On the Nature of Turbulence

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Results developed in collaboration with
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Maximum entropy production as admissibilty for physical solutions.

Let’s start with the first set of slides

Turbulence is the outstanding problem of classical physics.

Entropy is the log volume of a surface of constant energy

Energy is dependent on details of physics, consequently, so is entropy.

Maximization of entropy prodiction is a fundmental law of theoretical physics, but remains disputed among mathematicians.

Maximum entropy is well understood in 1D. Entropy maximizing shock waves are physical, other shock waves are not. Correction due to Hugoniot, corrects error of Riemann.

Shock wave entropy maximizes the energy dissipation rate associated with fluid particle positions. Dissipation is Fourier's law of viscous heat prodiction.

Energy depends on physics. For single fluid incompressible fluids, energy is the fluctuation of velocity and vorticity.

Maxium rate of entropy production refers to both. Maximum rate of energy dissipation refers to both.

Two fluid turbulence has an additional energy of mixing, related to Fick's law.

Nonphysical turbulent mixing flows can fail to maximize single fluid entropy.

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Nonphysical two fluid mixing flows are not claimed to maximize two fluid entropy. Maximization of single fluid entropy is not relevant for two fluid mixing flows and does not contradict maximum entropy production as an admissibility principle

Theorem: Lebesgue measure, as a linear functional on phase space, maximizes entropy production relative to comparison measures, including other solutions of the Euler/Navier-Stokes equations.

Proof (in a short outline):

Entropy is convex due to the log volume in its definition.

Log integral ≥ integral log

Questions?

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Convergence of Resummed Renormalized Perturbation Theory

Let's start with the first set of slides

Classical stochastic physics and quantum field theory (QFT) share an overlap in much of their structure. The three infinite even moments are common. The Lagrangians defined by the transport of energy are similar. Renormalized perturbation theory (RPT), which is a program to map the Lagrangian into the exponent, has many common features.

Classical stochastic physics has a finite

global gauge group, but in contrast to QFT, no local gauge group. The fluids gauge group is defined by the symmetry $u_i(x,t) \to -u_i x,t)$ in each spatial direction i. For generic physics, the gauge group is si,ple nonAbelian, and gives the three infinite moments. For incompessible fluids, only two moments are infinite.

The three (or two) infinite moments are evaluated in a Sobolev space of negative index. This allows use of mathematical tools in their analysis.

The converged resummed renormalized perturbation theory defines a mapping of statistical momemts of turbulence into closed oriented surfaces in 3-space.

Surface discontinuities produce a Gibbs phenomena and so RPT is asymptotic, not convergent. Thus we use resummation methods to obtain convergence. Iterated arithmetic means.

Some terms in RPT (the first two or three even moments) are infinite. These are evaluated in Sobolev spaces of negative index, whereupon they become finite.

The proof is quite technical.We expect to have an arXiv version shortly.

The conceptual basis for the proof depends on:

1. Prime sequences
2. Bell number expansions: Each term is a sum of products of prime sequences
3. Law of large numbers to control the Bell number expansion
4. Prime sequences have finite expectation values in some Sobolev norm

A sequence is a linear combination of products of quadratic terms $Du^{\mu\nu}Du_{\mu\nu}$ from the Lagrangian. It is prime if it cannot be written as a product to two subsequences.

Given a finite set $X_j = \{1, \ldots, j\}$, the Bell number B_j is the set of partitions of X_j into nonoverlapping subsets.

These subsets label prime sequence factors and the Bell number sum labels all possible factorizations.

Given a finite set $X_j = \{1, \ldots, j\}$, the Bell number B_j is the set of partitions of X_j into nonoverlapping subsets.

Combinatoric estimates of this sum depend on Borel resummation, the law of large numbers.

Anayltic estimates depend on convergence properties of vacuum expectation values for prime sequences in possibly negative index Sobolev norms.

Questions?

