Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2015

October 29, 2015

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Tables 1 and 2. Overall 179 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

	Numbers			Percentages %		
	2015	(2014)	(2013)	2015	(2014)	(2013)
Distinction	55	(55)	(55)	30.73	(30.9)	(30.9)
Pass	105	(103)	(103)	58.66	(57.87)	(57.87)
Partial Pass	13	(12)	(13)	7.26	(6.74)	(7.3)
Incomplete	0	(1)	(0)	0	(0.56)	(0)
Fail	6	(7)	(7)	3.35	(3.93)	(3.93)
Total	179	(178)	(178)	100	(100)	(100)

Table 2: Numbers in each class (Honour Moderations)

	Numbers		Percentages %	
	(2012)	(2011)	(2012)	(2011)
I	(60)	(64)	(30.77)	(32.49)
II	(123)	(119)	(63.08)	(60.41)
III	(4)	(9)	(2.05)	(4.57)
Pass	(0)	(1)	(0)	(0.51)
Honours (unclassified)	(1)	(0)	(0.51)	(0)
Fail	(7)	(4)	(3.59)	(2.03)
Total	(195)	(197)	(100)	(100)

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the Preliminary Examination in Mathematics.
- **Marking of scripts.**
  As in previous years, no scripts were multiply marked by Moderators; however all marking was conducted according to a detailed marking scheme, strictly adhered to. For details of the extensive checking process, see Part II, Section A.

B. New examining methods and procedures

This is the third year of the new Preliminary Examination in Mathematics, which has replaced Honour Moderations. The same methods and procedures were followed as in 2013 and 2014.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

Following the review of the Preliminary Examinations after two years, Teaching Committee has agreed the abolition of the Applications course. The Statistics course will be extended and will replace Optimization. Timetabling and syllabus changes have been made to Geometry, Dynamics, Introductory Calculus, Fourier Series and PDEs, and Multivariable Calculus.

The following changes will be implemented from 2015/16:

	MICHAELMAS	HILARY	TRINITY
PAPER I	Linear Algebra I	Linear Algebra II, Groups I	Groups II
PAPER II	Sequences and Series	Continuity and Differentiability	Integration
PAPER III	Introduction to Calculus Probability		Statistics
PAPER IV	Geometry	Dynamics	Constructive Maths
PAPER V		Fourier Series & PDEs Multivariable Calculus	

This will mean that from the 2016 examinations Paper III will be 3 hours long (students choosing 6 questions from 9) and Paper V will be 2 hours long (4 questions from 6). This also rationalizes the problems in Paper IV of setting 3 questions on Constructive Mathematics and Optimization.

D. Notice of examination conventions for candidates

The Notice to Candidates, containing details of the examinations and assessment, including the Examination Conventions, was issued to all candidates at the beginning of Trinity term. All notices and the Examination Conventions in full are available at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

Acknowledgements

The Moderators would like to thank the academic administration team, in particular Nia Roderick, Charlotte Turner-Smith and Helen Lowe, for all their work in running the examinations system. We thank Waldemar Schlackow and Helen Lowe for running the marks database. We also thank Dr Richard Earl for his advice.

We are very grateful to Dr Andrew Thompson for administering the Computational Mathematics projects. We would also like to thank the Assessors Dr Lino Amorim, Dr Robert Gaunt, Dr Chris Gill, Dr Cameron Hall, Dr Oliver Maclaren, Dr Michael Salter-Townshend, Dr Rolf Suabedissen, and Prof. Andy Wathen for their assistance with marking.

Timetable

The examinations began on Monday 22nd June at 2.30pm and ended on Friday 26th June at 12.00pm.

Medical certificates and other special circumstances

A subset of the Moderators attended a pre-board meeting to band the seriousness of circumstances for each application of factors affecting performance received from the Proctors' office. The outcome of this meeting was relayed to the Moderators at the final exam board. The moderators gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate.

See Section F for further detail.

Setting and checking of papers and marks processing

The Moderators first set questions, a checker then checked the draft papers and, following any revisions, the Moderators met in Hilary term to consider the questions on each paper. They met a second time to consider the papers at the end of Hilary term making further changes as necessary before finalising the questions. A meeting was held in early Trinity term for a final proof read. The Camera Ready Copy (CRC) was prepared and each Moderator signed off the papers. The CRC was submitted to Examination Schools in week 4 of Trinity term.

