Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2014

October 29, 2014

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Tables 1 and 2. Overall 178 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

	Numbers		Percentages %	
	2014	(2013)	2014	(2013)
Distinction	55	(55)	30.9	(30.9)
Pass	103	(103)	57.87	(57.87)
Partial Pass	12	(13)	6.74	(7.3)
Incomplete	1	(0)	0.56	(0)
Fail	7	(7)	3.93	(3.93)
Total	178	(178)	100	(100)

Table 2: Numbers in each class (Honour Moderations)

	Numbers			Percentages %		
	(2012)	(2011)	(2010)	(2012)	(2011)	(2010)
I	(60)	(64)	(61)	(30.77)	(32.49)	(29.9)
II	(123)	(119)	(125)	(63.08)	(60.41)	(59.12)
III	(4)	(9)	(11)	(2.05)	(4.57)	(6.08)
Pass	(0)	(1)	(0)	(0)	(0.51)	(0)
Honours (unclassified)	(1)	(0)	(0)	(0.51)	(0)	(0)
Fail	(7)	(4)	(7)	(3.59)	(2.03)	(3.87)
Total	(195)	(197)	(204)	(100)	(100)	(100)

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the Preliminary Examination in Mathematics.
- **Marking of scripts.**
  As in previous years, no scripts were multiply marked by Moderators; however all marking was conducted according to a detailed marking scheme, strictly adhered to. For details of the extensive checking process, see Part II, Section A.

B. New examining methods and procedures

This is the second year of the new Preliminary Examination in Mathematics, which has replaced Honour Moderations. The same methods and procedures were followed as in 2013.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

- This year, each Assessor was asked to submit estimated class boundaries for each question marked. It was felt these were difficult to estimate with any certainty and also less helpful than anticipated, as borderline scripts are considered at the final meeting anyway. We therefore do not recommend that this practice be continued.
- The Moderators discussed the requirement for shorter questions in the new Preliminary Examination compared with Honour Moderations. Several Moderators noted the difficulty of setting shorter questions that contain enough elementary material for the weaker candidates while still testing stronger candidates to an appropriate depth. These concerns have been raised with the Teaching Committee and will be discussed as part of the ongoing *Review of Prelims.*
- Following the recommendation of last year's Moderators, the Computational Mathematics Assessor was present at the start of the final Examiners' Meeting. This enabled the Moderators to ensure that the USM results in Computational Mathematics were appropriate and, in particular, to determine which candidates did not meet the requirements for a Pass in this assessment. We recommend that this practice be continued.

D. Notice of examination conventions for candidates

The Notice to Candidates, containing details of the examinations and assessment, including the Examination Conventions, was issued to all candidates at the beginning of Trinity term. All notices and the Examination Conventions in full are available at http://www.maths.ox.ac.uk/notices/undergrad.

Part II

A. General Comments on the Examination

Acknowledgements

The Moderators would like to thank the academic administration team, in particular Nia Roderick, Charlotte Turner-Smith and Helen Lowe, for all their work in running the examinations system. We also thank Waldemar Schlackow and Helen Lowe for running the marks database.

We are very grateful to Prof. Colin Macdonald for administering the Computation Mathematics projects. We would also like to thank the Assessors Dr Robert Gaunt, Dr Cameron Hall, Dr Arthur Lipstein and Dr Tomasz Lukowski for their assistance with marking.

Timetable

The examinations began on Monday 23rd June at 2.30pm and ended on Friday 27th June at 12.00pm.

Medical certificates and other special circumstances

Four medical certificates were received from the Proctors' office. The Moderators gave careful regard to each case, scrutinised the relevant candidates' marks and adjusted them where appropriate.

Setting and checking of papers and marks processing

The Moderators first set questions, a checker then checked the draft papers and, following any revisions, the Moderators met in Hilary term to consider the questions on each paper. They met a second time to consider the papers at the end of Hilary term making further changes as necessary before finalising the questions. A meeting was held in early Trinity term for a final proof read. The Camera Ready Copy (CRC) was prepared and each Moderator signed off the papers. The CRC was submitted to Examination Schools in week 4 of Trinity term.

The examination scripts were collected from Ewert House and delivered to the Mathematical Institute.

