

# Examiners' Report: Final Honour School of Mathematics

## Part II: Trinity Term 2004

### Part I

#### A STATISTICS

- Numbers and percentages in each class

	Number			Percentages %		
	2004	2003	2002	2004	2003	2002
I	37	(37)	(30)	54.4	(50)	(47.6)
II.1	20	(26)	(24)	29.4	(35.1)	(38.1)
II.2	8	(7)	(9)	11.8	(9.5)	(14.3)
III	2	(4)	(0)	0.03	(5.4)	(0)
P	1	(0)	(0)	0.12	0	(0)
F	0	(0)	(0)	0	(0)	(0)
Total	68	(74)	(63)	(100)	(100)	(100)

- **Numbers of vivas and effects of vivas on classes of result** Not applicable.
- **Marking of scripts** Numbers of scripts double marked: 10 (Dissertations). The remaining scripts were, in the customary way, single marked according to detailed marking schemes, strictly adhered to.
- Numbers taking each paper:

	Number
c1 & c2	68
c3 & c4	58
c5 & c6 (dissertation)	10

There were also two candidates for Part II Mathematics & Philosophy who took papers c1–c4.

#### B. New examining methods and procedures

University Standardised Marks were produced on a trial basis in 2003 but were not used in the final classification. This year USM marks were generated algorithmically from the raw marks in the same way as in the trial, except that marks were on a scale of 0–100 rather than being confined to the range 20–100. Classes were determined from the USMs according to the published examining conventions and marks were reported in USM form only.

**C. Changes in examining methods and procedures currently under discussion or contemplated for the future**

This was the penultimate examination under the present course structure and regulations. We do not anticipate any significant changes for the 2005 examination.

**D. Notice of examination conventions for candidates**

The candidates were given details of the examining conventions both in a supplement to their Course Handbooks and in the notices that were sent out by the examiners [attached].

## HONOUR SCHOOL OF MATHEMATICS PART II 2004

### EXAMINERS' REPORT PART II

#### Section A. General Comments on the Examination

The internal Examiners record their very warm thanks to:

- The External Examiners, Professors Fokas and MacPherson, for their helpful advice throughout the year and for their wise guidance and good humour during the classification process;
- all Assessors;
- the staff at the Mathematical Institute, in particular Maria Moreno, to whom we owe a huge debt of gratitude for the very efficient way in which she administered a highly complex operation, and Catherine Goodwin, for her support and guidance;
- Elliott Nichol for his cooperation and patience in managing the database and in assisting with the processing of data and the graduate students who acted as checkers for the entry of marks;
- Examiners from previous years, whose records and guidance, especially concerning the database, were invaluable.

#### Timetable

The papers c1–c4 were taken on Tuesday morning, Wednesday morning, and Thursday morning and afternoon of 6th week of Trinity Term, in numerical order. As usual, candidates for c5 were incarcerated over lunch on Thursday to permit the sharing of questions between c3, c4 and c5; this arrangement regrettably forces most candidates to take two of their papers on the same day.

#### Setting and checking of papers and marks processing

We make the following points in relation to the examining process:

1. Following the usual pattern, lecturers provided draft questions which were then vetted by selected members of the appropriate panels and overseen by an Examiner. The lecturers also acted as Assessors, marking the questions on their courses.
2. The work involved, both academically and administratively, in delivering complete and accurate camera-ready copy of the papers by the due date is immense. (This year there were 37 options and 35 setters, affiliated to three different departments.) We re-iterate comments which have been made year after year by our predecessors: the timetable for producing, checking and typesetting for CRC the questions for Part II is very tight, and any significant delay at any stage makes it very difficult for the Examiners to do a professional job of vetting and editing the questions and for the External Examiners to be given adequate time for considered comments. We are pleased to report that this year the number of setters and checkers seriously overshooting the deadlines was extremely small, and we offer our sincere thanks to all those who successfully strove to get their questions in (almost) on time.