The examination scripts were collected from Ewert House and delivered to the Mathematical Institute.

Once the scripts had been marked and the marks entered, a team of graduate checkers, under the supervision of Nia Roderick, sorted all the scripts for each paper of the examination. They carefully cross checked against the marks scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. A number of errors were corrected, with each change checked and signed by an Examiner, at least one of whom was present throughout the process. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The candidates under consideration are Mathematics and Mathematics & Statistics candidates, 179 in total. We do not distinguish between them as they all take the same papers.

Marks for each individual paper are reported in university standardised form (USM) requiring at least 70 for a Distinction, 40–69 for a Pass, and below 40 for a Fail.

As last year the Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average proportion in each class over the past five years, together with recent historical data for Honour Moderations.

The raw marks were recalibrated to arrive at the USMs reported to candidates, adopting the procedures outlined below. These procedures are similar to the ones used in previous years.

To ensure equal weightings across all subjects, papers were first standardised to have similar proportions of candidates attaining each class. A piecewise linear mapping was adopted to produce a USM from a raw mark. The default algorithm for each paper works as follows.

1. Candidates' raw marks for a given paper are ranked in descending order. Here the population data used is the set of marks for all candidates in Mathematics or Mathematics & Statistics.

2. The default percentages p_1 of Distinctions and p_2 of nominal upper seconds (USM 60–69) in this population are selected, these percentages

being similar to those adopted in previous years.

3. The candidate at the p_1-th percentile from the top of the ranked list is identified and assigned a USM of 70. Let the corresponding raw mark be denoted by R_1.

4. Similarly, the candidate at the $(p_1 + p_2)$-th percentile from the top of the list is assigned a USM of 60 and the corresponding raw mark is denoted by R_2.

5. The line segment between $(R_1, 70)$ and $(R_2, 60)$ is extended linearly to the USMs of 72 and 57 respectively. Denote the raw marks corresponding to USMs of 72 and 57 by C_1 and C_2 respectively. Line segments are then drawn connecting $(C_1, 72)$ to $(100, 100)$ and connecting $(C_2, 57)$ to $(0, 0)$.

Thereby a piecewise linear map is constructed whose vertices, at $\{(0,0), (C_2, 57), (C_1, 72), (100, 100)\}$, are located away from any class boundaries.

A first run of the outlined scaling algorithm was performed. It was confirmed that the procedure resulted in a reasonable proportion of candidates in each class. The Moderators then used their academic judgement to make adjustments where necessary as described below. The Moderators were not constrained by the default scaling map and were able, for example, to insert more vertices if necessary.

To obtain the final classification, a report from each Assessor was considered, describing the apparent relative difficulty and the general standard of solutions for each question on each paper. This information was used to guide the setting of class borderlines on each paper.

The scripts of those candidates in the lowest part of each ranked list were scrutinised carefully to determine which attained the qualitative class descriptor for a pass on each paper. The gradient of the lower section of the scaling map was adjusted to place the pass/fail boderline accordingly.

Careful consideration was then given to the scripts of candidates at the Distinction/Pass boundary.

Adjustments were made to the scaling maps where necessary to ensure that the candidates' performances matched the published qualitative class descriptors.

The Computational Mathematics assessment was considered separately. In consultation with the relevant Assessor it was agreed that no recalibration was required, so the raw marks (out of 40) were simply multiplied by 2.5 to produce a USM.

Finally, the class list for the cohort was calculated using the individual paper USMs obtained as described above and the following rules:

Distinction: both $Av_1 \geq 70$ and $Av_2 \geq 70$;

Pass: not meriting a Distinction and a USM of at least 40 on each paper;

Partial Pass: a USM of less than 40 on one or two papers;

Fail: a USM of less than 40 on three or more papers.

Here Av_2 is the average over the five written papers and Av_1 is the weighted average over these papers together with Computational Mathematics (counted as one third of a paper). The Moderators verified that the overall numbers in each class were in line with previous years, as shown in Tables 1 and 2.

The vertices of the final linear model used in each paper are listed in Table 3, where the x-coordinate is the raw mark and the y-coordinate the USM.

