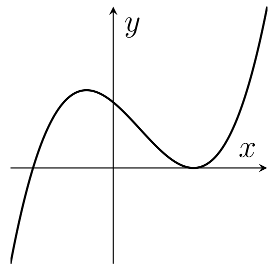
MAT 2008 Q3

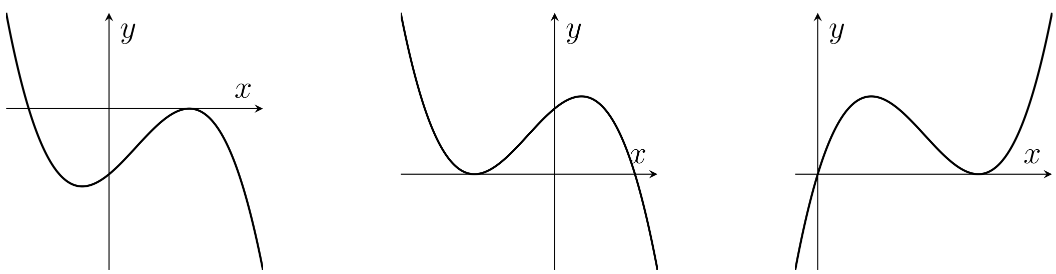
(i) The graph $y = f(x)$ of a certain function has been plotted below.



On the next three pairs of axes (A), (B), (C) are graphs of

$$y = f(-x), \qquad f(x-1), \qquad -f(x)$$

in some order. Say which axes correspond to which graphs.



(A) (B) (C)

(ii) Sketch, on the same axes, graphs of *both* of the following functions

$$y = 2^{-x^2} \qquad \text{and} \qquad y = 2^{2x-x^2}.$$

Carefully label any stationary points.

(iii) Let c be a real number and define the following integral

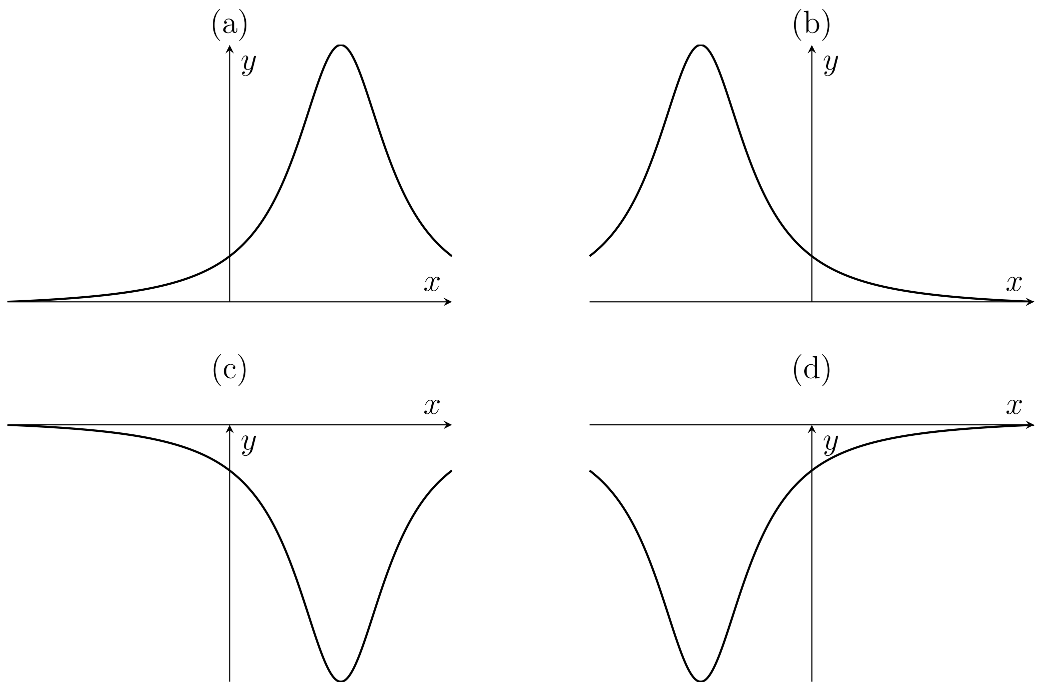
$$I(c) = \int_0^1 2^{-(x-c)^2}\,\mathrm{d}x.$$

State the value(s) of c for which $I(c)$ is largest. Briefly explain your reasoning. [Note you are not being asked to calculate this maximum value.]