3. Both the External Examiners scrutinised all the draft questions with remarkable care, even those in areas far beyond their specialist expertise, and spotted a number of errors which might otherwise have appeared in the printed papers. We are extremely grateful to them for the thoroughness with which they carried out a very daunting task.
4. Clearly set out, well-annotated solutions greatly assist the Examiners, and in particular the External Examiners, in assessing questions for appropriateness of length and standard and in detecting possible errors, and facilitate the checking of marks. By no means all setters wrote out specimen solutions so that someone not conversant with the material could easily make sense of them. An administrative grouse from the Chairman: despite clear exhortations to do so, almost all setters failed to label and date the iterations of their questions and solutions so that these could be still be identified once they were separated from the log-in sheets. This omission greatly adds to the work involved in getting complete and properly labelled bundles of drafts prepared for circulation to the examining team and in particular the External Examiners.
5. Only a small proportion of setters supplied draft questions T<sub>E</sub>X-ed with the exam style file, as the instructions request. We would not wish to recommend that it should be compulsory that setters do this. However, we do draw attention to the work involved in converting questions into the standard format and urge that secretarial assistance should be available to get all questions (and not just the small number of handwritten ones) to this state of readiness.
6. The Examiners were granted permission this year to get each candidate to fill in a departmental logsheet for each paper recording a list all questions attempted; these logsheets provided a valuable cross-check that all scripts had been correctly logged in and accounted for at all stages of the process. (The instructions to candidates printed on the script booklets are not well worded for use with multi-marker examinations in which the number of questions or sections to be attempted is not prescribed.)
7. The checking of marks entry was conducted in the same way as in the past few years. This worked smoothly and effectively.

### **Determination of University Standardised Marks**

USM marks for each of the Part I papers were awarded last year. As in the trial run in 2003, USMs were awarded for c1+c2, c3+c4, c5 and c6, and an overall USM mark for Parts I and II together arrived at as an average (in the manner described in the First Notice to Candidates).

Each dissertation was double-marked, and an agreed USM mark derived in each case from the marks awarded by the two Assessors.

For Papers c1–c5, raw marks in sumsquare form were used. The conversion from raw marks to USMs was carried out using the algorithm which was employed for Part I last year (this is described in the Examiners' report for that examination) and which was run experimentally on last year's Part II data. The only modification made to the algorithm was to change to a mark range of 0–100 (from one of 20–100), as we had been instructed to do; to achieve this, an additional corner, at 37, was introduced, to calculate marks at the bottom end of the range.

The algorithmic process is such that, once the two parameters which identify the top two borderlines on the sumsquare-ranked list have been fixed, the USMs, and hence the class awarded, are determined without the Examiners having scope to deviate from these (except at the bottom end, where the Examining Conventions allow them discretion). It should be stressed that the algorithm generates USMs which measure performance in a relative, rather than an absolute, way.

## Classification and Class Percentages

Many trial values of the parameters were input to the algorithm and the associated provisional USM-based classification then scrutinised to see if the performance of candidates on either side of the resulting borderlines was, in the Examiners' academic judgement, in accordance with the published qualitative class descriptors. In interpreting the descriptors, the assessment of the External Examiners played an important part. Account was also taken of the measures of performance that can be gleaned from the full profile of a candidate's raw marks, and comparisons made with classification in previous years, when classification was driven by such indicators.

The Examiners were fully satisfied that, in the class list finally agreed, all candidates were awarded the classes that their performances merited. No candidate was demoted as a result of the 'weak paper' rules set out in the Examining Conventions. As was to be expected on the basis of Part I performance and confirmed by the Assessors' reports for Part II, a large number of firsts was awarded. The percentage of firsts, 54.4%, is the highest in the past five years, but lower than that, 62.1%, in 1998, the first year the MMath was awarded. It was disappointing to see a relatively high number of poor performances (II.2 8, III 2, Pass 1). All of these candidates had performed relatively poorly in Part I. All of the bottom three were awarded overall a class one lower than that corresponding to their Part I average; on Parts I and II together, none of these candidates was close to a borderline.

A circular was sent in June to Chairmen of Examiners in subjects under the aegis of the Mathematical and Physical Sciences Board drawing attention to variations in class percentage figures across the Division. Note was taken of this by the Part II Examiners, but it did not influence their decisions, if only because the Part I marks had already been set last year and because it is inappropriate to consider the MMath and the Mathematical Sciences BA separately. Indeed, we should prefer to see the entire cohort classified together at the end of the third year, with a separate classification being awarded for the fourth year for those taking the MMath. Such an arrangement would, we believe, result in a fairer and more meaningful assessment of all the candidates at each stage, and would make the task of the Examiners easier.

We are aware of disappointment and frustration amongst some college tutors and students at the limited information divulged on individual performances by comparison with previous years. In releasing only USM marks we were, of course, following University instructions. We could, and with hindsight should, have deviated from the previous practice of generating a single mark for each pair of papers c1/c2 and c3/c4; in the absence of detailed information on numbers of alpha and beta questions, etc., four separate marks would have been helpful. It should be noted that the method for deriving USM marks is necessarily crude, and, as observed above, gives a measure of attainment relative to that of other candidates. Given the high level of performance of the majority of the candidates, the Part II USM marks of those few who performed, in relative terms, very badly on Part

II papers were very low; this is simply a consequence of the way the algorithm operates. Also, some anomalies in individual USM marks, in both Part I and Part II, inevitably result when the number of candidates involved is small and untypical.