Table 3: Vertices of final piecewise linear model

Paper	Positions of vertices				
I	(0,0)	(30,37)	(44.4,57)	(78,72)	(100,100)
II	(0,0)	(23,37)	(32,57)	(72.6,72)	(100,100)
III	(0,0)	(38,37)	(50,57)	(79,72)	(100,100)
IV	(0,0)	(29,37)	(40.9,57)	(66.4,72)	(100,100)
V	(0,0)	(18.5,37)	(27,57)	(55.5,72)	(100,100)
CM	(0,0)				(40,100)

Table 4 gives the rank list of average USM scores, showing the number and percentage of candidates with USM greater than or equal to each value.

Table 4: Rank list of average USM scores

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	%
91	1	2	1.12
88	3	3	1.68
86	4	5	2.79
85	6	7	3.91
82	8	8	4.47
79	9	11	6.15
78	12	13	7.26
77	14	16	8.94
76	17	17	9.5
75	18	19	10.61
74	20	26	14.53

Table 4: Rank list of average USM scores (continued)

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	%
73	27	32	17.88
72	33	39	21.79
71	40	47	26.26
70	48	55	30.73
69	56	63	35.2
68	64	71	39.66
67	72	84	46.93
66	85	89	49.72
65	90	102	56.98
64	103	110	61.45
63	111	115	64.25
62	116	125	69.83
61	126	133	74.3
60	134	140	78.21
59	141	148	82.68
58	149	153	85.47
57	154	154	86.03
56	155	156	87.15
56	155	156	87.15
55	157	158	88.27
54	159	161	89.94
54	159	161	89.94
50	162	162	90.5
49	163	163	91.06
48	164	164	91.62
47	165	165	92.18
46	166	169	94.41
45	170	171	95.53
44	172	172	96.09
40	173	173	96.65
38	174	174	97.21
34	175	176	98.32
28	177	177	98.88
26	178	178	99.44
11	179	179	100

B. Equal opportunities issues and breakdown of the results by gender

Table 5 shows the performances of candidates broken down by gender.

Table 5: Breakdown of results by gender

	Total		Male		Female	
Class	Number	%	Number	%	Number	%
Distinction	55	30.73	48	36.64	7	14.58
Pass	105	58.66	71	54.20	34	70.83
Partial Pass	13	7.26	9	6.87	4	8.33
Incomplete	0	0	0	0	0	0
Fail	6	3.35	3	2.29	3	6.25
Total	179	100	131	100	48	100

C. Statistics on candidates' performance in each part of the Examination

The performance statistics for each individual assessment are given in the tables below: Paper I in Table 6, Paper II in Table 7, Paper III in Table 8, Paper IV in Table 9, Paper V in Table 10 and Computational Mathematics in Table 11. The number of candidates who received a failing USM of less than 40 on each paper is given in Table 12 on page 11.

Table 6: Statistics for Paper I

Question	Average Mark		Std	No. of Attempts	
Number	All	Used	Dev	Used	Unused
Q1	14.73	14.73	3.32	179	0
Q2	12.05	12.09	3.41	152	1
Q3	12.53	12.53	4.27	145	0
Q4	13.32	13.32	5.10	56	0
Q5	11.97	11.97	4.00	164	0
Q6	11.71	11.71	4.54	59	0
Q7	12.53	12.53	2.65	127	0

Table 7: Statistics for Paper II

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	11.01	11.08	3.38	137	1
Q2	12.04	12.04	3.59	163	0
Q3	11.34	11.48	4.09	58	1
Q4	10.80	10.87	5.26	155	2
Q5	10.78	11.00	5.28	44	1
Q6	8.40	8.46	3.21	158	3
Q7	11.70	11.70	6.56	169	0

Table 8: Statistics for Paper III

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	12.34	12.42	3.99	158	2
Q2	14.34	14.48	5.18	96	1
Q3	14.54	14.54	2.96	104	0
Q4	14.03	14.03	4.30	124	0
Q5	13.68	13.68	3.47	75	0
Q6	13.64	13.72	4.12	158	1
Q7	11.97	11.97	3.94	177	0

Table 9: Statistics for Paper IV

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	8.07	8.07	4.66	68	0
Q2	11.06	11.06	4.04	175	0
Q3	7.56	7.56	4.65	158	0
Q4	11.73	11.73	3.93	121	0
Q5	14.07	14.18	3.37	127	2
Q6	13.02	13.05	2.91	171	1
Q7	14.23	14.23	4.37	60	0

