Warm-up (not MAT)
Some quick area practice; skip these if you want to.

- An equilateral triangle has side length a. Find its area.
- An equilateral triangle has all three corners on a circle of radius a. Find its area.

Short question (MAT 2009 Q1B)
The point on the circle

$$x^2 + y^2 + 6x + 8y = 75$$

which is closest to the origin, is what distance from the origin?

(a) 3, (b) 4, (c) 5, (d) 10.

Short question (MAT 2007 Q1D)
The point on the circle

$$(x-5)^2 + (y-4)^2 = 4$$

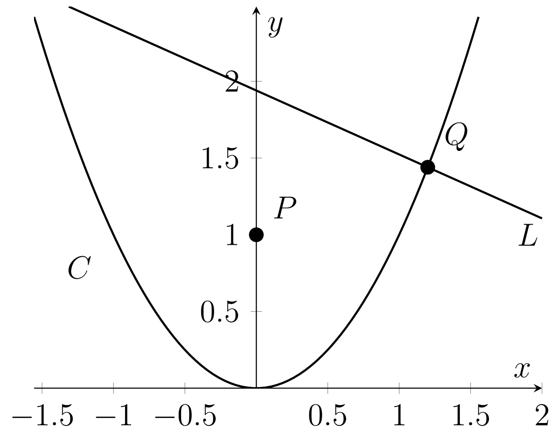
which is closest to the circle

$$(x-1)^2 + (y-1)^2 = 1$$

is

(a) (3.4,2.8), (b) (3,4), (c) (5,2), (d) (3.8,2.4).

Long question (MAT 2009 Q4)
As shown in the diagram below: C is the parabola with equation $y = x^2$; P is the point $(0, 1)$; Q is the point (a, a^2) on C; L is the normal to C which passes through Q.



(i) Find the equation of L

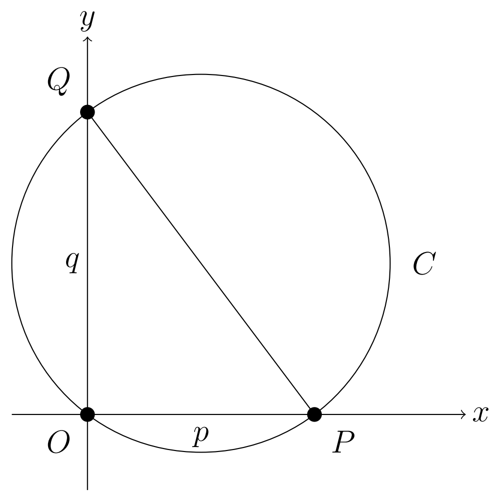
(ii) For what values of a does L pass through P?

(iii) Determine $|QP|^2$ as a function of a, where $|QP|$ denotes the distance from P to Q.

(iv) Find the value of a for which $|QP|$ is smallest.

(v) Find a point R in the xy-plane but not on C, such that $|RQ|$ is smallest for a unique value of a. Briefly justify your answer.

Long question (MAT 2008 Q4, modified slightly)
Let p and q be positive real numbers. Let P denote the point $(p, 0)$ and let Q denote the point $(0, q)$.



(i) Show that the equation of the circle C which passes through P, Q, and the origin O is

$$x^2 - px + y^2 - qy = 0.$$

Find the centre and area of C.

(ii) Show that

$$\frac{\text{area of circle } C}{\text{area of triangle } OPQ} \geqslant \pi$$

(iii) Find all possible values of $\tan \angle OPQ$ if

$$\frac{\text{area of circle } C}{\text{area of triangle } OPQ} = 2\pi$$

Bonus question (not MAT)
Let's say that you've got a circle of radius 1. The largest triangle that fits inside that circle is an equilateral triangle with all three corners on the circle (perhaps you could think about how you would prove this).

What if, instead of a circle, you had a pentagon of side length 1? Can you describe the largest triangle that fits inside that pentagon?