Warm-up (based on MAT 2008 Q1B)
Let $\alpha = \ln \pi$. Write each of the following expressions in terms of α

$$\sqrt{2\ln(\pi^2)}, \qquad 2\left(\frac{1}{\ln \pi}\right)^3, \qquad \frac{1}{4\ln\sqrt{\pi}}. \tag{$*$}$$

Use the fact that $\pi > e$ to find an inequality for α.

Use the facts that $\pi < 4$ and $e > 2$ to show that $\alpha < 2$.

Use these two inequalities for α to decide which of the three expressions in $(*)$ is the largest, and which is the smallest.

Short question 1 (MAT 2011 Q1H)
The number of *positive* values x which satisfy the equation

$$x = 8^{\log_2 x} - 9^{\log_3 x} - 4^{\log_2 x} + \log_{0.5} 0.25$$

is

(a) 0, (b) 1, (c) 2, (d) 3, (e) 4.

Short question 2 (MAT 2013 Q1F)
Three *positive* numbers a, b, c, satisfy

$$\log_b a = 2, \qquad \log_b (c-3) = 3, \qquad \log_a (c+5) = 2.$$

This information

(a) specifies a uniquely.

(b) is satisfied by exactly two values of a

(c) is satisfied by infinitely many values of a

(d) is contadictory.

Extension: Unless you're saving it as a timed past paper, try MAT 2019 Q1G.

Long question (Very slightly adapted from MAT 2013 Q2)

(i) Suppose that k is a real number not equal to 1 or -1. The function $f(t)$ satisfies the identity

$$f(t) - kf(1-t) = t$$

for all values of t. By replacing t with $1-t$, determine $f(t)$.

(ii) Now consider instead the identity

$$f(t) - f(1-t) = g(t). \qquad (\star)$$

(a) Show that if $g(t) = t$ then no function $f(t)$ satisfies $(\star)$.

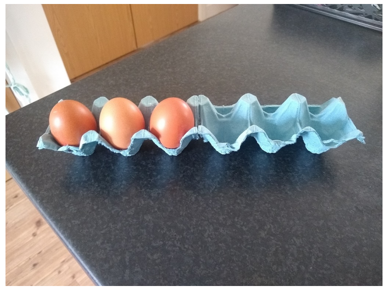
(b) Find a condition that the function $g(t)$ must satisfy in order for there to be a function $f(t)$ which satisfies $(\star)$

(c) Show that $g(t) = t$ does not obey your condition, but that $g(t) = (2t-1)^3$ does.

(d) If $g(t) = (2t-1)^3$, find a function $f(t)$ which satisfies $(\star)$.

Extension: If $g(t) = (2t-1)^3$ and $f(t)$ is a cubic in t, find all possible functions $f(t)$.

Bonus problem (not MAT)
I've got three eggs and a long thin eggbox, which has six spaces for eggs in a row.



There are $\binom{6}{3} = 20$ ways that the eggs could be put in the box, and I'd like to take photos of all the possibilities. I'm worried about breaking the eggs, so in between photos I will only move one egg, and I will move it exactly one space along in the eggbox. I'd also like to avoid unnecessary moves, so no photos should be repeated.

Can you find a sequence of 20 photos that follows these rules, or prove that it's impossible?

Eggs-tension: What if I had two eggs instead of three? What if I had four eggs?