Table 10: Statistics for Paper V

Question Number	Average Mark All	Used	Std Dev	No. of Attempts Used	Unused
Q1	8.23	8.38	4.69	117	3
Q2	10.33	10.37	3.83	112	1
Q3	8.81	8.84	3.66	126	2
Q4	8.32	8.32	4.00	151	0
Q5	7.50	7.63	4.42	119	4
Q6	10.23	10.23	4.78	87	0
Q7	7.93	7.93	4.88	168	0

Table 11: Statistics for Computational Mathematics

Question Number	Average Mark All	Used	Std Dev	No. of Attempts Used	Unused
Q1	16.49	16.49	3.45	177	0
Q2	14.73	14.73	3.34	71	0
Q3	18.65	18.65	1.92	110	0

Table 12: The number of failures on each paper

Paper	Number	%
I	8	4.5
II	8	4.5
III	8	4.5
IV	10	5.6
V	14	7.8
CM	3	1.7

D. Recommendations for Next Year's Examiners and Teaching Committee

As the Moderators noted in the past it does not seem appropriate to appoint examiners both for part A optional courses and Prelims. This year two of the Prelims examiners, Prof. Papazoglou and Prof. McGerty, were also assessors for optional courses in Part A. The overlap of the marking for the two years meant these examiners had little more than a week to mark a very large number of scripts. The only way to keep even extended deadlines under this schedule is to work long hours during the week and keep marking throughout the weekend. It hardly leaves any time for other departmental or College related obligations that in fact both assessors did have. We think that the unfortunate scheduling made it much harder than should be the case to give every script its due consideration and increases the risk of a lapse of judgement. There seems to be two natural things which could be done to assist an examiner in this position in the future, either:

1. Their Part A exam scheduled in week 8 so as to give them time to mark it before the Prelims examinations in week 9.

2. Or have additional assessors appointed to assist them in marking Prelims.

The examiners noted the hope that Nominating Committee had taken into consideration the change of syllabus when sourcing an appropriate split of expertise for the exam board next year.

The examiners would like to highlight that the rubric provided by the Department of Computer Science on their exam papers did not specify that candidates should answer each question in a separate booklet. This had been problematic when marking the 3 Probability questions, as the questions had been split across 2 assessors.

The examiners also recommended changing the Examination Conventions to allow questions to be formed of two to four parts, rather than two to three parts.

E. Comments on papers and on individual questions

Paper I

Question 1

A generally high standard of answers, particularly in the proof of the rank nullity theorem. Common errors in the bookwork were to confuse subgroups with subspaces, not showing non-emptiness of subspaces, and not taking images of the added basis vectors in the proof of the rank nullity theorem. Also problematic was linearity of maps, both of the requested examples and of the map S in (c)(iii). In the last two parts of (c), a good number of candidates claimed that the existence of S amounted to T being invertible, despite the question clearly hinting that this was not the case. In the last part a number of candidates assumed that T had a specific very easy matrix representation without explicitly choosing a basis.

Question 2

Most candidates chose this question, half of them only managing part (a) (bookwork) and (b) (mechanical), not getting anywhere with part (c) which requires a thorough conceptual understanding.

Question 3

Very few complete answers were given and although generally well done, a number of attempts were very disappointing. Overall slightly lower standard to Question 1.

As expected, some candidates had difficulties translating the geometry into a matrix, but this was generally well done. Apart from a few computational errors, part (b) caused a lot of difficulty: a lot of candidates did not even make any attempt to show that there are no other invariant subspaces than the obvious ones. Most of those who did claimed it was 'geometrically obvious' or erroneously assumed that all planes through 0 were either the xy, yz or xz-plane. On the other hand there were some very good answers showing a lot of mathematical understanding and creativity in this part. In part (c) the most common source of errors was to not realize that the underlying field was $\mathbb{C}$ and not $\mathbb{R}$, typically manifesting itself by claiming that $(0, 0, 1)^T$ and $(0, 0, i)^T$ were linearly independent or that $\mathbb{C}^3$ had a basis consisting of 6 vectors. Finally, in deriving the eigenvectors a number of candidates did not use all equations and did not check that their supposed eigenvector was in fact one.

Question 4

Around a third of the candidates took this question and many of them did very well.

Question 5

Most candidates attempted this. Parts a.i, a.ii were generally well done. Several candidates used the coset equality lemma for part a.i even though it was not necessary. In part a.ii some candidates didn't show that $|gH| = |H|$. In part a.iii several candidates did not show that the cardinality of $< g >$ is equal to the order of g. Candidates who used that $g^{|G|} = e$ without proof received no marks.

