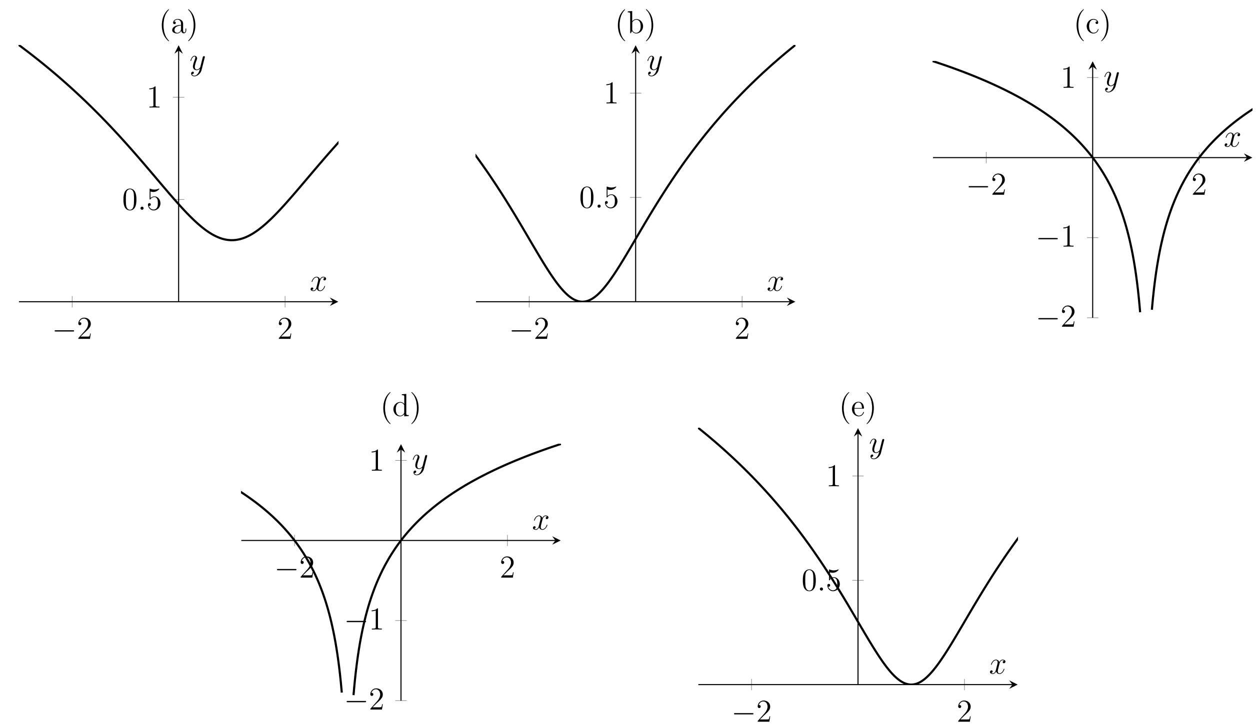
Short question (MAT 2014 Q1B, modified slightly)
A super-spy is tracking down an evil villain. He knows that the villain loves the function

$$y = \log_{10}\left(x^2 - 2x + 2\right).$$

He sees five vans drive by with the following graphs sketched on them. Which one should he follow to find the villain's lair?



Extension: Guess functions for the other four graphs.

Short question (MAT 2013 Q1B, modified slightly)
In the lair, the super-spy finds himself trapped in a room full of mirrors. He sees that the graph of $y = \sin x$ has been reflected first in the line $x = 180^\circ$, and then reflected in the line $y = 2$. It is vitally important that he works out what the equation of the resulting graph is, so that he can escape from the mirror room. The equation of the resulting graph is

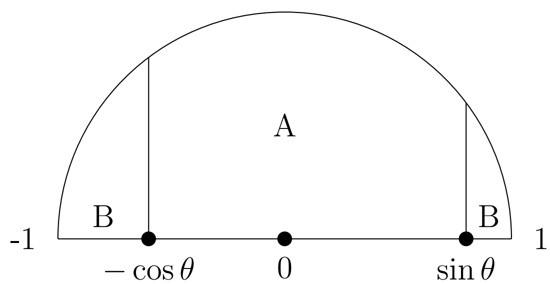
(a) $y = \cos x$ (b) $y = 2 + \sin x$ (c) $y = 4 + \sin x$ (d) $y = 2 - \cos x$.