Once the scripts had been marked and the marks entered, a team of graduate checkers, under the supervision of Nia Roderick and Charlotte Turner-Smith, sorted all the scripts for each paper of the examination. They carefully cross checked against the marks scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. A number of errors were corrected, with each change checked and signed by an Examiner, at least one of whom was present throughout the process. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The candidates under consideration are Mathematics and Mathematics & Statistics candidates, 178 in total. We do not distinguish between them as they all take the same papers.

Marks for each individual paper are reported in university standardised form (USM) requiring at least 70 for a Distinction, 40–69 for a Pass, and below 40 for a Fail.

As last year the Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average proportion in each class over the past five years, together with recent historical data for Honour Moderations.

The raw marks were recalibrated to arrive at the USMs reported to candidates, adopting the procedures outlined below. These procedures are similar to the ones used in previous years.

To ensure equal weightings across all subjects, papers were first standardised to have similar proportions of candidates attaining each class. A piecewise linear mapping was adopted to produce a USM from a raw mark. The default algorithm for each paper works as follows.

1. Candidates' raw marks for a given paper are ranked in descending order. Here the population data used is the set of marks for all candidates in Mathematics or Mathematics & Statistics.

2. The default percentages p_1 of Distinctions and p_2 of nominal upper seconds (USM 60–69) in this population are selected, these percentages being similar to those adopted in previous years.

3. The candidate at the p_1-th percentile from the top of the ranked list is identified and assigned a USM of 70. Let the corresponding raw mark be denoted by R_1.

4. Similarly, the candidate at the $(p_1 + p_2)$-th percentile from the top of the list is assigned a USM of 60 and the corresponding raw mark is

denoted by R_2.

5. The line segment between $(R_1, 70)$ and $(R_2, 60)$ is extended linearly to the USMs of 72 and 57 respectively. Denote the raw marks corresponding to USMs of 72 and 57 by C_1 and C_2 respectively. Line segments are then drawn connecting $(C_1, 72)$ to $(100, 100)$ and connecting $(C_2, 57)$ to $(0, 0)$.

Thereby a piecewise linear map is constructed whose vertices, at $\{(0,0), (C_2, 57), (C_1, 72), (100, 100)\}$, are located away from any class boundaries.

A first run of the outlined scaling algorithm was performed. It was confirmed that the procedure resulted in a reasonable proportion of candidates in each class. The Moderators then used their academic judgement to make adjustments where necessary as described below. The Moderators were not constrained by the default scaling map and were able, for example, to insert more vertices if necessary.

To obtain the final classification, firstly a report from each Assessor was considered, describing the apparent relative difficulty and the general standard of solutions for each question on each paper. Moderators gave estimated class boundaries for each question based on the candidates' overall performance relative to the published qualitative class descriptors. This information was used to guide the setting of class borderlines on each paper.

The scripts of those candidates in the lowest part of each ranked list were scrutinised carefully to determine which attained the qualitative class descriptor for a pass on each paper. The gradient of the lower section of the scaling map was adjusted to place the pass/fail boderline accordingly.

Careful consideration was then given to the scripts of candidates at the Distinction/Pass boundary and at the USM 59/60 boundary (nominally the 2.1/2.2 borderline). Adjustments were made to the scaling maps where necessary to ensure that the candidates' performances matched the published qualitative class descriptors.

The Computational Mathematics assessment was considered separately. In consultation with the relevant Assessor it was agreed that no recalibration was required, so the raw marks (out of 40) were simply multiplied by 2.5 to produce a USM.

Finally, the class list for the cohort was calculated using the individual paper USMs obtained as described above and the following rules:

Distinction: both $Av_1 \geq 70$ and $Av_2 \geq 70$;

Pass: not meriting a Distinction and a USM of at least 40 on each paper;

Partial Pass: a USM of less than 40 on one or two papers;

Fail: a mark of less than 40 on three or more papers.

Here Av_2 is the average over the five written papers and Av_1 is the weighted average over these papers together with Computational Mathematics (counted as one third of a paper). The Moderators verified that the overall numbers in each class were in line with previous years, as shown in Tables 1 and 2.