Part b was more challenging. In part b.i many candidates attempted to use the coset equality lemma, however this is only valid for subgroups. Some candidates managed to do part b.ii (using the previous part) even though they did not do b.i.

Question 6

This question was also chosen by about a third of the candidates with a wide range of levels of performance. Only one candidate gained full marks, very few gave adequate reasons in part (b) for why there are precisely two groups of order 6 (up to isomorphisms).

Question 7

Many candidates attempted this.

Part a was generally well done. In part a.i some candidates did not give a complete proof that the stabiliser is a subgroup getting only partial credit.

In part a.ii several omitted to show that the map defined is 1-1 or well defined.

Part b.i was attempted by many candidates. Students that realized that they had to use the orbit-stabilizer theorem either answered the question completely or received partial credit for their efforts. Part b.ii was quite challenging and only a handful of students produced complete or nearly complete solutions. Several candidates seemed to have a good intuitive understanding but failed to give a correct definition of the function f.

Paper II

Question 1

This was a popular question. Parts (a)(i)-(ii) took the candidates through a proof of Bolzano Weierstrass theorem. This standard piece of bookwork was generally well done, and allowed diligent students to pick up good marks.

Marks were most commonly lost for an inadequate justification of the inductive step in the proof of (i). The rest of the question was, however, found to be more difficult by many of the students. Parts (a)(iii) and (b) were accessible to many students, but only the best candiates were able to score decent marks on the more challenging (a)(iv) and (c). There were some very good solutions here, though.

Question 2

This was a popular question. The bookwork of part (a) was mostly very well done. In part (iii), many candidates recalled the elementary proof of convergence via Bernoulli's inequality that was given in lectures, but those that didn't often struggled to give a satisfactory proof.

Part (b) tested the candidates' understanding of convergence of sequences, and discriminated well between the students. The weaker candidates often struggled to pick up more than a couple of marks here, whilst the stronger students showed a very good understanding of the concept of convergence of sequences and scored good marks. Many students used an incorrect application of the algebra of limits to 'prove' that (ii) is true.

Question 3

Part a.i was generally well done. A few candidates however said that the series converges if the partial sums have an upper bound which is of course incorrect. Part a.ii was generally well done but several candidates, unsuccessfully, tried to tackle this using the definition rather that the Cauchy convergence criterion. Part a.iii was done by most candidates either directly or using the ratio test.

In part b.i many candidates made mistakes in the definition of radius of convergence.

In part b.ii some candidates stated wrongly the comparison test omitting some hypothesis. In the second part some only showed that $R \geq 1$ but did not show that $R \leq 1$ too.

In part b.iii several candidates assumed that the series converged absolutely at z_0 giving invalid proofs.

Part c.i was partially done by many candidates. Fewer candidates did the last part of c.i. In both parts c.i and part c.ii several candidates did not realized that the a_n's had to be natural numbers and not just reals.

Question 4

This question was generally well-answered. The first part was pure bookwork, with a number of students losing marks for incorrectly stating the definition of continuity or uniform continuity because they jumbled the order of logical quantifiers. Part $b)i)$ was successfully attempted by most students, using at least two valid strategies. Part $b)ii)$ was the most challenging. Quite a few students missed that the example asked for was required to take values in $[1, \infty)$.

Question 5

This question was not very popular and attempted by roughly a third of the candidates. Candidates did better in part (a), which was entirely bookwork. Many candidates forgot the absolute value in the statement of the M-test and/or failed to see the relevance of absolute convergence in (iii). Part (b) caused more difficulties especially (ii). Most mistakes in (i) came from incorrectly applying the M-test. In (ii) many candidates were unable to provide a correct example and of the ones who did only a few were able to properly show uniform convergence of the resulting series, even though this could be done directly from the definition. In the last part many students failed to correctly utilize the condition of convergence of the series of absolute values.

Question 6

The bookwork for this question was generally well done, but the rest of the question appears to have been quite challenging. Many candidates failed to carefully check that the function at the end of part $a)$ was differentiable at 0. Part $b)i)$ was poorly attempted in most cases, despite being an easier version of a problem sheet question. Most students were unable answer $b)ii)$, though the students who did solve it came up with quite a few different strategies, most different from that proposed in the hint.