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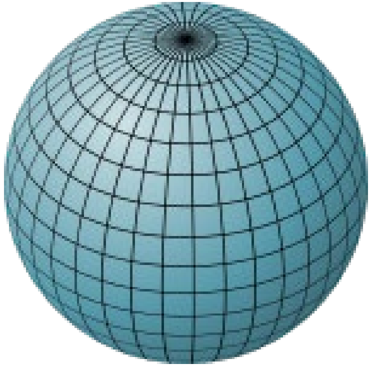
Twisted

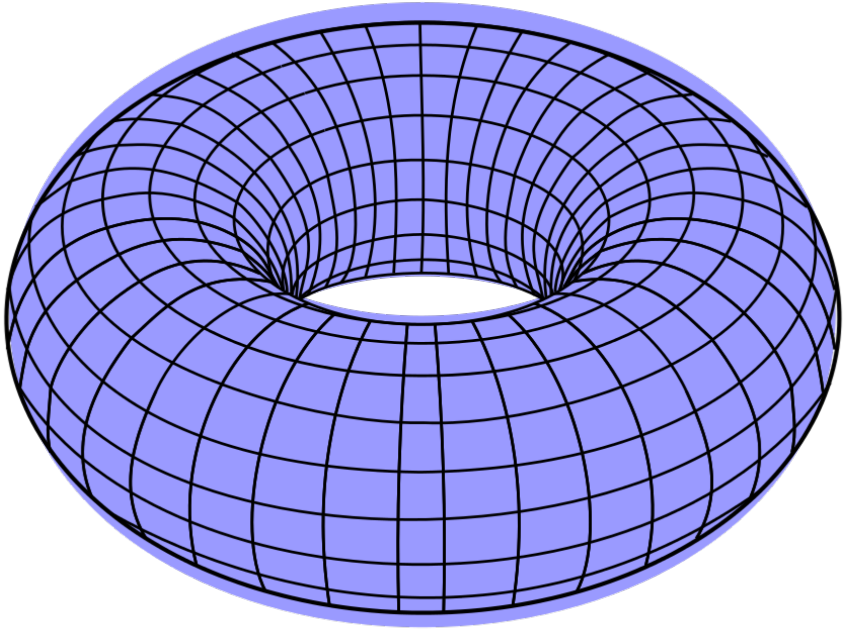
States of fully developed turbulence as vorticity spheres and tori (knotted) of various genus.