The vertices of the final linear model used in each paper are listed in Table 3, where the x-coordinate is the raw mark and the y-coordinate the USM.

Table 3: Vertices of final piecewise linear model

Paper	Positions of vertices			
I	(0,0)	(52.8,57)	(73.8,72)	(100,100)
II	(0,0)	(45.7,57)	(77.2,72)	(100,100)
III	(0,0)	(42.3,57)	(70.8,72)	(100,100)
IV	(0,0)	(45,57)	(75,72)	(100,100)
V	(0,0)	(35,57)	(63.2,72)	(100,100)
CM	(0,0)	(40,100)		

Table 4 gives the rank list of average USM scores, showing the number and percentage of candidates with USM greater than or equal to each value.

Table 4: Rank list of average USM scores

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	%
93	1	1	0.56
91	2	2	1.12
88	3	3	1.69
87	4	4	2.25
86	5	5	2.81
85	6	7	3.93
84	8	11	6.18
82	12	12	6.74
80	13	13	7.30
79	14	16	8.99
78	17	17	9.55
77	18	18	10.11
76	19	21	11.80
75	22	24	13.48
74	25	30	16.85
73	31	33	18.54
72	34	40	22.47
71	41	47	26.40

Table 4: Rank list of average USM scores (continued)

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	%
70	48	55	30.90
69	56	61	34.27
68	62	71	39.89
67	72	81	45.51
66	82	89	50.00
65	90	97	54.49
64	98	107	60.11
63	108	115	64.61
62	116	121	67.98
61	122	127	71.35
60	128	133	74.72
59	134	138	77.53
58	139	143	80.34
57	144	146	82.02
56	147	149	83.71
55	150	153	85.96
54	154	155	87.08
52	156	158	88.76
51	159	160	89.89
50	161	162	91.01
49	163	164	92.13
48	165	167	93.82
46	168	169	94.94
44	170	170	95.51
43	171	171	96.07
42	172	172	96.63
39	173	173	97.19
37	174	174	97.75
35	175	175	98.31
34	176	176	98.88
25	177	177	99.44
0	178	178	100

B. Equal opportunities issues and breakdown of the results by gender

Table 5 shows the performances of candidates broken down by gender.

Table 5: Breakdown of results by gender

Class	Total Number	Total %	Male Number	Male %	Female Number	Female %
Distinction	55	30.9	43	34.96	12	21.82
Pass	103	57.87	64	52.03	39	70.91
Partial Pass	12	6.74	10	8.13	2	3.64
Incomplete	1	0.56	1	0.81	0	0
Fail	7	3.93	5	4.07	2	3.64
Total	178	100	123	100	55	100

C. Statistics on candidates' performance in each part of the Examination

The performance statistics for each individual assessment are given in the tables below: Paper I in Table 6, Paper II in Table 7, Paper III in Table 8, Paper IV in Table 9, Paper V in Table 10 and Computational Mathematics in Table 11. The number of candidates who received a failing USM of less than 40 on each paper is given in Table 12 on page 11.

Table 6: Statistics for Paper I

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	11.38	11.38	4.47	148	0
Q2	11.69	11.69	3.77	160	0
Q3	15.85	15.85	3.36	152	0
Q4	13.99	13.99	3.90	73	0
Q5	11.74	11.74	3.33	145	0
Q6	13.63	13.63	3.51	158	0
Q7	11.31	11.31	3.72	51	0

Table 7: Statistics for Paper II

Question	Average Mark		Std	No. of Attempts	
Number	All	Used	Dev	Used	Unused
Q1	12.12	12.12	3.18	167	0
Q2	12.36	12.36	4.02	56	0
Q3	12.32	12.32	3.39	131	0
Q4	11.13	11.15	5.50	71	1
Q5	14.06	14.06	3.99	118	0
Q6	14.24	14.25	4.10	165	1
Q7	11.47	11.47	5.52	165	0

Table 8: Statistics for Paper III

Question	Average Mark		Std	No. of Attempts	
Number	All	Used	Dev	Used	Unused
Q1	12.98	12.98	4.56	163	0
Q2	8.38	8.38	4.93	65	0
Q3	10.92	10.98	4.02	126	1
Q4	12.03	12.03	3.75	134	0
Q5	10.31	10.31	4.72	111	0
Q6	14.86	14.86	4.09	107	0
Q7	11.12	11.12	5.12	176	0