Question 7

The solutions to this question varied in quality. Parts (a) and (b) are standard bookwork from the Analysis III course. For part (a), a surprising number of students failed to provide a satisfactory answer, whilst others ignored that a continuous function on a closed, bounded interval is bounded, instead proving that the integral was differentiable, hence continuous. When they chose to use it, a minority of students showed a worrying lack of understanding of epsilon-delta definitions of continuity at this stage of the question. The attempts at Part (b) were generally good, with some perfect solutions justifying every step and going into great detail. Most of the problems at this stage seemed to be due to reproducing the skeleton of a memorised proof and missing justifications, such as not considering the case when g is identically zero. Part (c) applied similar notions to part (b), attracting some excellent solutions. Almost all attempts used the hint given, but far too many students failed to check that K is continuous before applying the Intermediate value theorem, and many solutions included errors in the form of reversed inequalities.

Paper III

Question 1

(a) was answered perfectly by most.
(b) The word 'derive' indicates that the solution should not be just stated, but many students only did this.
(c) Most, but not all, students spotted that $y = x$ is a solution, so solutions of the form $y = ux$ can be tried. There are two challenges to the solution. First, obtaining u': While it appears attractive to cancel common factors, it is easier to spot the integral if no cancelling has been done. Second, partial fractions are needed to integrate: Typically students either got the full answer, got halfway, or were unable to spot the right method.

Question 2

(a) It was surprising that a proportion of students didn't realise that the Jacobian needs to be inverted to obtain $\dfrac{\partial v}{\partial y}$.
(b) is bookwork, and was well answered.
(c) The reversal or the order of the terms in the two parts threw several students. For the last part, an unexpected differential equation drops out.

Question 3

(a) This was found almost universally easy.
(b) The geometric perspective is about the level sets of f needing to be tangential to the curve g - this wasn't answered well by many.
(c) The most challenging thing about this question is the unpleasant surds which crop up. The easiest route is to solve for x by eliminating λ, and then for y. Care is needed with alternating signs which distinguish the solutions.

Question 4

Candidates generally performed well on this question, with considerable numbers getting almost all of the marks available. Common problems were: inadequate explanations in (b)(i) (or, indeed, not understanding the origin of the -1 on the RHS); and not remembering how to solve a difference equation (and/or calculation errors) in (b)(ii). (c)(ii) discriminated well between candidates who were and weren't able to spot the symmetry of the distribution of

$$\sum_{i=1}^{m} X_i$$

(although few gave really convincing explanations).

Question 5

The standard of solutions was generally good, although there were also considerable numbers of candidates who essentially gave up after the mostly standard material in (a). Common difficulties included: not recalling for which values of s the probability generating function is defined; and not being able to correctly differentiate e^{s-1}. Parts (b)(ii)-(iv) discriminated well between candidates. There were very few excellent explanations given in (b)(ii), nor truly careful solutions to (b)(iv).

Question 6

This was a popular question, and the stronger students often scored high marks. Part (a) was a standard piece of bookwork, but candidates often lost marks for failing to correctly state when the mean and variance existed. In part (a)(ii), many students lost a mark by assuming linearity of expectation, rather than working with the integral definition for expectation. Part (b) was generally very well done.

Part (c) was found to be quite challenging by the weaker students, but the stronger students often performed very well. Many students found the derivation of the c.d.f. in part (i) to be difficult, and there was often insufficient justification of various steps: to score full marks the students needed to appeal to the fact that Y is a non-negative random variable (and is hence supported on the positive real line) and that the standard normal density is symmetric about the origin. It was surprising how many students were unable to use the chain rule to find the p.d.f. of Y. Those that did were often able to score well on part (iii).

Question 7

Candidates lost marks for notation that was confusing or conflicting, particularly in (b)(i); for example, using theta and theta-hat interchangeably or setting the log-likelihood to be zero everywhere. Full marks were only awarded in (b)(i) when the MLE was clearly shown to maximise the log-likelihood: many students stated the second derivative and simply declared it to be negative without simplifying, which was not a convincing argument.

Paper IV

Question 1

The geometry question was attempted by only about 40% of candidates. Quite a few students had trouble with establishing the equation of a plane through a given point and line – perhaps due to some confusion about the distinction between a vector lying on a plane and being parallel to the plane when the plane does not pass through the origin. Candidates had no problem however applying the formula in the second part, but surprisingly few candidates were able to establish the necessary condition in the final part of the question, and even fewer were able to decide whether that condition was sufficient.