We include, as requested by the Mathematics Teaching Committee, a percentile table for overall USMs.

Av USM	no.	%
91	1	1.5
90	2	2.9
85	3	4.4
84	4	5.9
82	7	10.3
79	8	11.8
78	11	18.3
76	14	20.6
75	19	27.9
74	22	32.4
73	27	39.7
72	29	42.6
71	32	47.1
70	37	54.4
69	39	57.4
68	43	63.2
67	45	66.2
66	49	72.1
65	51	75.0
64	53	72.9
63	56	82.4
62	57	83.8
59	59	86.8
56	60	88.2
55	62	91.2
53	63	92.6
52	64	94.1
51	65	95.6
47	66	97.1
44	67	98.5
35	68	100.0

This table does give an overall impression of the distribution of marks. But, in view of the caveats above about placing too much weight on individual USMs, we cannot emphasise too strongly that a linear rank order derived from this table would not accurately reflect the achievement of individual candidates in most cases. (The same would be true even of a linear ranking based on average USMs before rounding up.)

### **B. Equal opportunities issues and breakdown of the results by gender**

The table below shows percentages of male and female candidates in Mathematics Part II.

	I	II.1	II.2	III	P	F	Total
Male	30(44.1%)	9(13.2%)	7(10.3%)	1(0.02%)	1(0.02%)	0	48(70.6%)
Female	7(10.3%)	11(16.2%)	1(0.02%)	1(0.02%)	0	0	20(29.4%)
Total	37(54.4%)	20(29.4%)	8(0.32%)	2(0.04%)	1(0.02%)	0	68(100%)

### C. Detailed numbers on candidates' performance in each part of the exam

#### Statistics

##### Paper c1

Section	no. of candidates	no. of q. attempts	average mark	std dev.
A	11	20	20.25	4.54
B	9	14	18.57	3.11
C	10	14	11.14	9.36
D	9	13	8.23	6.15
E	2	3	10.66	4.04
F	19	30	20.20	6.46
G	0	0		
H	10	18	20.67	5.42
I	13	21	15.57	8.08
J	23	40	18.88	7.67
K	6	7	10.57	6.43
L	7	14	16.57	5.14
M	12	22	16.55	4.87
N	7	11	20.36	7.23
O	20	36	19.17	4.67
P	9	16	14.38	7.73
Q	0	0		

**Paper c2**

Section	no. of candidates	no. of q. attempts	average mark	std dev.
A	11	21	21.52	4.42
B	5	8	20.13	7.20
C	6	12	12.25	7.34
D	11	16	16.13	5.14
E	2	4	15.00	10.92
F	19	30	19.77	6.08
G	1	1	1.00	
H	10	16	18.31	6.06
I	13	22	20.36	5.58
J	24	43	17.98	5.55
K	7	11	15.16	9.05
L	7	13	19.92	7.22
M	14	25	16.92	6.14
N	6	8	15.38	6.99
O	21	41	17.32	6.72
P	8	14	12.79	7.65
Q		0		

**Paper c3**

Section	no. of candidates	no. of q. attempts	average mark	std dev.
AA	9	14	20.29	3.75
BB	11	20	17.90	7.23
CC	10	17	14.29	7.47
DD	8	13	13.62	8.94
EE	10	18	22.33	4.56
FF	0	0		
GG	4	6	20.50	3.83
HH	16	28	18.54	6.14
II	16	21	15.95	7.50
JJ	6	10	17.30	3.68
KK	8	15	19.33	4.10
LL	2	2	21.00	5.66
MM	6	12	19.42	4.70
NN	12	18	18.17	4.57
OO	4	6	18.83	6.37
PP	3	6	22.33	1.51
QQ	7	10	9.20	4.44
RR	3	6	5.83	5.67
SS	5	9	21.56	3.32
TT	1	1	21.00	



### Paper c4

Section	no. of candidates	no. of q. attempts	average mark	std dev.
AA	9	18	21.78	5.13
BB	10	18	18.56	4.45
CC	10	18	22.06	2.94
DD	6	11	13.55	9.07
EE	10	18	16.56	7.10
FF	0	0		
GG	4	4	19.00	4.97
HH	15	27	20.26	7.28
II	14	23	15.70	5.92
JJ	7	12	17.33	4.36
KK	8	16	18.81	4.98
LL	2	4	18.25	5.38
MM	6	12	16.58	7.78
NN	12	22	16.50	5.23
OO	4	7	18.00	3.67
PP	3	6	18.67	6.44
QQ	7	11	11.46	7.30
RR	5	6	10.67	6.68
SS	6	11	20.55	4.85
TT	0	0		