Table 9: Statistics for Paper IV

Question	Average Mark		Std	No. of Attempts	
Number	All	Used	Dev	Used	Unused
Q1	6.41	6.63	4.77	65	4
Q2	13.65	13.65	2.89	176	0
Q3	11.05	11.14	4.07	161	3
Q4	14.52	14.52	3.96	129	0
Q5	13.28	13.35	4.17	159	1
Q6	8.55	8.93	5.12	54	4
Q7	12.89	13.03	4.06	140	2

Table 10: Statistics for Paper V

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	10.61	10.61	5.32	163	0
Q2	9.82	9.87	4.01	131	1
Q3	6.69	6.81	4.59	58	1
Q4	10.45	10.45	3.26	159	0
Q5	11.62	11.62	4.37	60	0
Q6	12.03	12.12	4.55	132	2
Q7	9.43	9.43	5.21	161	0

Table 11: Statistics for Computational Mathematics

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	16.34	16.34	3.53	170	0
Q2	15.54	15.54	3.89	72	0
Q3	18.27	18.27	1.80	108	0

Table 12: The number of failures on each paper

Paper	Number	%
I	8	4.5
II	11	6.2
III	9	5.1
IV	8	4.5
V	12	6.8
CM	2	1.1

D. Recommendations for Next Year's Examiners and Teaching Committee

Failure rate. The overall numbers of Failures and of Partial Passes were in line with last year. We agreed with last year's Examiners that the new Partial Pass classification is a positive development and gives weaker candidates a good chance to improve over the long vacation.

Comments on the syllabus. The Examiners discussed the difficulty of Paper V, which was again failed by more candidates than any other paper.

The material on three-dimensional calculus and applications has historically been challenging and unpopular, and is now covered in fewer lectures. The Geometry question on Paper IV also attracted particularly low marks. These observations will be fed back to the ongoing *Review of Prelims*.

Workload. The Examiners discussed the timescale for marking. It was generally agreed that, without the help of assessors, it was too tight to turn around the marks in time for script checking. It was suggested that 1.5 working days per full question is a reasonable timescale to allow careful marking while respecting the terms of good practice. However, the Examiners were concerned that the examination period should not extend further into the long vacation, and it was therefore suggested that more consistent use should be made of Assessors.

It was also noted that marking Prelims and Part A simultaneously is very difficult. It was proposed that the Department should avoid asking Part A lecturers to be Prelims examiners.

E. Comments on papers and on individual questions

Paper I

Question 1
This was attempted by the vast majority of candidates. Part (a) was generally done well. A good number of candidates did well on part (b), but many did not seriously attempt it or did so and made elementary mistakes (such as defining spaces via unions and complements of other spaces).

Question 2
Most candidates did this question. Part (a) was done very well, but the application of the rank-nullity formula in b)(i) often led to some confusion with inequalities. Part b)(ii) though unseen was done quite well.

Question 3
Attempted by almost all candidates this question, leaving aside some computational slips in part (b), was done very well.

Question 4
Only a minority of candidates tried this question. Part (a) and the calculations in (c) were done well, but (b), although it had a simple solution, was done less well.

Question 5
Many candidates attempted this. Part a.i was generally well done but some candidates gave imprecise or incorrect arguments. An answer saying for

example that a power of g is equal to g since G is finite received no marks. Part a.ii was generally well done. In part a.iii some candidates failed to justify why the order of the group generated by g is equal to the order of G and some gave a proof of Lagrange's theorem- which was not required. Many candidates did not manage to do the second part of b.i, a common mistake was to implicitly assume that the homomorphism is injective.

Several candidates used the 1st isomorphism theorem to answer b.ii rather than the results on the earlier part of the question. They generally did well but sometimes they failed to give a complete argument only guessing the correct number of homomorphisms. Overall part b.ii was more challenging and in particular very few candidates managed to do the last part of this.

Question 6
Most attempted this and many found the question accessible.

Parts a.i, a.ii were generally well done but some candidates did not show that the centre is a subgroup getting only partial credit for this part. Some failed to show that if an element is in the centre its inverse is in the centre too.