Question 2

Almost all candidates attempted this question. On average the marks were fairly low.
(a) Many candidates lost marks for not carefully demonstrating all the requirements for the plane of motion, and for not stating the physical principles expressed by each constant. Most students could derive the stated equations.
(b) Most students struggled to use the final equation of (a) to complete this part of the question. Very few were able to correctly deduce that the particle reaches the origin in finite time.

Question 3

The majority of the candidates answered this question. Marks were very low in general.
(a) Many candidates forgot to explicitly use the fact that $\theta = \omega t$ in writing down an expression for $\vec{r}$. Most candidates could correctly use NL2 to write down the equation of motion, but many failed to take the appropriate dot product to deduce the stated equation for $r(t)$.
(b) Very few candidates correctly linearised the last equation of (a) about $r = a$ and so many marks were lost in this part of the question. Many candidates could find the steady states in the final part, but only a small fraction of those correctly evaluated their linear stability.

Question 4

Fewer candidates (though still most) attempted this question. Marks were, on average, higher than for the other two questions, in part as the bookwork parts were standard.
(a) Most candidates gained close to full marks on this part.
(b) The majority of candidates struggled with showing that E is constant.
(c) Very few candidates attempted this part (though it could be done without having completed other parts). Of those that got close to a final answer, rather than just rearranging the expression for κ, many stated conditions for which the quadratic has real roots.

Question 5

Question 5 was generally done well and, in particular, most of the candidates got the bookwork in (a) substantially correct. In part (b), the most common errors were incorrect justifications in (b)(ii) (including failing to observe that $y_1, y_2 \geq 0$) and incorrect identification of the feasible region in (b)(iii). Of those who did identify the feasible region, many did not justify the identification of the optimum. (b)(iv) was generally well done, even by those with incorrect answers to (b)(iii).

Question 6

Almost all candidates attempted this question on Euclid's algorithm and Diophantine equations. Most substantially completed part (a) and (b), but were challenged to provide a proof in (c). Nobody scored full marks on this question despite the numbers attempting it: those that were able to provide a correct proof in (c) made at least small steps/omissions in earlier parts.

Question 7

Was attempted by about 1/4 to 1/3 of candidates and several scored full marks. All who seriously attempted the question gained reasonable scores.

Paper V

Question 1

This question caused serious problems for all but the strongest candidates. In part (a) many stated correctly the integral identity in Stokes' Theorem, but omitted or muddled the hypotheses; many of the weaker candidates confused the hypotheses with those of the Divergence Theorem. The derivation of the identity in part (b) was reasonably well done, though many weaker candidates lost marks for stating incorrectly the definition of curl in Cartesian coordinates or for applying incorrectly the chain rule. While there were a reasonable number of Distinction level solutions to part (c), many candidates were unable to parametrise correctly the relevant surface and curves or to compute correctly the integrands; there were more successful attempts from those that stuck with cylindrical polar coordinates than those that introduced spherical polar coordinates: only a handful of candidates were able to navigate successfully the use of both coordinate systems in one computation.

Question 2

This question was well done by those that had learnt the bookwork. All but a minority of the weaker candidates scored full or nearly full marks in part (a) on the derivation of Green's 1st and 2nd integral identities; that minority largely lost marks for being unable to state and apply the correct definitions of the gradient and divergence in part (a)(ii). This trend continued for parts (b)(i) and (b)(ii). In part (b)(i) the majority arrived upon the counter example by proving $\boldsymbol{\nabla} u = \mathbf{0}$ and derived from scratch the key integral identity rather than appealing to Green's 1st integral identity as intended. There were a small number of Distinction level solutions to parts (b)(iii) and (b)(iv): more were able to give a counterexample for (b)(iii) than a proof for (b)(iv), though there were a number of elegant solutions to (b)(iv) that exploited correctly Green's 2nd integral identity and adapted correctly the proof of (b)(ii).