MAT 2008 Q1G

Which of the graphs below is a sketch of

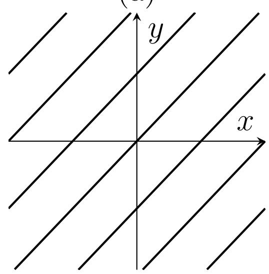
$$y = \frac{1}{4x - x^2 - 5} \quad ?$$



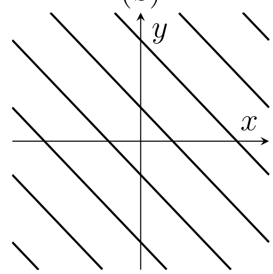
MAT 2009 Q1G

The graph of all those points (x, y) in the xy-plane which satisfy the equation $\sin y = \sin x$ is drawn in

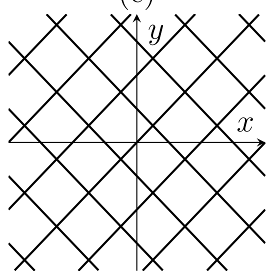
(a)



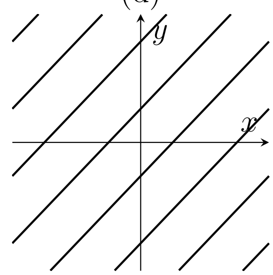
(b)



(c)



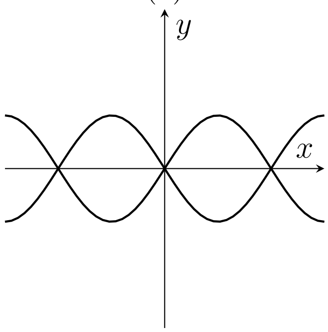
(d)



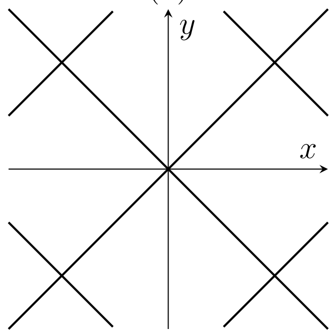
MAT 2015 Q1G

The graph of $\cos^2(x) = \cos^2(y)$ is sketched in

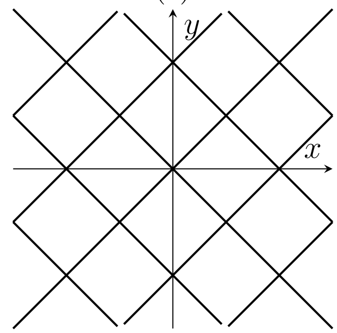
(a)



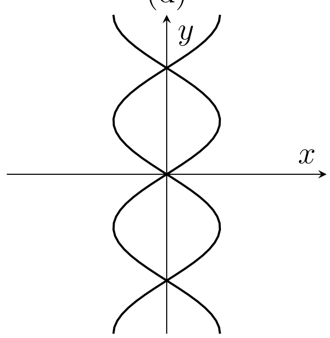
(b)



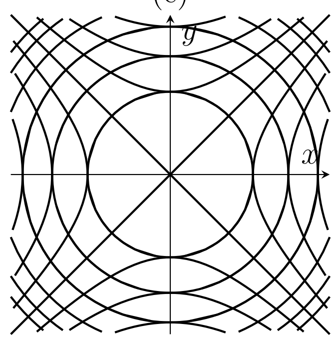
(c)



(d)



(e)



Bonus question (not MAT)

In some car racing games or sports, you have a sort of power-boost you can use for a short amount of time. This question will consider a driver with two different speed boosts available; should they use both at the same time or one after the other?

Suppose that the car has power P normally, and has two ways to briefly increase this. While using the first power-boost, the power is increased to $P + P_1$ for T seconds. While using the second power-boost, the power is increased to $P + P_2$ for T seconds. If both power-boosts are used simultaneously, the power is increased to $P + P_1 + P_2$ for T seconds. The velocity of the car in any case is the cube root of its power. (Footnote for Physics fans[1]).

Should the driver use one power-boost and then immediately use the other, or would it be better to use both power-boosts together, and then drive normally for T seconds– which of these options makes the car go further over the course of $2T$ seconds? (Footnote for car racing fans[2]).

[1]This isn't dimensionally consistent; there should be some sort of pre-factor constant, but it turns out that the decision in this question doesn't depend on the constant. The idea behind the cube root is that the drag on the car is roughly proportional to the square of the velocity, and this should balance the force from the engine (ignoring any time it takes to accelerate up to the new top speed). Power is force times velocity, so will be roughly proportional to the cube of the velocity.

[2]This is just looking at straight-line speed, ignoring corners or overtaking; in fact, we're ignoring any effects from other cars entirely.