Part b.i was done by most candidates but some supplied a proof which was not required. In part b.ii some failed to list correctly the conjugacy classes. Many candidates attempted b.iii. However some used the fact that two permutations have the same cycle type iff they are conjugate. This was not shown earlier in the question so some points were taken off. Some candidates using this failed to explain precisely why it implies that the centre is trivial. Even though many candidates clearly had an idea why the centre is trivial they often gave incomplete arguments forgetting some cases and they only got partial credit.

Question 7
This was not a very popular question.

Part a.i was generally well done. In part a.ii some candidates showed that the map is 1-1 but did not prove that it is onto. Several failed to see that they had to show that the map is 1-1 and onto for every g.

The second part of a.ii was generally done well.

The first part of b.i was done by many candidates but several failed to see the point of the second part showing that the kernel is a subgroup (of G) rather than showing that it is contained in H. On part b.ii some showed that the group admits a homomorphism to S_3 but did not manage to give a correct argument for the cardinality of the image.

Paper II

Question 1
This was a very popular question. Part (a) was mostly bookwork and was generally well done. Part (iii) was more challenging and tested the candidates' understanding of the concepts of infimum and convergence.

Almost all candidates could show that the sequence x_n was increasing, but many struggled on the more challenging parts (b)(ii) and (b)(iii). Roughly half of the candidates deduced that x_n is bounded and that x_n therefore converges. However, many attempts stopped there, and only the very best candidates were able to prove that the limit is 0. In part (iii), many candidates proved that c_n is Cauchy and then used the fact that a real sequences converges if it is Cauchy to deduce that c_n converges. Unless this fact was proved, no credit was given. Indeed, there is a simple self-contained proof, which uses results that were proved earlier in the question.

Question 2
This was attempted by a minority of candidates. Only a)(ii) in part (a) caused any difficulties. Part (c) was generally done well. Candidates struggled somewhat with (b) although a number of different solutions were found.

Question 3
Many candidates attempted this.

Part a.i was generally well done but some candidates failed to do the second part. A small number showed that Cauchy is equivalent to convergent which was not required.

Surprisingly many candidates had difficulties with a.ii. Some did this directly without invoking the Cauchy condition but still got full marks. A smaller number used the integral test not giving a full justification for this and had a mark taken off. Part a.iii was generally well done.

Most candidates found b.i challenging and either gave a wrong answer or failed to justify their answer. Some used the limit form of the comparison test but failed to explain how they calculated the limit and a had a point taken off for this.

Many candidates attempted b.ii and had the right ideas but quite a few made mistakes in their calculations and had points taken off for this. Some tried to use the comparison test rather than Leibniz's test giving an incorrect justification for their answer and did not get any credit.

Question 4
The first part of this question asked for a proof of the continuous inverse function theorem. This was generally answered well, although a minority

of students were unable to provide full details of the ϵ-δ proof. The second half asked for a proof of the fact that a continuous injective map from an interval to the real line is strictly monotone. With the hint, this is an easy application of the Intermediate Value Theorem, and was generally very well answered. The final two parts were also done quite well. They were most easily answered by applying the previous result, but could alternatively be handled using the IVT.

Question 5
This question was based on the Weierstrass M-test. In the first part, the students were required to state and prove this test, and in the second part, they had to apply it. The final sub-part provided the most challenge. Many students failed to spot that the two series in (b)(i) and (b)(ii) are actually the same. Instead, they mistakenly attempted to apply the M-test by comparing the series to one with alternating signs.

Question 6
The first part was completely standard bookwork on Rolle's theorem and the Mean Value Theorem. This was one area where the weakest students were able to pick up some marks. In the second part, the students were asked to provide proofs or counterexamples to statements relating to the positivity of the derivative of a function. All three statements were true. Fortunately, only a few students made the error of asserting that a strictly increasing function must have strictly positive derivative.

Question 7
The first two parts consisted of standard bookwork, and both were answered very well by the majority of students. The final part was considerably more challenging, and less than a quarter of the students were able to give satisfactory answers. The simplest solution is a function that is identically zero except when $x = 1/n$.

