MAT 2010 Q1A

The values of k for which the line $y = kx$ intersects the parabola $y = (x-1)^2$ are precisely

(a) $k \leqslant 0$, (b) $k \geqslant -4$, (c) $k \geqslant 0$ or $k \leqslant -4$, (d) $-4 \leqslant k \leqslant 0$.

MAT 2014 Q2

Let a and b be real numbers. Consider the cubic equation

$$x^3 + 2bx^2 - a^2x - b^2 = 0 \qquad (*)$$

(i) Show that if $x = 1$ is a solution of $(*)$ then

$$1 - \sqrt{2} \leqslant b \leqslant 1 + \sqrt{2}.$$

(ii) Show that there is no value of b for which $x = 1$ is a repeated root of $(*)$.

Given that $x = 1$ is a solution, find the value of b for which $(*)$ has a repeated root.

For this value of b, does the cubic

$$y = x^3 + 2bx^2 - ax^2 - b^2$$

have a maximum or a minimum at its repeated root?

MAT 2015 Q1F

Let n be a positive integer. Then $x^2 + 1$ is a factor of

$$(3 + xs^4)^n - (x^2 + 3)^n(x^2 - 1)^n$$

for

(a) all n, (b) even n, (c) odd n, (d) $n \geqslant 3$ (e) no values of n.