### Paper c5

Section	no. of candidates	no. of q. attempts	average mark	std dev.
AA	2	7	21.29	6.90
BB	1	2	14.50	0.71
CC	1	4	20.75	4.03
DD	0	0		
HH	0	0		
II	1	4	19.00	2.16
LL	0	0		
MM	3	9	10.67	4.61
NN	1	3	13.33	2.89
OO	1	2	21.50	3.54
QQ	0	0		

## Section D. Comments on sections and on individual questions

### A Model Theory

This year the course has been reasonably popular. 11 candidates attempted it in c1 and 6 candidates attempted in c2 papers with a good rate of success (40% alphas and 50% betas). This is higher than average but there are no reasons to consider the questions too

easy.

Each question has been attempted by at least one candidate.

### **B Axiomatic Set Theory**

There were 15 candidates for section A, Axiomatic Set Theory of whom one answered two questions, one answered three, but all the rest made serious attempts at all four questions. There were a very large number of alphas and one could say that the questions were too easy. Probably there was too much bookwork. However, it was not completely predictable, and the students had clearly worked hard at learning and understanding it and all of them could reproduce it very efficiently.

There were no problems with the exam (at least, I wasn't contacted during it) nor with the marking. There was one candidate (03567) who, I am almost certain, misread question A3 and did not provide a PROOF of the Koenig Inequality (only the statement). This lost him 10 marks in an otherwise perfect answer.

### **C Group Theory**

The questions appeared to discriminate effectively between the able and the less able students. There were some excellent accounts of bookwork, and a few perfect solutions to riders: it was a pleasure to award full marks for two solutions. The question on nilpotency attracted no serious solutions: the apparent difficulty of this topic will be taken into account in next year's course.

I was particularly pleased with the attempts at the final question on CZ-groups. Since the very last part of it proved too hard I readjusted the marking scheme (awarding up to three extra marks for a good introduction to CZ-groups).

### **D Lie Algebras**

These questions allowed plenty of scope for students to demonstrate their competence with the material; they were perhaps found on the difficult side. I was perplexed to see that what I regarded as the easiest question, Question 1, was rather poorly done: it required little more than basic linear algebra. The other questions, with more bookwork, fared better, and in particular Question 3 attracted some excellent solutions.

### **E Differentiable Manifolds**

**E1:** 2 attempts, neither very serious apart from the bookwork.

**E2:** 1 attempt, the candidate knew what he/she was doing but made a sign error and lacked either time or stamina to proceed.

**E3:** 2 attempts, one an alpha.

**E4:** 2 attempts, one full marks, the other definitions only.

### **F Analytic Topology**

**F1:** Generally well-done. One or two people gave an example of a connected space that was not path-connected, which was not to the point.

- F2:** Also generally well-done. Some candidates were prone to make set-theoretic slips with inclusion relations or equations involving  $f^{-1}$ ,  $f^*$ , etc. In the proof that the inverse image  $X$  under a proper map of a compact space  $Y$  is compact, it is vital to define a cover of  $X$  the small images of which also cover  $Y$ : some people forgot this.
- F3:** The bookwork part of the question centred round the proof of Stone's Theorem; substantial credit was available for knowing this. The remainder people found to be hard. It might be worth pointing out that what the question is asking one to do is prove that the Long Line is not paracompact, and is therefore not metrisable. (You are providing an open cover where for any refinement, by using rule (ii), you can find a point  $b$  belonging to infinitely many elements of the refinement.)
- F4:** This question was often done well. Some people had difficulty showing that  $\pi \circ k$  was a homeomorphism of  $X$  onto its image (in the third paragraph).

## G Function Spaces for Applications

Only two candidates took the course, and there was only one attempt at a question, which scored only one mark. I do not believe the questions were particularly difficult.

## H Functional Analysis

There were 14 undergraduate students who attended the lectures for this year's course; questions were attempted by 12 candidates.

Three of the questions were done very well by the majority of those who attempted them, as the following table shows. The other (H3) was based on bookwork which evidently was not found congenial (the geometric form of the Hahn-Banach separation theorem for topological vector spaces).