Paper III

Question 1
This question was popular and generally well done. Most students managed part (a) without difficulty, although some weaker students showed a very flimsy grasp of the chain rule while attempting to perform the suggested change of variable. In part (b), instead of spotting that the left-hand side is already a perfect derivative, many laboriously calculated an integrating factor, often introducing errors on the way. The few students who mistakenly interpreted $\tan^{-1} x$ as $\cot x$ rather than $\arctan x$ were given credit for otherwise correct working. In part (c), many students made silly algebraic slips which turned a straightforward integration into a very complicated one. A

small penalty was applied to those who omitted the final inversion to obtain the explicit solution $y(x)$ as directed.

Question 2
This question was not popular and caused serious problems for many weaker students, who appeared not to understand even the most basic facts about partial differentiation and the chain rule.

Many fallacious "proofs" were given in part (a): no marks were given to those who claimed that $u_x = 1/x_u$, etc. Easy marks were available in part (b) just for applying the chain rule twice, but even this was too much for many. In part (c), partial credit was given to those who verified that the given function satisfies the PDE instead of obtaining it as the general solution.

Question 3
This question was quite popular but on the whole quite poorly done. In part (a), many students struggled with the basic analysis needed to identify the critical points correctly, and some had the classification criteria completely muddled. In part (b)(i), many thought that the relevant direction was $\boldsymbol{\nabla} f$ rather than $-\boldsymbol{\nabla} f$, and some instead tried to find the normal to the surface $z = f(x, y)$. A significant number failed even to compute $\boldsymbol{\nabla} f$ accurately.

Many of the weaker students did not even start part (b)(ii), and most of those who did were unable to express mathematically the concept of a curve with a given tangent direction. Credit was given to those stronger students showed that the level sets of $f(x, y)$ and $xy\left(x^2 - y^2 - 1\right)$ are orthogonal and argued from there, rather than by writing down a differential equation as the Examiner had intended.

Question 4
The bookwork parts (a)(i) and (ii) were done well, though some candidates didn't even attempt (forgot?) to prove the law of total probability. In (a)(iii), *both* $\mathbb{P}(A_i \cap A_j) = \mathbb{P}(A_i)\mathbb{P}(A_j)$ for all (i, j)-pairs *and* $\mathbb{P}(A_1 \cap A_2 \cap A_3) = \mathbb{P}(A_1)\mathbb{P}(A_2)\mathbb{P}(A_3)$ are required for independence. Many candidates gave one of these conditions but not the other. So to show the non-independence of A_1, A_2, A_3 it is sufficient to *show* that any one of these relationships does not hold – but some candidates wrote down intuition about why A_1, A_2, A_3 are not independent rather than *showing* they are not independent. There were many correct answers to (b)(ii)-(iv), including from candidates who lost quite a few marks earlier in the question.

Question 5
Part (a) was done well. In (b)(i) the question states that standard properties of a Binomial(n, p) can be assumed, so expectation $= np$ and variance $= npq$, from which $\mathbb{E}[Y \,|\, X = k]$ and $\mathbb{E}[Y^2 \,|\, X = k]$ are easily found because conditional on $X = k$ we have $Y \sim$ Binomial$(k, \frac{1}{2})$. However many candidates in

effect *proved* that $\mathbb{E}[\text{Binomial}(n,p)] = np$, etc, rather than *using* standard properties.

Very few candidates could get far with the variance of $X+Y$ in (b)(iii). Since X and Y are not independent, this involves calculating $\text{cov}(X,Y)$ or $\mathbb{E}[XY]$. Only a few candidates could write down the correct expression for $\text{var}(X+Y)$ in terms of $\text{var}(X)$, $\text{var}(Y)$ and $\text{cov}(X,Y)$, and even fewer could calculate $\text{cov}(X,Y)$ or $\mathbb{E}[XY]$.

Question 6
Many candidates attempting this question got good marks. Part (b)(i) was done well, and (b)(ii) often was too, though there were some errors in determining the right variance to use in (b)(ii).

Question 7
Part (a) was standard and done well, though many candidates lost a mark for not checking that their MLE was a maximum. In (b)(v) the question is to calculate a suitable sample size n *before* any data is collected, so the required value of n cannot depend on n_1, n_2 (nor on any estimate $\widehat{p}$ of p) since $n_1, n_2, \widehat{p}$ are unknown before any data is collected – many candidates missed this point.

