

- **Title:** 'A short introduction to the Fractional Laplacian' by Dr. David Gómez-Castro (University of Oxford)
- **Aimed at:** D. Phil students
- **Course Length:** 2 hours with a comfort break
- **Dates and Times:** 10<sup>th</sup> November 2021 10am
- **Course Overview:** In this lecture we will give a brief introduction to some equations involving the fractional Laplacian in the whole space, mainly the Laplace and heat equations. This operator has attracted the interest of some of the best specialists in PDEs over the last decades (e.g. Luis Caffarelli, Alessio Figalli, Juan Luis Vázquez). Firstly, we will introduce several (equivalent) definitions of the fractional Laplacian Fourier transform, as the infinitesimal generator associated to stochastic processes with long-range interactions, then through to the classical heat equation, and the surprising formula of Caffarelli-Silvestre. In order to present the notion and properties of weak solutions, we present the Sobolev space of fractional order. Finally we will introduce the notion of viscosity solution, and conclude the lecture with a short discussion of the different non-equivalent fractional Laplacian in a bounded domain.

- **Bibliography**

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[2] Di Nezza, E., Palatucci, G., & Valdinoci, E. (2012). Hitchhiker's guide to the fractional Sobolev spaces. *Bulletin Des Sciences Mathématiques*, 136(5), 521–573. <https://doi.org/10.1016/j.bulsci.2011.12.004>

[3] Caffarelli, L., & Silvestre, L. (2007). An extension problem related to the fractional laplacian. *Communications in Partial Differential Equations*, 32(8), 1245–1260. <https://doi.org/10.1080/03605300600987306>

[4] Ros-Oton, X., & Serra, J. (2014). The Dirichlet problem for the fractional Laplacian: Regularity up to the boundary. *Journal Des Mathématiques Pures et Appliquées*, 101(3), 275–302. <https://doi.org/10.1016/j.matpur.2013.06.003>

Dr David Gómez-Castro  
 Mathematical Institute, University of Oxford,  
 Oxford OX2 6GG, UK  
[www.maths.ox.ac.uk/people/david.gomezcastro](http://www.maths.ox.ac.uk/people/david.gomezcastro)