- H1** Some candidates gave incomplete definitions of the direct product, failing to specify the norm or define the algebraic operations on it.
- H2** Only a couple of candidates gave the intended proof of the last part, *via* the identification of the weak\* and the weak topologies on the unit ball of the dual of a reflexive space; others used the Hahn-Banach theorem (but were not penalised for this).
- H3** Many candidates stated the required criterion for continuity correctly and then wasted time by proving it. Conversely, some candidates applied the Hahn-Banach theorem correctly but failed to give its statement (as instructed). Those who completed the final part rarely took the quickest route (i.e., take  $A$  to be the open ball with centre  $y$  and radius  $\varepsilon > 0$ , suppose  $A$  and  $B$  are disjoint and apply the separation result).
- H4** The statement of the Banach-Mazur theorem given was often incomplete. Several candidates saw the quick way to deduce the continuity of characters (prior working shows that a character is the composition of two continuous maps) which was not highlighted in lectures.

## **I Mathematics and the Environment**

**I1:** Not too well done, although straightforward

**I2:** OK

**I3:** Perhaps too easy

**I4:** OK

## **J Perturbation Methods**

In general the answers were very good. The questions were mostly relatively straightforward, and in most cases an indication of the answer one was aiming for was given in the question.

**J1** Most candidates were able to justify all their approximations. Only one or two got part (iv) correct.

**J2** There were a lot of good answers to this question. Although it was an interior layer rather than a boundary layer, the question was fairly straightforward, and good students found it easy to get an alpha. On the other hand, the question was not trivial, and the answers showed good understanding of the technique—a lot of students managed to do the matching correctly.

**J3** There were few marks above 21 on this question, since everybody made the same 2 mistakes. Very few people checked that the solution given in part (i) satisfied the boundary conditions. Nobody got the rider in part (iv) right. Answers to the remaining parts were good.

**J4** Strangely most people that lost marks on this question did so on part (i), the supposedly easier part. They had trouble in applying the boundary conditions to the multiple scales solution, whereas in part(ii) they didn't have to worry about boundary conditions since they were looking for periodic solutions. Quite a few mistakenly assumed that  $t \lesssim O(1/\epsilon)$  meant that  $t\epsilon \ll 1$ .

## **K Further Quantum Theory**

The response to the questions was disappointing. Of the four questions three gave rise to at least one solution. On the other hand three candidates returned essentially perfect solutions for just one problem and then attempted no other suggesting that the problems were perhaps rather long.

## **L General Relativity I**

Only a small group (7) took this option, but the standard was pleasingly high. Almost all had studied the material fully and carefully, and wrote high standard answers, which demonstrated a good understanding of difficult material. Almost all of the seven candidates made substantial attempts at all of the questions, and there was a higher proportion

of alpha marks than in previous years. The only question that seemed to give serious difficulty (four high betas, but no alphas) was L3, which was on tensor manipulation in special relativity. Determinants still seem to be an Achilles heel even at fourth-year level.

### **M Complexity and Cryptography**

- M1** This was the least popular question and also the one which caused most difficulty. Very few successfully completed the reasonably straightforward reductions in the last part of the question.
- M2** Most of the difficulty centred on part (ii) where candidates had problems showing it was easy to decipher and in proving the difference equation.
- M3** This was, somewhat surprisingly, the most popular question and also received the best answers. Several candidates went wrong on the last part by treating the discrete logarithm problem as a language.
- M4** This was quite popular and was well done on the whole

### **N Martingales and Stochastic Analysis**

An error occurred in part b (ii) because  $T(w)$  should have been the first time  $X$  crossed zero (i.e.  $X_n(w) = 0$  should have read  $X_n(w) \leq 0$  or the word positive should have been inserted before the word martingale on line 2 of the question).

One candidate identified the error & corrected it- he had lost a couple of marks elsewhere in the question. But I gave them back to him as credit for this observation.

A number of candidates gave good answers to this bit module the fact that they swept over the fact that  $X_{T_{nn}}$  need not be positive, or got stuck with their calculation at the end because they realised they could not assume it. By a good answer, I mean they need the statement of the upraising lemma and how to use it & put themselves in a positive where they would certainly have got the correct answer if the question had been possible! I gave them full marks for this part of the question.

### **O Mathematical Genetics**

- O1** This was a question about the coalescent process in Mathematical Genetics and was generally well done, with bookwork parts answered correctly. Part (iv) was not worked through completely by most candidates. The Poisson distribution needed justification, as well as the mean being correct. The unseen part (vi) was reasonably attempted by candidates, but not always completed correctly.
- O2** This was a question about recombination, testing Hardy-Weinberg equilibrium and sequence alignment. It was generally well done, with the bookwork part mostly answered correctly. Part (iii) was well answered by most candidates. In part (iv) most candidates knew how to approach the question, but some did not take enough care with computation of the test statistic. Most students successfully found the optimal alignment in part (v), while a very small number made some computational errors in applying the sequence alignment algorithm.
- O3** The tree construction parts of the question (i), (ii), (v) and (vi) were well done. Finding the probability of the sample of sequences from the underlying coalescent

tree in (iii) was attempted by most, though candidates didn't do the algebraic simplification carefully and well.

