Warm-up (based on MAT 2014 Q1G)
Expand $(1 + x + x^2)^2$.
What is the coefficient of x^2 in the expansion of $(1 + x + x^2)^3$?
Now let n be a positive integer.

- What is the coefficient of x^2 in the expansion of $(1 + x + x^2)^n$?
- What is the coefficient of x^3y^5 in the expansion of $(1 + x + y)^n$?
- What is the coefficient of x^3y^5 in the expansion of $(1 + xy + y^2)^n$?

MAT 2008 Q1A
The function

$$y = 2x^3 - 6x^2 + 5x - 7$$

has

(a) no stationary points;

(b) one stationary point;

(c) two stationary points;

(d) three stationary points.

Extension: Find a condition on a, b, c, and d, for the cubic $y = ax^3 + bx^2 + cx + d$ to have two stationary points.

MAT 2008 Q1E
The highest power of x in

$$\left\{\left[\left(2x^6+7\right)^3+\left(3x^8-12\right)^4\right]^5+\left[\left(3x^5-12x^2\right)^5+\left(x^7+6\right)^4\right]^6\right\}^3$$

is

(a) x^{424}, (b) x^{450}, (c) x^{500}, (d) x^{504}.

MAT 2008 Q1D
When

$$1+3x+5x^2+7x^3+\cdots+99x^{49}$$

is divided by $x-1$ the remainder is

(a) 2000, (b) 2500, (c) 3000, (d) 3500.

MAT 2009 Q1I
The polynomial

$$n^2x^{2n+3}-25nx^{n+1}+150x^7$$

has x^2-1 as a factor

(a) for no values of n;

(b) for $n=10$ only;

(c) for $n=15$ only;

(d) for $n=10$ and $n=15$ only.

MAT 2007 Q1F
The equation

$$8^x + 4 = 4^x + 2^{x+2}$$

has

(a) no real solutions;

(b) one real solution;

(c) two real solutions;

(d) three real solutions.

MAT 2008 Q1H
The equation

$$9^x - 3^{x+1} = k$$

has one or more real solutions precisely when

(a) $k \geqslant -\frac{9}{4}$, (b) $k > 0$, (c) $k \leqslant -1$, (d) $k \geqslant \frac{5}{8}$.

Bonus question (not MAT)
A triangle ABC has side lengths $AB = 3$, $BC = 5$, $CA = 7$. Find the angle $\angle ABC$.

Find another triangle with integer side lengths and the same angle at B.

Extension: Find another triangle with integer side lengths and half the angle at B.

For more information or to check your answers, search for "Eisenstein triple".