Warm-up (not MAT)
The first three terms of an arithmetic series are 1, x, and 4. Find x.

The first three terms of a geometric series are 1, x, and 4. Find x.

Three numbers are in arithmetic progression if they have a common difference. Three numbers are in geometric progression if they have a common ratio. What can you say if three numbers are in arithmetic progression and in geometric progression?

MAT 2010 Q1B
The sum of the first $2n$ terms of

$$1, \quad 1, \quad 2, \quad \frac{1}{2}, \quad 4, \quad \frac{1}{4}, \quad 8, \quad \frac{1}{8}, \quad 16, \quad \frac{1}{16}, \quad \ldots$$

is

(a) $2^n + 1 - 2^{1-n}$, (b) $2^n + 2^{-n}$, (c) $2^{2n} - 2^{3-2n}$, (d) $\dfrac{2^n - 2^{-n}}{3}$.

MAT 2009 Q2

A list of real numbers $x_1, x_2, x_3, \dots$ is defined by $x_1 = 1$, $x_2 = 3$ and then for $n \geqslant 3$ by

$$x_n = 2x_{n-1} - x_{n-2} + 1.$$

So, for example,

$$x_3 = 2x_2 - x_1 + 1 = 2 \times 3 - 1 + 1 = 6.$$

(i) Find the values of x_4 and x_5.

(ii) Find values of real constants A, B, C such that $\underline{\text{for } n = 1, 2, 3}$,

$$x_n = A + Bn + Cn^2. \qquad (*)$$

(iii) Assuming that equation $(*)$ holds true $\underline{\text{for all } n \geqslant 1}$, find the smallest n such that $x_n \geqslant 800$.

(iv) A second list of real numbers $y_1, y_2, y_3, \dots$ is defined by $y_1 = 1$ and

$$y_n = y_{n-1} + 2n$$

Find, explaining your reasoning, a formula for y_n which holds for $n \geqslant 2$.

What is the approximate value of x_n/y_n for large values of n?

MAT 2008 Q2

(i) Find a pair of positive integers, x_1 and y_1, that solve the equation

$$(x_1)^2 - 2(y_1)^2 = 1.$$

(ii) Given integers a, b, we define two sequences $x_1, x_2, x_3, \ldots$ and $y_1, y_2, y_3, \ldots$ by setting

$$x_{n+1} = 3x_n + 4y_n, \qquad y_{n+1} = ax_n + by_n, \qquad \text{for } n \geqslant 1.$$

Find *positive* values for a, b such that

$$(x_{n+1})^2 - 2(y_{n+1})^2 = (x_n)^2 - 2(y_n)^2.$$

(iii) Find a pair of integers X, Y which satisfy $X^2 - 2Y^2 = 1$ such that $X > Y > 50$.

(iv) Using the values of a and b found in part (ii), what is the approximate value of x_n/y_n as n increases?

Bonus question (not MAT)
I've been watching some best-of-five esports matches. This is a series of five games between two teams, with the first team to win three games being declared the winner of the match. I don't support any of the teams in particular, but I'm interested in the different narrative arcs that you can get out of a best-of-five match. I suppose really it's a first-to-three match, because they stop playing games once one of the teams has won three of the best-of-five.

As a simpler example to show you what I mean, let's think about a best-of-three match between team A and team B. It might be the case that team A wins both the first and second games (and the third game doesn't get played). Alternatively, it might be the case that team B wins both the first and second games, but because I don't support either team in particular, I count that as the same narrative arc. The other narrative arcs are;

- one team wins the first game, the other team draws level in game two, but then the team that won the first game wins the third game
- the "reverse sweep"; one team wins the first game, but then the other team wins two in a row

Find and describe all of the possible narrative arcs for a best-of-five match.

Extension: How many possible narrative arcs are there for a best-of-$(2n+1)$ match?