- O4** The bookwork in parts a(i) and b(i) were generally well answered. Most students correctly computed the probability for part b(ii), but some had some difficulty in correctly applying Bayes rule to compute the probability in part b(iii). A few students made some computational errors in applying the Delta method in part b(ii), but most students made a reasonable attempt at this question.

## **P Numerical Linear Algebra and Approximation**

8 candidates achieved a wide range of marks on the questions on this course: the top performer dropped only 5 marks on all 4 questions with 2 perfect full marks solutions. Whereas there were a number of scrappy bits which attended only single figure marks from some of the other candidates.

Questions 1 and 4 seemed the most popular but the slightly more non-standard question 3 and question 2 on Multigrid were also attempted by most of the candidates.

## **Q Domain Theory**

No attempts

## **AA Gödel Incompleteness Theorems**

**AA1:** People seemed to find this the hardest question on c3/c4 (with fewest attempts). People tended to consider more cases than were necessary to prove  $\Sigma_0$  completeness, making some answers rather long.

**AA2:** Done well, most people had the right idea for the last part.

**AA3:** There were two necessary assumptions missing from this question, that the formulas  $\phi_0$  and  $\phi_1$  are  $\Sigma_0$  and that  $S$  is consistent. However everyone understood that this was what was intended. This question was possibly too easy (although not everyone saw the link between the last part and the second incompleteness theorem).

**AA4:** Many people got full marks on this question. I think the bookwork did a good job of testing knowledge of provability predicates, but it could have done with a harder final part to distinguish amongst the candidates better.

## **BB Representation Theory**

**Q1** (Semi-simple modules) This was not as good as I had expected. Several candidates make the usual mistake with direct sums of more than one terms, or even say  $L \subset A \oplus B$  and  $L \cap A = \{0\}$  implies  $L \subseteq B$  (the last part of the question shows this to be false).

**Q2** (Maschke's Theorem) Generally well done.

**Q3** (Local algebras, indecomposability) Basically fine.

Q4 (quiver algebras of finite type) The first part was fine. The second part could have done with a hint. The composition series part was rather hard and last part was fine.

### CC Algebraic Number Theory

Overall 11 students attempted CC questions. There was a surprising difficulty with routine algebraic manipulations in question 1.

Question 3 seems to have been the least discriminating, with nearly all marks very high. However the students on this course were mostly very strong, and the generally high marks should be fair.

Students had a tendency to quote and prove entire theorems when only a small part, or simpler case, was necessary. They would still gain full marks but this led to wasted time and some replication between questions. One or two students even commented on the apparent replication without noticing that they had proved a much stronger result than necessary.

Factoring ideals and calculating the class number (questions 3 and 4) was generally very well done, possibly reflecting the fairly standard nature of these problems and large amount of practice given on the example sheets.

### DD Algebraic Topology

Generally the questions when seriously attempted were solved quite well with some exceptionally good answers. However, the low number of attempts and the distribution of alpha marks indicates that the questions were too long and hard (and the course had been too full).

DD1 and DD2 seemed very long, partially because candidates proved more than was asked for, partially because they provided more detail to proofs than expected, and partially because the questions were quite long. For DD1, some candidates had trouble with the precise definition of *homotopic as paths*. The last part on the other hand presented little difficulties. In DD2, the concept of a *normal* covering presented difficulties though most candidates had clearly the right idea. All who tried seriously managed to get alpha marks on DD3 though few were able to prove that in the odd case, any even degree could occur. Question DD4 only attracted a couple of serious attempts. Interpreting the induced maps in the Mayer-Vietoris long exact sequence for the mapping torus was hard. Only one candidate went on to the application.

### EE $C^*$ -algebras

There appear to have been ten candidates who regarded this section as a first priority, and about three or four others attended the lectures and classes, but whose main interests lay elsewhere. The generally high standard last year was improved upon this year, and a large number of very good solutions were produced.

- ee.1 Every candidate obtained an alpha on this question, thereby displaying a firm knowledge of the basic structure of the spectrum of elements of a unital  $C^*$ -algebras.
- ee.2 Of the eight attempts five were alphas and two were betas. This was a gratifying result, since the question did not cover material presented lectures, and, therefore re-

quired rather more original thought than is often the case. There was a typographical error in one of the equations and this was pointed out by three of the candidates.

- ee.3 Most candidates were able to make some headway with the first part of the question, but a complete solution to a rather testing problem eluded all except one candidate.
- ee.4 Of the nine attempts to this questions, seven received alpha marks. This once again demonstrated that candidates had obtained a firm grasp of the ideas involved in the study of states of a  $C^*$ -algebras.

