ and the x-axis.
- Find $\int_{-1}^{1} 1 + x + x^2 + x^3 + x^4 + x^5 + x^6\,\mathrm{d}x$

Graphs

- Sketch graphs of
$$y = x^2 + 4x + 3, \quad y = x^3 + 4x^2 + 3x, \quad y = 2^x, \quad y = \log_2 x \quad \text{on separate axes.}$$
- Sketch graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ on the same axes.

Logarithms and powers

- Simplify $\log 3 + \log 4$ into a single term.
- Expand $(e^x + e^{-x})(e^x + e^{-x})$
- Solve $2^x = 3$.

Transformations

- Let $f(x) = x^2 + 4x + 3$. If you didn't sketch a graph of this before, sketch one now.
- Sketch a graph of $y = f(x + 2)$.
- Sketch a graph of $y = 3f(2x)$.

Geometry

- Add the vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.
- Find the equation of the line through $(1, 0)$ and $(0, -1)$.
- Find the equation of the line through $(1, 2)$ with gradient 3.
- A circle has centre $(-1, 4)$ and radius 3. Write down an equation for the circle.
- What's the area of this circle?
- Points A and B lie on a circle with centre O and radius 1. The angle $\angle AOB$ is $120°$. Find the length of the arc between A and B. Find the area enclosed by that arc and the radii OA and OB.

Trigonometry

- Solve $\sin x = \frac{1}{2}$.
- Solve $\tan x = 1$.
- Expand and simplify $(\sin x + \cos x)^2$
- Simplify $\cos(450° - x)$
- A triangle ABC has side lengths $AB = 3$ and $BC = 2$, and the angle $\angle ABC = 120°$. Find the remaining side length AC, the area of the triangle, and an expression for $\sin \angle BCA$.

Sequences and series

- A sequence is defined by $a_0 = 1$, $a_1 = 1$, $a_2 = 1$, and
$$a_n = a_{n-1} + a_{n-2} + a_{n-3} \quad \text{for } n > 2.$$
Find a_{10}.
- A sequence has first term 3 and each subsequent term is 5 more than the previous term. Find the sum of the first four terms.
- A sequence has first term 4 and each subsequent term is 6 times more than the previous term. Find the sum of the first four terms.
- When does the sum $1 + x^3 + x^6 + x^9 + x^{12} + ...$ converge? Simplify it in the case that it converges.