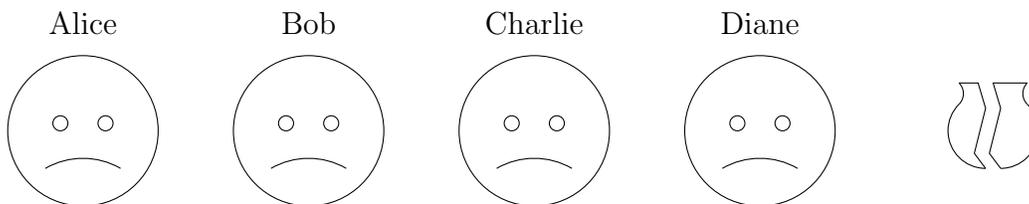


Long question (MAT 2011 Q6)



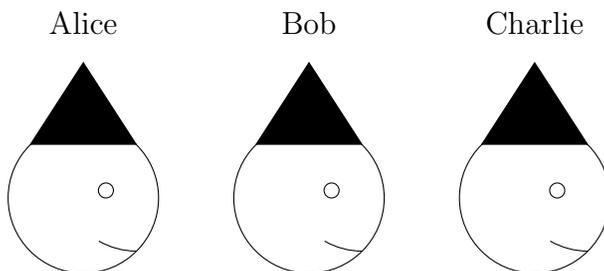
Alice, Bob, Charlie, and Diane are playing together when one of them breaks a precious vase. They all know who broke the vase. When questioned they make the following statements:

- Alice : It was Bob.
Bob : It was Diane.
Charlie : It was not me.
Diane : What Bob says is wrong.

Each statement is either true or false.

- (i) Explain why at least one of the four must be lying.
- (ii) Explain why at least one of them must be telling the truth.
- (iii) Let us suppose that exactly one of the four is lying, so the other three are telling the truth. Who is lying? Who did break the vase? Explain your answer.
- (iv) Let us now suppose that exactly one of the four is telling the truth, so the other three are lying. Who is telling the truth? Who did break the vase? Explain your answer.
- (v) Let us now suppose that two of the statements are true and two are false. List the people who might now have broke the vase. Justify your answers.
- (vi) Hence show that if we don't know how many of the four statements are true, then any one of the four could have broken the vase.

Long question (MAT 2012 Q6)

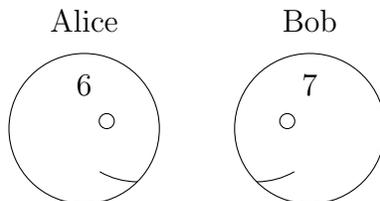


Alice, Bob, and Charlie are well-known expert logicians; they always tell the truth. They are sat in a row, as illustrated above. In each of the scenarios below, their father puts a red or blue hat on each of their heads. Alice can see Bob's and Charlie's hats, but not her own; Bob can see only Charlie's hat; Charlie can see none of the hats. All three of them are aware of this arrangement.

- (i) Their father puts a hat on each of their hats and says "Each of your hats is either red or blue. At least one of you has a red hat." Alice then says "I know the colour of my hat." What colour is each person's hat? Explain your answer.
- (ii) Their father puts a new hat on each of their heads and again says: "Each of your hats is either red or blue. At least one of you has a red hat." Alice then says "I don't know the colour of my hat." Bob then says "I don't know the colour of my hat." What colour is Charlie's hat? Explain your answer.
- (iii) Their father puts a new hat on each of their heads and says: "Each of your hats is either red or blue. At least one of you has a red hat, and at least one of you has a blue hat." Alice says "I know the colour of my hat." Bob then says "Mine is red." What colour is each person's hat? Explain your answer.
- (iv) Their father puts a new hat on each of their heads and says: "Each of your hats is either red or blue. At least one of you has a red hat. and at least one of you has a blue hat." Alice then says "I don't know the colour of my hat." Bob then says "My hat is red". What colour is Charlie's hat? Explain your answer.
- (v) Their father puts a new hat on each of their heads and says: "Each of your hats is either red or blue. Two of you who are seated adjacently both have red hats." Alice then says "I don't know the colour of my hat." What colour is Charlie's hat? Explain your answer.

Long question (MAT 2013 Q6)

Alice, Bob, and Charlie are well-known expert logicians; they always tell the truth.



In each of the scenarios below, Charlie writes a whole number on Alice and Bob's foreheads. The difference between the two numbers is one: either Alice's number is one larger than Bob's, or Bob's number is one larger than Alice's. Each of Alice and Bob can see the number on the other's forehead, but can't see their own number.

- (i) Charlie writes a number on Alice and Bob's foreheads, and says "Each of your numbers is at least 1. The difference between the numbers is 1."

Alice then says "I know my number."

Explain why Alice's number must be 2. What is Bob's number?

- (ii) Charlie now writes new numbers on their foreheads, and says "Each of your numbers is between 1 and 10 inclusive. The difference between the numbers is 1. Alice's number is a prime." (A prime number is a number greater than 1 that is divisible only by 1 and itself.)

Alice then says "I don't know my number."

Bob then says "I don't know my number."

What is Alice's number? Explain your answer.

- (iii) Charlie now writes new numbers on their foreheads, and says "Each of your numbers is between 1 and 10 inclusive. The difference between the numbers is 1."

Alice then says "I don't know my number. Is my number a square number?"

Charlie then says "If I told you that, you would know your number."

Bob then says "I don't know my number."

What is Alice's number? Explain your answer.

Bonus question (not MAT)

I've got the Ace (A), Jack (J), King (K), and Queen (Q) of each of Clubs (\clubsuit), Diamonds (\diamond), Hearts (\heartsuit), and Spades (\spadesuit), and I'm going to deal these cards out to form a 4×4 grid. I'd like one of each suit (\clubsuit , \diamond , \heartsuit , \spadesuit) in each row and in each column, and one of each face card (A , J , K , Q) in each row and each column. Find a way to do this.

$A\spadesuit$	$K\heartsuit$	$Q\clubsuit$	$J\diamond$
$J\clubsuit$	$A\clubsuit$	$K\spadesuit$	$Q\diamond$
$Q\heartsuit$	$J\spadesuit$	$A\diamond$	$K\clubsuit$
$Q\spadesuit$	$K\diamond$	$J\heartsuit$	$A\heartsuit$

The arrangement pictured here is not a solution because it's got two \clubsuit in the second row, and two Q in the first column (and several other issues).