### **FF Calculus of Variations**

No attempts

### **GG Self-adjoint and Unitary Transformations in Hilbert Space**

Four candidates attempted these questions: one candidate was clear alpha quality; the other three were beta quality. The candidates concentrated on the two easier questions: on functional calculus; on position and momentum operators. These questions dealt with early material in the sections on bounded and unbounded operators respectively. The questions on one parameter semi-groups had one good solution, the question on Sturm-Liouville theory was ignored by all candidates.

It is difficult to make any inferences from the statistics available. However I think that the theory of unbounded self-adjoint operators, although very important, is probably too difficult for all but the best candidates. The course will not be repeated next year.

### **HH Applied Solid Mechanics**

Most candidates performed very well on these straightforward questions

### **II Mathematical Physiology**

- II1. The answers to II1 were good in general. Most common error was an inability to determine the stability of a steady state in the phase plane.
- II2. Not so many candidates attempted this question as attempted II1. Most got betas. I would have expected them to do better.
- II3. The candidates found this question quite hard. Quite a few gammas.
- II4. This question was fine.

### **JJ General Relativity II**

These questions attracted a good set of attempts all but 2 at beta level and above with each member of the cohort of 7 attempting at least half the questions. Perhaps there should have been more alphas to reflect the quality of the cohort.

- Q1: No one saw that to get through to the final curvature identities it was necessary to use the Ricci identities in the previous expression.



- Q2: No one remembered that the conserved quantity associated with a boost Killing vector tells you that the centre of mass moves with the velocity corresponding to the 4-momentum.
- Q3: This covered the routine theory of the Kerr solution. The argument for identifying the mass from the motion of far geodesics escaped all but one person.
- Q4: This was generally well done and caused few problems.

### **KK Quantum Field Theory**

Eight candidates attempted this section, of whom seven tried all four questions, and one tried three of them. Overall the questions were well done, with every candidate getting at least one  $\alpha$ , and one obtaining  $\alpha$ s on all four questions. Everybody managed  $\alpha$  or high  $\beta$  marks on questions KK2 and KK3. Questions KK1 and KK4 proved a little harder, or perhaps just longer, so that fewer completed them. (In particular, nobody produced a full answer to KK4.) There was a missing factor of 1/2 in the answer to the last part of KK1, so I marked that part generously. There was an unintended ambiguity in the penultimate paragraph of KK3, (which should have specified that the minimum was for varying  $w$ ), but fortunately slips on that part were due to miscalculation rather than misinterpreting the question.

### **LL Brownian Motion and Stochastic Calculus**

A small number of students took this exam (two), but that seems much better than previous years (zero apparently).

The union of the answers of the two students would have obtained a mark close to the perfect one. One student did very well to the 3 questions he attempted, but probably had no time to attempt the 4th one. The other one was often quite close to the right answer, showing that he had worked quite hard for the exam, but failed to get the last trick which would have given him a much better mark.

The whole lecture went quite well, with, for such a course, a reasonable attendance from undergraduates and visiting students, and a few post-graduate students from various areas.

### **MM Randomness and Complexity**

- MM1 Most of the problems were with the second part; in particular some candidates did not define parity  $P$  as a class of languages and this clearly causes difficulties in doing well on the rest of the question.
- MM2 This was the question which got the best answers. In one sense this is surprising as the underlying ideas are the hardest in the course. However, most of the question is either bookwork or very close to it.
- MM3 The main problems in this question were in proving the union of two languages belonging to  $IP$  is also in  $IP$ .
- MM4 Part (i) of this question caused most difficulty to those who attempted it. In particular, proving that the sum of the even coefficients of the chromatic polynomials is  $P$ -hard was not well done. The second part of the question, which was bookwork contained a small error but it did not cause a problem to those candidates who got that far.

## **NN Bayesian Statistics**

These questions were generally well done, with a fair range of marks and some excellent solutions on each question.

## **OO Pattern Recognition and Expert Systems**

Overall the standard was very good, bookwork was reproduced well and good attempts were made at unseen material. The attempts were spread reasonably evenly over the questions. The main areas where marks were lost are given below.

OO1 In part (ii) few candidates stated explicitly the independence assumptions made implicitly in their answers. In part (iii) some failed to note that a linear rule is obtained.

OO2 The bookwork was produced well and most candidates were able to adjust the classification rule in part (iv) and to suggest weighting observations as a strategy in growing the tree.