Paper IV

Question 1
The standard of the answers to this question was surprisingly low. The reasons for this are unclear. Geometry is not a particularly popular option, and so the majority of students decided to attempt the other two questions from Section A. Very few students were able to express the transformation T in correct matrix form, and as a result, most students could not deduce that, when $|a| = 1$, T is a rotation of order 4.

Question 2
The first part of the problem was attempted and solved properly by almost all candidates. Apart from few mistakes in using the chain rule and few incorrect signs, everybody was able to find the right equations of motion. The only but very common difficulty, which was also the main problem in part (b), was the proper use of initial conditions. In the second part most of candidates found the correct solution to the equations of motion. However, more than a half used wrong initial conditions, which led them to conclude that the trajectory of particle is elliptic (compact) instead of hyperbolic (non-compact).

Question 3
There were a couple of unfortunate errors in the form of the question sat

by the candidates, although this did not put too many people off and was easily accommodated in the mark scheme.

Question 4
This was a straightforward question for those that knew the material and attracted some high marks.

Question 5
This question was popular and generally done well. Most candidates managed to avoid numerical errors when carrying out the simplex algorithm. Part (a)(ii) asked candidates to explain what they were doing, but some did not. Part (a)(ii) also asked for the solution to the problem P given in the question, but some solutions stopped at a simplex tableau and did not say (as they should have) what the solution to P was – and some did not say how they knew they had reached an optimal point. Many candidates were unable to justify that their solution was unique. Solutions to (b)(ii) sometimes just said that the previous solution is no longer optimal whereas it is possible to say much more – there is no optimal solution in (b)(ii), the objective function is unbounded (above).

Question 6
This question was relatively unpopular and I got the impression that for many candidates it was attempted as a last resort. The weaker candidates appeared not to have learned even the basic definitions and bookwork underpinning the question, and displayed poor skills in basic algebraic manipulation.

Virtually all could show that $f(x)$ has a root in $(0, 1)$ but several failed to establish uniqueness. Relatively few managed the straightforward algebraic manipulations needed to show that $g : [0, 1] \to [0, 1]$. Although most candidates could quote the Contraction Mapping Theorem more-or-less correctly, very few convincingly obtained the required bound on $|g'|$. Part (c) was either done well (by those who had clearly learned the bookwork) or barely started.

Question 7
This was a straightforward question for those that knew the material and attracted some high marks.

Paper V

Question 1
This question was popular but caused many problems for weaker students. Although most could state the basic identity of Stokes' Theorem, the hypotheses were frequently muddled or just omitted. Some students seemed to

think that S refers to a region rather than a surface. This confusion continued into part (b), where a substantial number of students tried to integrate over the *region* $\{(x, y, z) : y^2 + z^2 \leq 4,\ z > 0,\ x^2 + y^2 \leq 1\}$ rather than over the given surface.

There were a large number of Distinction level solutions displaying a good grasp of the underlying theory and the ability to apply it. At the other end, many students were completely unable to identify or to parameterise the relevant surface and curve. A significant number failed even to compute $\boldsymbol{\nabla} \wedge \mathbf{f}$ accurately.

Question 2

Part(a)(i) was mostly done well, although many candidates failed to state all of the conditions necessary for the divergence theorem to hold, or neglected to define $\mathbf{n}\mathrm{d}S$ as outward-facing.

Part(a)(ii) proved more of a struggle, especially the part on divergence. Most candidates approached this using the product rule rather than stating and using the definition of divergence, but several of these candidates confused themselves by omitting the dot products and taking

$$\boldsymbol{\nabla} \cdot \mathbf{e}_r = \frac{\partial}{\partial x}\mathbf{e}_r + \frac{\partial}{\partial y}\mathbf{e}_r + \frac{\partial}{\partial z}\mathbf{e}_r$$

which is not a scalar.

Part(b) was bookwork, which candidates either did well or did not attempt. A few attempted to work in 1D rather than 3D, or neglected the fact that k depends on $\mathbf{r}$.

