Something a bit different this week; we're revisiting the bonus problems, especially the ones we haven't really talked about on stream. These questions are not MAT questions, they're just extra bits of maths that I wanted to share with you.

Bonus problem (11 June)
When you roll two normal fair six-sided dice, the sum is one of the following totals with the following probabilities;

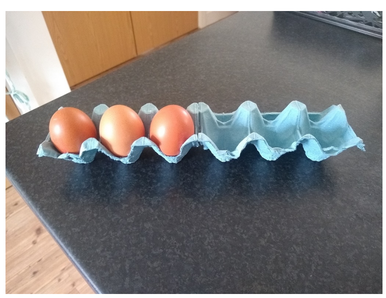
Total	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

I've got two six-sided dice, and each has a whole number greater than or equal to 1 on each side. My two dice are not the same, but they are each 'fair' in the sense that each of the six sides is equally likely to be rolled. When I roll my two dice, the sum might be any one of the same totals as above with the same probabilities as above. What are the numbers on my dice?

If you want to check your solution, search for "Sicherman dice"

Extension 3D-print these dice and use them to spice up Monopoly or D&D.

Bonus problem (18 June)
I've got three eggs and a long thin eggbox, which has six spaces for eggs in a row.



There are $\binom{6}{3} = 20$ ways that the eggs could be put in the box, and I'd like to take photos of all the possibilities. I'm worried about breaking the eggs, so in between photos I will only move one egg, and I will move it exactly one space along in the eggbox. I'd also like to avoid unnecessary moves, so no photos should be repeated.

Can you find a sequence of 20 photos that follows these rules, or prove that it's impossible?

Eggs-tension: What if I had two eggs instead of three? What if I had four eggs?

Bonus question (25 June)
Exactly one of the following numbers is prime. Which one? No calculators allowed!

$$91, \quad 3599, \quad 80111, \quad 125729, \quad 1010101, \quad 104060401 \quad 987654321.$$

Find a non-trivial factor of each of the other numbers (not 1 or the number itself).

Bonus question (02 July)
Let's say that you've got a circle of radius 1. The largest triangle that fits inside that circle is an equilateral triangle with all three corners on the circle (perhaps you could think about how you would prove this).

What if, instead of a circle, you had a pentagon of side length 1? Can you describe the largest triangle that fits inside that pentagon?

Bonus question (09 July)
I've got the Ace (A), Jack (J), King (K), and Queen (Q) of each of Clubs ($\clubsuit$), Diamonds ($\diamondsuit$), Hearts ($\heartsuit$), and Spades ($\spadesuit$), and I'm going to deal these cards out to form a 4×4 grid. I'd like one of each suit ($\clubsuit$, $\diamondsuit$, $\heartsuit$, $\spadesuit$) in each row and in each column, and one of each face card (A, J, K, Q) in each row and each column. Find a way to do this.

$$\begin{array}{cccc} A\spadesuit & K\heartsuit & Q\clubsuit & J\diamondsuit \\ J\clubsuit & A\clubsuit & K\spadesuit & Q\diamondsuit \\ Q\heartsuit & J\spadesuit & A\diamondsuit & K\clubsuit \\ Q\spadesuit & K\diamondsuit & J\heartsuit & A\heartsuit \end{array}$$

The arrangement pictured here is not a solution because it's got two $\clubsuit$ in the second row, and two Q in the first column (and several other issues).

Bonus question (16 July)
A super-spy is in a casino with £100 of HM Government's money to gamble. He is offered the following fair (but boring) game;

The player flips a fair coin. If they flip tails, they lose 40% of their money. If they flip heads, they win 50% of their money.

The spy would never consider gambling anything other than all of his money, given half a chance. Is this a good idea?

Later in the evening, the super-spy is offered the following amped-up version of the game;

The player flips a fair coin one thousand times in a row. Each time they flip tails, they lose 40% of their money. Each time they flip heads, they win 50% of their money.

- Should we let the spy play this game, or should we get him out of the casino now?
- How much money do you expect him to have after playing this game?
- What's the most likely amount of money that he will have after playing this game?

Bonus question (23 July)
This is a game involving three players (P_1, P_2, and P_3) and three cakes. Initially each player has one cake.

Taking it in turns, starting with the first player P_1, each player chooses someone and redistributes their cake among the players. For example, I might choose someone else to give all of their cake to me. Or I might be nice and choose someone to split their cake in the ratio 70:20:10, giving the big piece to me, giving the middle-sized piece to the other player as a sort of bribe, and letting my victim keep 10% of their cake for themselves (or any other ratio). Here's another example of how the game might progress;

- Each player starts with one cake.
- P_1 decides (greedily) that P_3 should give their entire cake to P_1.
- P_1 now has two cakes, P_2 has one cake, and P_3 has no cake.
- P_2 sees that P_1 has the most cake, but is worried that P_3 will steal all their cake. They choose P_1 and take half a cake for themselves, but let P_1 keep the remaining $1\frac{1}{2}$ cakes.
- P_1 now has $1\frac{1}{2}$ cakes, P_2 has $\frac{1}{2}$ a cake, and P_3 has no cake.
- Finally, P_3 takes revenge on P_1 by choosing them and redistributing all their cake to P_3.
- At the end of the game, P_1 has no cake, P_2 has $\frac{1}{2}$ a cake, and P_3 has $1\frac{1}{2}$ cakes.

Each player is aware of the rules and would like to end the game with as much cake as possible. What should they do?

Extension: (very hard) What if there are more than 3 players?

Bonus question (30 July)
You might have seen that the derivative of e^x is e^x. In other words, e^x is an example of a function $f(x)$ with $f'(x) = f(x)$.

The derivative of e^{kx} is ke^{kx}. Can you find two functions $f(x)$ and $g(x)$ with

$$f'(x) = g(x), \quad g'(x) = f(x) \quad ?$$

If you've seen the derivative of trigonometric functions, then you already know examples of four functions $f(x)$, $g(x)$, $h(x)$, $j(x)$ with

$$f'(x) = g(x), \quad g'(x) = h(x), \quad h'(x) = j(x), \quad j'(x) = f(x).$$

Can you find three functions $f(x)$, $g(x)$, $h(x)$ with

$$f'(x) = g(x), \quad g'(x) = h(x), \quad h'(x) = f(x) \quad ?$$

Bonus bonus problem (06 August)
Base-d is a way of writing numbers where the place value of each digit is given by powers of d. The digits are normally allowed to be from 0 to $d-1$ inclusive, making up extra symbols for extra digits beyond 9 if we need to (usually capital letters first). For example, in base 10 (decimal) the place values are powers of 10, and in base 2 (binary) the place values are powers of 2. It's a true fact that any positive whole number can be written in binary (that is, written as a sum of some powers of two).

Consider base-4, with allowed digits 0, 1, 2, and 3. Write the first few whole numbers in base-4. What's the relationship between binary and base-4?

Consider base-minus-3, with allowed digits 0, 1, and 2. The place values are 1, -3, 9, -27, and so on. Does this work: can you write all positive whole numbers in this system?

Consider base-$\sqrt{2}$, with allowed digits 0 and 1. Does this work?

Consider base-i, with allowed digits 0 and 1. Does this work?