OO3 The second part of part (i) is quite difficult to explain but all attempts at this were good. In part (ii) all candidates were able to identify the main points but full credit was given only to those candidates who explained *why* computations under the saturated model are prohibitive.

OO4 Only one candidate was able to moralise the directed acyclic graph correctly - others missed out one or more connections. Most candidates made a small calculation error of some sort but since the correct approach was taken in all cases such errors received only minimal penalty.

## **PP Finite Element Methods for Partial Differential Equations**

Three candidates attempted these questions. Overall, the answers were of very high quality.

PP1 The question concerned the weak formulation of a two-point boundary value problem with mixed Neumann-Robin boundary conditions, and the construction and error analysis of the resulting piecewise linear finite element approximation. All candidates produced  $\alpha$ -quality answers.

PP2 The question concerned the connection between the weak formulation of an elliptic boundary value problem and the associated minimisation problem, and the relationship of these to their respective finite element approximations for the PDE  $-\Delta u + u = f$  with nonhomogeneous Neumann boundary condition. All three candidates attempted the question and produced  $\alpha$ -quality answers.

PP3 The question concerned the *a posteriori* error analysis of the finite element approximation of a two-point boundary value problem with Neumann boundary conditions. Two candidates produced  $\alpha$ -quality answers; the third candidate was only partially successful in adapting the theory from the lectures to the deal with the situation in the question.

PP4 The question concerned the construction and the stability analysis of the  $\theta$ -scheme for the finite element approximation of the heat equation. There was one low  $\alpha$ -quality solution, one very high  $\beta$ -quality solution, and one half-hearted attempt which only gained 7 points.

### **QQ Introduction to Continuous Optimisation**

Seven students answered questions from this option on papers c3 & c4, none on paper c5. The lowest mark was 1, the highest 25. I think that the questions were pitched at about the right level, all containing  $\gamma$ ,  $\beta$  and  $\alpha$  parts and receiving a fair share of attempts. The most successful question seems to have been QQ4, perhaps because this was the shortest problem and made the students feel comfortable because it contained several inductive proofs. The question least attempted was QQ2, although those who attempted it were pretty successful at it, which shows that it wasn't any more difficult than the other problems.

### **RR Categories, Proofs and Games**

Five candidates answered questions, of whom only one attempted all four questions. This candidate produced one excellent and one good answer, the attempts on the other parts being incomplete. Other candidates were weaker, the answers in some cases being perfunctory.

The answers on the question relating to the logic strand of the course were rather disappointing.

### **SS Lambda Calculus**

6 candidates attempted questions on this topic. The overall standard was extremely good, with almost all attempts receiving marks around or above 20 per question. Candidates displayed a very thorough understanding of the whole syllabus and the ability to lay out proofs economically and correctly. This performance was very pleasing; the high marks can be attributed to a strong year although perhaps the questions contained a little too much familiar material to stretch the most able. Most of the lost marks were due to minor errors.

### **TT Direct Methods for Sparse Linear Systems**

No report received. Only one question attempt in total.

### **Dissertations**

Ten candidates opted to write dissertations. The range of topics was very wide. We note that commentaries on classic papers seemed to provide particularly suitable vehicles for candidates to display and exploit mathematical knowledge and understanding. The overall standard was high, and the External Examiners commented favourably on the work submitted.

## **E. Comments on performance of identifiable individuals**

## **F. Names of members of the Board of Examiners**

Prof P Candelas, Dr P Clifford, Prof A S Fokas (external), Prof N J Hitchin, Dr P D Howell, Prof H D MacPherson (external), Dr H A Priestley (Chairman), Prof B Zilber.

### **Assessors for Papers c1–c5**

Prof S Abramsky, Prof J M Ball, Dr A Belton, Prof S J Chapman, Dr M J Collins, Prof L Duff, Dr C M Edwards, Dr K Erdmann, Dr A C Fowler, Prof R C Griffiths, Dr K C Hannabuss, Dr R A Hauser, Dr S Howson, Dr A D Ker, Dr B Kirchheim, Dr R Knight, Prof T J Lyons, Dr P J Northrop, Dr J Ockendon, Dr G M Reed, Dr J A Scott, Prof E Süli, Dr N Thapen, Prof U Tillmann, Dr N Victoir, Dr G F Vincent-Smith, Mr W Vos, Dr A Wathen, Prof D J A Welsh, Prof A J Wilkie, Prof J S Wilson, Dr N M J Woodhouse.

### **Assessors for dissertations**

Prof M P F du Sautoy, Prof D R Heath-Brown, Dr R Hinch, Dr S D Howison, Prof C J H McDiarmid, Dr P M Neumann, Dr J Norbury, Dr D R Stirzaker, Prof D J A Welsh