MAT 2009 Q1A
The smallest value of

$$I(a) = \int_0^1 \left(x^2 - a\right)^2 \, \mathrm{d}x,$$

as a varies, is

(a) $\dfrac{3}{20}$, (b) $\dfrac{4}{45}$, (c) $\dfrac{7}{13}$, (d) 1.

MAT 2007 Q3
Let

$$I(c) = \int_0^1 \left((x-c)^2 + c^2\right) \, \mathrm{d}x$$

where c is a real number.

(i) Sketch $y = (x-1)^2 + 1$ for the values $-1 \leqslant x \leqslant 3$ and show on your graph the area represented by the integral $I(1)$.

(ii) Without explicitly calculating $I(c)$, explain why $I(c) \geqslant 0$ for any value of c.

(iii) Calculate $I(c)$.

(iv) What is the minimum value of $I(c)$ (as c varies)?

(v) What is the maximum value of $I(\sin\theta)$ as θ varies?

MAT 2007 Q1H

Given a function $f(x)$, you are told that

$$\begin{aligned}\int_0^1 3f(x)\,\mathrm{d}x + \int_1^2 2f(x)\,\mathrm{d}x &= 7,\\ \int_0^2 f(x)\,\mathrm{d}x + \int_1^2 f(x)\,\mathrm{d}x &= 1.\end{aligned}$$

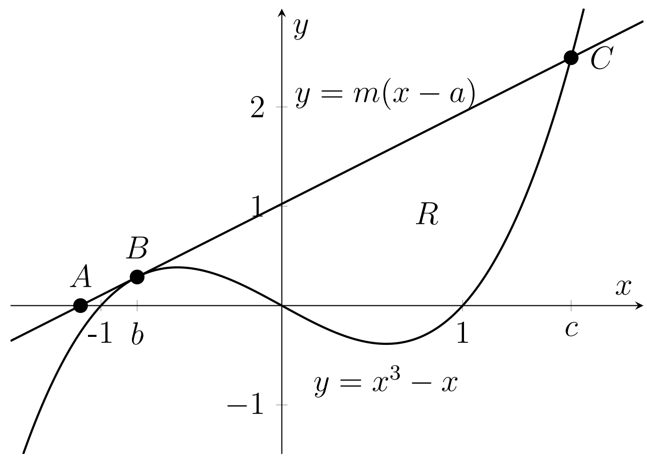
It follows that $\int_0^2 f(x)\,\mathrm{d}x$ equals

(a) -1, (b) 0, (c) $\frac{1}{2}$, (d) 2.

MAT 2011 Q3

The graphs of $y = x^3 - x$ and $y = m(x - a)$ are drawn on the axes below. Here $m > 0$ and $a \leqslant -1$.

The line $y = m(x - a)$ meets the x-axis at $A = (a, 0)$, *touches* the cubic $y = x^3 - x$ at B and intersects again with the cubic at C. The x-coordinates of B and C are respectively b and c.



(i) Use the fact that the line and cubic *touch* when $x = b$, to show that $m = 3b^2 - 1$.

(ii) Show further that

$$a = \frac{2b^3}{3b^2 - 1}.$$

(iii) If $a = -10^6$, what is the approximate value of b?

(iv) Using the fact that

$$x^3 - x - m(x - a) = (x - b)^2 (x - c)$$

(which you need not prove), show that $c = -2b$.

(v) R is the finite region bounded above by the line $y = m(x - a)$ and bounded below by the cubic $y = x^3 - x$. For what value of a is the area of R largest?

Show that the largest possible area of R is $\frac{27}{4}$.

Bonus question (not MAT)
I've been thinking about applying effects to a sine wave (which might cause your voice to sound like a robot). My idea is to do something to "lower the resolution" of the sine wave.

Let $[x]$ be the integer closest to x, so $[\pi] = 3$ and $[-2] = -2$. We'll round numbers midway between integers upwards, so $[-1.5] = -1$ and $[0.5] = 1$.

- Sketch $y = [\sin x]$
- Sketch $y = \dfrac{1}{3}[3 \sin x]$
- Sketch $y = \dfrac{4}{3}\left[\dfrac{3}{4} \sin x\right]$

Extension: Send these waves through a speaker. Work out how to do this for waves that aren't sine waves. Does it sound like a robot, or was this just a rubbish idea?