Vacuum state, multiple vortex spheres



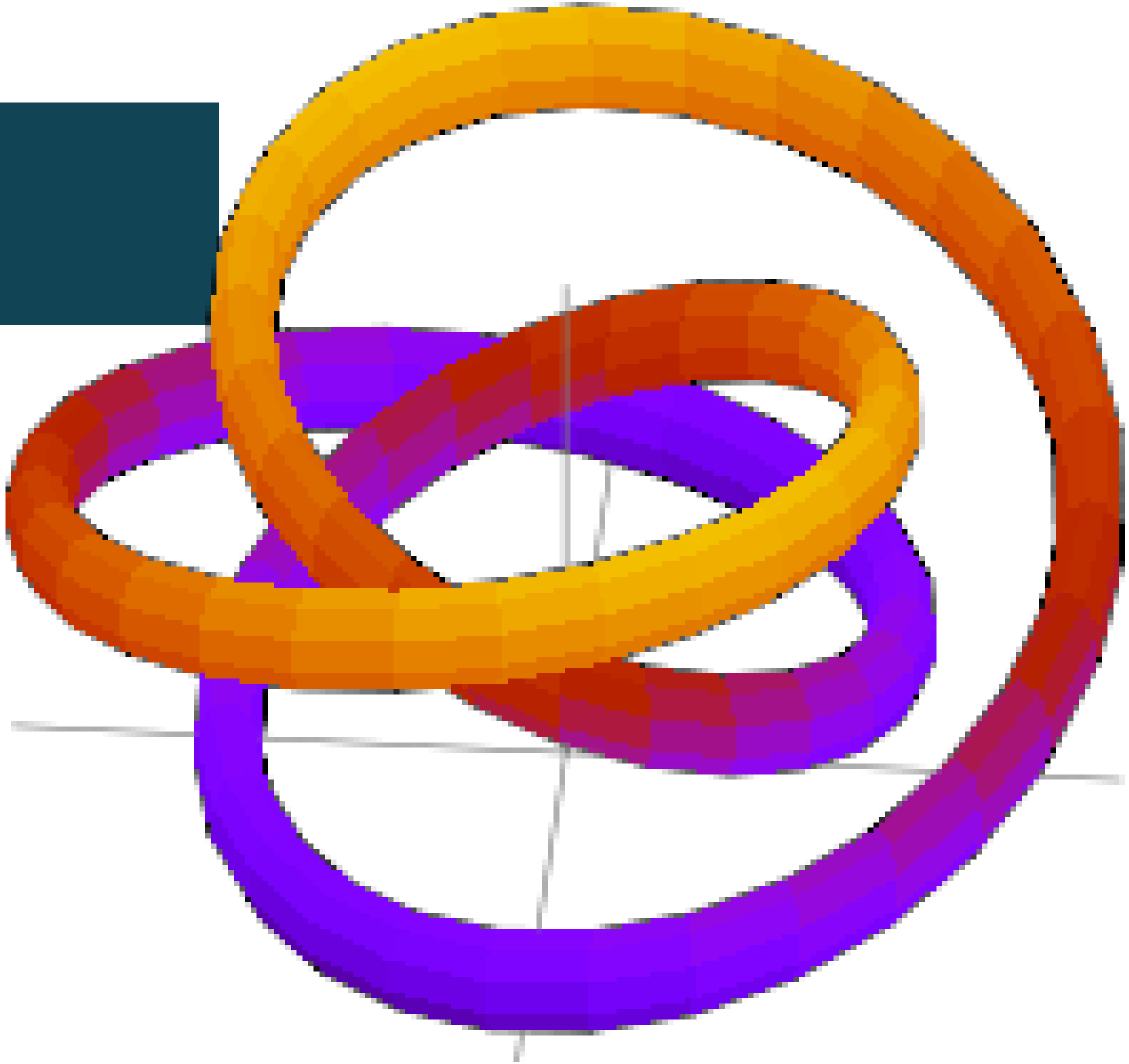
The vacuum state is fully developed turbulence.

A torus



Vortex rings are commonly observed turbulent structures. They are the first excited state of turbulence

A knotted torus



Low energy states of turbulence : vacuum (vortex sphere), torus (vortex ring), higher genus tori

Genus 2 torus

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Conjectured Phase Transitions (motivated by lore from fluids)

Systematic across fluids ---
quantum quarks and Yang-Mills
gauge fields ---
and quantized general relativity

Fluid phase transitions

- Laminar to turbulent transition
- Finit e gauge group, no gauge group related phase transition
- Fluid is a matter field.
- Helicity transition

Yang-Mills + quarks, phase transitions

- Quantum mechanics and atoms to elementary particles transition
- Gauge group chiral symmetry breaking. Plasma to strong particles transition
- Matter field chiral symmetry breaking. Plasma to weak particles transtion

Quantum General Relativity String theory 2D conformal quantum fields

- Dark matter to quantum general relativity with strings
- Gauge group chiral symmetry breaking. Plasma to strong particles transition
- Matter field chiral symmetry breaking. Plasma to weak particles transtion

Summary

- 1. Maximun entropy production as admissibility condition.
- 2. Renormalized perturbation theory: resumed, convergent
- 3. Turbulent states as vortex spheres, tori
- 4. Fluids as a window into QFT, quantum general relativity phase transitions. Dark matter

Questions?

Thanks