Extension: Describe reflections that would produce the other equations.

Long question (MAT 2013 Q4, modified slightly)
(i) Let $a > 0$. Sketch the graph of

$$\frac{a+x}{a-x} \qquad \text{for} \quad -a < x < a$$

(ii) Let $0 < \theta < \pi/2$. In the diagram below, a half-disc is given by $x^2 + y^2 \leqslant 1$ and $y \geqslant 0$, and this is a floor-plan of the evil villain's lair; there is a shark tank on each side of the lair, and a complicated system of platforms in the middle, on which it is safe to stand. The region A consists of those points with $-\cos\theta \leqslant x \leqslant \sin\theta$ where it's safe to stand (for a particular value of θ). Outside of that area is the region B, which is the remainder of the half-disc, where the shark tanks are. Find the area of A in terms of θ.



(iii) Assuming *only* that $\sin^2\theta + \cos^2\theta = 1$, show that $\sin\theta\cos\theta \leqslant 1/2$.

(iv) The super-spy gets control of lair's platforms, allowing him to adjust θ between 0 and $\pi/2$. He would like to make as much of the floor safe as possible. What is the largest value that the ratio

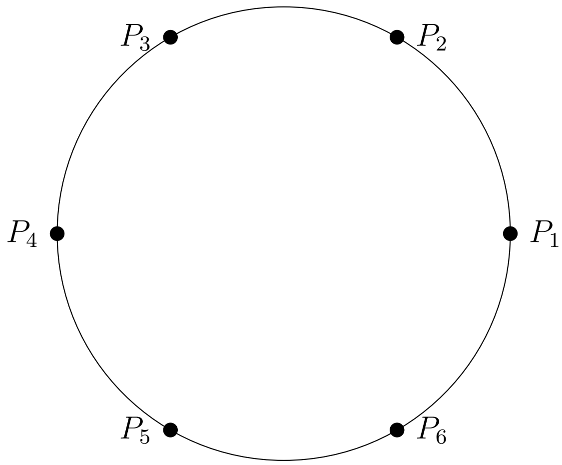
$$\frac{\text{area of } A}{\text{area of } B}$$

can take, as θ varies?

Extension: The evil villain seizes back control of the platforms. He would like to make as much of the floor dangerous as possible. What is the smallest value that the ratio in part (iv) can take, as θ varies?

Long question (MAT 2010 Q6, modified slightly)
In the questions below, the people involved make statements about each other. Each person is either a super-spy (S) who always tells the truth, or an evil villain (V) who always lies.

(i) Six people, $P_1, P_2, \ldots, P_6$ sit in order around a circular table with P_1 sitting to P_6's right, as shown in the diagram below.



(a) Suppose all six people say "the person directly opposite me is telling the truth". One possibility is that all six are lying. But, in total, how many different possibilities are there? Explain your reasoning.

(b) Suppose now that all six people say "the person to my left is lying". In how many different ways can this happen? Explain your reasoning.

(ii) Now n people $Q_1, Q_2, \ldots Q_n$ sit in order around a circular table with Q_1 sitting to Q_n's right.

(a) Suppose that all n people make the statement "the person on my left is lying *and* the person on my right is telling the truth". Explain why everyone is lying.

(b) Suppose now that every person makes the statement "either the people to my left and right are both lying *or* both are telling the truth". If at least one person is lying, show that n is a multiple of three.

Bonus question (not MAT)
A super-spy is in a casino with £100 of HM Government's money to gamble. He is offered the following fair (but boring) game;

The player flips a fair coin. If they flip tails, they lose 40% of their money. If they flip heads, they win 50% of their money.

The spy would never consider gambling anything other than all of his money, given half a chance. Is this a good idea?

Later in the evening, the super-spy is offered the following amped-up version of the game;

The player flips a fair coin one thousand times in a row. Each time they flip tails, they lose 40% of their money. Each time they flip heads, they win 50% of their money.

- Should we let the spy play this game, or should we get him out of the casino now?
- How much money do you expect him to have after playing this game?
- What's the most likely amount of money that he will have after playing this game?

The characters in the worksheet are fictional, and any resemblance to any person (living, dead, or the intellectual property of Ian Fleming/ EON Productions) is purely coincidental.