Part(c) was more conceptual, and many candidates struggled, especially with determining and implementing the correct boundary conditions. A common mistake was to impose flux as $\partial T/\partial t$, or to reason that $T = 0$ at $r = b$ implies $\partial T/\partial r = 0$ at $r = b$. A disappointingly large number of candidates felt that $\boldsymbol{\nabla} \cdot \mathbf{q} = 0$ implies $\mathbf{q}$ is constant. The best candidates saw (c)(i) immediately, and made good progress with (c)(ii), but no candidate obtained the correct final result.

Question 3

In part (a) the majority of attempts made the correct change of variables, though many lost marks for computing incorrectly the Jacobian or the subsequent double integral. There were only a handful good attempts at the triple integral in part (b) despite it being similar to three examples in the lecture notes. The majority of attempts were doomed by ignoring the hint, using instead scaled spherical polar coordinates, or followed the hint and made the correct scaling of the coordinates, but then failed to make the correct rotation.

Question 4
The bookwork in part (a) was well done on the whole, though the majority of attempts did not use a substitution to justify their manipulation of the integrals in parts (i) and (iii). In part (b) the calculation of the Fourier cosine coefficients was not well done on the whole: the majority of attempts did not consider both of the two special cases or got bogged down with an unnecessary simplification of their expressions. In the tail of part (b), a sizeable number of attempts correctly set $p = 1$ and $x = \pi/2$, but then failed to deduce the sum either by failing to correct their cosine series or by evaluating incorrectly the Fourier coefficients.

Question 5
The bookwork in part (a) was very well done on the whole, only a minority failing to state the correct boundary conditions. Only a handful of attempts found efficiently the functions f and g in part (b), the majority employing an unnecessary separation of variables argument and getting lost in the subsequent evaluation of the integration constants. While the separation of variables in part (c) was well done, the majority failed to derive integral expressions for the Fourier coefficients, and only one candidate made good progress with the tail.

Question 6
The bookwork in part (a) was very well done on the whole, though a significant minority either failed to state or to use the assumption that $|\partial y/\partial x| \ll 1$. Part (b) was well done on the whole, though a number of attempts failed to differentiate correctly D'Alembert's formula. Part (c) was well done on the whole, the majority of attempts demonstrating a good understanding of the geometric interpretation of D'Alembert's formula, though a sizeable number of attempts evaluated incorrectly the solution in four out of ten of the relevant regions.

Question 7
Most candidates did well on part (a), which asked them to state Gauss' Law and derive it from Poisson's equation, but had difficulty with parts (b) and (c). In part (b), many students took the electric field inside the sphere to be proportional to $1/r^2$ rather than r^2, as the question indicated. In part (c), many students were able to compute the electric potential outside the sphere, but had difficulty computing the electric potential inside the sphere.

Computational Mathematics

Technical issues
Most submissions seemed to go well technically. As always, there are a few students that upload a link to their file instead of the file itself, or leave out

their figures or otherwise don't follow the directions (see below).

A major exception this year was Project 3, due to a bug in Sage which occasionally prevented downloading a PDF file. Happily, all students were able to submit either a PDF, a .sagews or in some cases, screen-shots, print-to-file, etc. So assessment was still possible in all cases. For next year, there is a workaround which I will add to the notes.

Something to fix for next year is that candidates are unsure about labelling their project as 1 or 2 by submission date (no) or by project number (yes). So for next year, they should be called Project A, B and C.

Following instructions
I'm always surprised at how poorly the directions are followed! Roughly 5% of candidates include their name. Many do not follow the required folder structure. Many name their functions different from what was requested.

Comments about individual projects
Project 3 was probably a bit easier than the others but this was intentional and advertised as such. This was because candidates no assistance with Sage (for example, they could talk to demonstrators about Matlab).

F. Comments on performance of identifiable individuals

Removed from the public version of the report.

G. Names of members of the Board of Examiners

- **Examiners: Prof. P. Howell (Chair), Prof. M. Lackenby, Prof. A. Lauder, Dr N. Laws, Prof. L. Mason, Prof. J. Oliver, Prof. P. Papazoglou.**
- **Assessors: Dr R. Gaunt, Dr C. Hall, Dr A. Lipstein, Dr T. Lukowski, Prof C. Macdonald.**