Subject Panel: Mathematical Methods and Applications

Suggested title of dissertation: Models for phase separation

Dissertation supervisor: Andreas Münch

Maximum number of students (at least 4): 4

Description of proposal:
Phase separation is one of the most important patterning processes in nature and in technical applications, where, for example, a material system with two components separates into spatial domains occupied by different phases, i.e. regions of distinct composition. It usually begins rapidly, for example from a nearly homogeneous mixture, and slows down as the domains grow and eventually reach a steady state. The domains are separated by interface regions, where the material composition varies from one phase to another. The process can be derived by phase-field equations, of which the Cahn-Hilliard equation is one of the most basic models. This project will start from the early onset of the phase separation but will mostly focus on the late stages when the interface layers are thin compared to the size of the domains.

Possible avenues of investigation:
After acquiring and overview of the Cahn-Hilliard equation / phase separation e.g. through reading the introductory chapter [1], a good starting point is the linear stability analysis of the Cahn-Hilliard model and a derivation of the sharp interface limit by a variation of the the technique in [2]. After this, students can follow one of several other avenues, for example: They can consider other, more complex models, e.g. patterning in diblock-copolymers [3] or models coupling phase separation and elasticity [2]; or they can choose to explore sharp interface models numerically, for example via boundary integral methods [1].

Pre-requisite knowledge:
Essential: B5.2 Applied Partial Differential Equations, A6: Differential Equations 2
Recommended: C5.5 Perturbation Methods,

Useful reading:

[1] A. Novick-Cohen. Chapter 4 The Cahn–Hilliard Equation. In C. M. Dafermos and M. Pokorny, editors, *Handbook of Differential Equations:*

Evolutionary Equations, volume 4, pages 201–228. North-Holland, Jan. 2008.

[2] R. L. Pego. Front Migration in the Nonlinear Cahn-Hilliard Equation. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, 422(1863):261 –278, Apr. 1989.

Further references:

[1] H. J. Jou, P. H. Leo, and J. S. Lowengrub. Microstructural Evolution in Inhomogeneous Elastic Media. *Journal of Computational Physics*, 131(1):109–148, Feb. 1997.

[2] P. H. Leo, J. S. Lowengrub, and H. J. Jou. A diffuse interface model for microstructural evolution in elastically stressed solids. *Acta Materialia*, 46(6):2113–2130, Mar. 1998.

[3] Y. Nishiura and I. Ohnishi. Some mathematical aspects of the micro-phase separation in diblock copolymers. *Physica D: Nonlinear Phenomena*, 84(1–2):31–39, June 1995.