The theme this week is 'tricky multiple-choice questions'. You have been warned!

MAT 2013 Q1H

The area bounded by the graphs

$$y = \sqrt{2 - x^2} \quad \text{and} \quad x + \left(\sqrt{2} - 1\right) y = \sqrt{2}$$

equals

(a) $\dfrac{\sin\sqrt{2}}{\sqrt{2}}$, (b) $\dfrac{\pi}{4} - \dfrac{1}{\sqrt{2}}$, (c) $\dfrac{\pi}{2\sqrt{2}}$, (d) $\dfrac{\pi^2}{6}$.

MAT 2009 Q1J

The number of *pairs* of *positive integers* x, y which solve the equation

$$x^3 + 6x^2y + 12xy^2 + 8y^3 = 2^{30}$$

is

(a) 0, (b) 2^6, (c) $2^9 - 1$, (d) $2^{10} + 2$.

MAT 2012 Q1H

In the region $0 < x \leqslant 2\pi$, the equation

$$\int_0^x \sin(\sin t)\,\mathrm{d}t = 0$$

has

(a) no solution, (b) one solution, (c) two solutions, (d) three solutions.

MAT 2007 Q1I

Given that a and b are positive and

$$4\left(\log_{10} a\right)^2 + \left(\log_{10} b\right)^2 = 1,$$

then the greatest possible value of a is

(a) $\frac{1}{10}$, (b) 1, (c) $\sqrt{10}$, (d) $10^{\sqrt{2}}$.

MAT 2014 Q1J

For all real numbers x, the function $f(x)$ satisfies

$$6 + f(x) = 2f(-x) + 3x^2 \left(\int_{-1}^{1} f(t)\, \mathrm{d}t \right).$$

It follows that $\int_{-1}^{1} f(x)\, \mathrm{d}x$ equals

(a) 4, (b) 6, (c) 11, (d) $\frac{27}{2}$, (e) 23.

MAT 2008 Q1J

In the range $0 \leqslant x < 360°$ the equation

$$(3 + \cos x)^2 = 4 - 2\sin^8 x$$

has

(a) 0 solutions, (b) 1 solution, (c) 2 solutions, (d) 3 solutions.

MAT 2010 Q1I
For a positive number a, let

$$I(a) = \int_0^a \left(4 - 2^{x^2}\right) \, \mathrm{d}x.$$

Then $\dfrac{\mathrm{d}I}{\mathrm{d}a} = 0$ when a equals

(a) $\dfrac{1+\sqrt{5}}{2}$, (b) $\sqrt{2}$, (c) $\dfrac{\sqrt{5}-1}{2}$, (d) 1.

MAT 2011 Q1I
In the range $0 \leqslant x < 360°$ the equation

$$\sin^8 x + \cos^6 x = 1$$

has

(a) 3 solutions , (b) 4 solutions, (c) 6 solutions, (d) 8 solutions.

Bonus question (not MAT)
This is a game for two players (player 1 and player 2). First each player will choose a sequence three coin flips; either heads (H) and tails (T). Then we will flip a fair coin until one of the chosen sequences happens. For example, if player 1 chooses "HTT" and player 2 chooses "THH" and then the coin flips land as "HTHTHH" then player 2 wins.

Each player must choose a sequence of three results, with player 1 choosing first, and then player 2 choosing a different sequence of three results. Show that player 2 can choose strategically based on player 1's choice, so that they have more than 50% chance to win the game.

This is called Penney's game, and the solution to the question above is on Wikipedia.