Question 3

The first half of this question was very well done on the whole, while the second half caused serious problems even for the strongest candidates. All but a minority of weaker candidates scored full or nearly full marks on part (a); a sizeable minority lost a mark for sketching incorrectly the curve C (usually by ignoring the limits on t and sketching a helix) or for not stating the correct integral expressions for the length of a curve or the surface area of a surface. While there were many good attempts at computing the volume of the region in part (b) using a variety of different parameterisations, many lost marks for exploiting incorrectly the symmetry of the region. Only a handful of the stronger candidates made progress at computing the surface area of the boundary of the region in part (c) and there were no complete solutions, so it was marked generously.

Question 4

Almost all candidates attempted this question. Scores were not generally high with a significant fraction obtaining less than half the available marks.

(a) A number of candidates lost marks for not carefully defining the series on the domain [0,L] (rather [-L,L] or $[-\pi, \pi]$) and missing various related scale factors. Many more struggled to identify the proper expression to which the series converge and in particular to carefully consider the behaviour at the end points.
(b) Many candidates made careless manipulation or integration errors (some struggled to do the integration at all). A large number did not draw or incorrectly drew the required sketches, in particular struggling with the end points. Few mentioned the Gibbs phenomena and even fewer the rate of convergence.
(c) Very few candidates managed this question. Many focused on behaviour at x=0 without considering what happens at x=L. This led many to consider odd functions without considering their periodic extensions. A few gave non-polynomial examples.

Question 5

This question was slightly more popular than Question 6 and much less than Question 4. Again the standard was not particularly high.

(a) Many candidates could set up the uniqueness problem in terms of a difference of two solutions, identify the new boundary conditions and carry out the proof for the $\alpha = 0$ case, but most could not do the $\alpha > 0$ case as

well. A few candidates tried to incorrectly use essential boundary or initial conditions from the original problem rather than the derivative condition from the difference problem. A few tried to (incorrectly) modify the energy function for the damped problem.
(b) Many candidates failed to use the given form of y, i.e. $y = e^{(-\alpha t)}F(t)sin(nx)$, instead working with a generic separable form $G(t)H(x)$. A few of those that did this managed to eventually separate out F, but struggled more than those who worked with the given form directly. Many did not apply the initial conditions.
(c) Very few candidates made much progress on this question. Only a few could write down the series correctly and even fewer could derive the expressions for the coefficients.

Question 6

The fewest number of candidates attempted this question. A number of candidates did well on this question relative to the other questions, though there were still a number who failed to obtain half of the available marks.

(a) Most candidates gained close to full marks on this part. A few didn't correctly distinguish between heat/heat flow and temperature.
(b) A number of those that attempted this question did well on it, though the working was generally messy and sometimes 'hand-wavy' (perhaps due to knowing the answer to be obtained).
(c) This was the hardest part for most candidates and most struggled to get the correct a and b values. This seemed to generally be due to difficulties applying the heat flux condition in the centre.

Question 7

On the whole, this question was not particularly well done, but there was also a very broad spread of marks.

Most candidates failed to get full marks for part a) because they omitted to mention the physics associated with Gauss's flux theorem (i.e. they failed to mention the electric flux or the total charge), or they omitted to make any reference to the fact that normals must be outward facing, or they omitted to mention that $\nabla \times \mathbf{E} = 0$ only implies the existence of a potential function because $\mathbb{R}^3$ is simply connected. A significant minority of candidates tried to derive a proof for Gauss's flux theorem by starting from Poisson's equation or by considering the electric field of a collection of point charges: this led to proofs that (while normally containing the essential elements required for the marks) were both more complicated and less coherent than proofs that used the equations of electrostatics given in the exam question.

The volume integrals required in Part b) i) seemed to confuse some candidates – a large number of candidates picked incorrect regions of integration:

some treated the case of $r > a$ as if they should only integrate in a spherical shell rather than over the entire sphere, while others integrated each shell region in its entirety, leading to evaluations of the volume integrals that were completely independent of r. More worryingly, a significant number of candidates failed to give their answer for $\mathbf{E}$ as a vector, or were generally careless with the distinction between vectors and scalars in their manipulations of φ and $\mathbf{E}$. Several candidates solved Poisson's equation rather than using Gauss's flux theorem, creating extra work for themselves and losing themselves marks if they failed to derive Poisson's equation from Gauss' flux theorem. Few candidates completed enough of b) i) to be able to successfully attempt b) ii) or b) iii), but those few that did often displayed a clear physical insight into the meaning of their solutions.

Computational Mathematics

The students chose two projects out of three (two Matlab-based, Projects A and B; one Sage-based, Project C), and each was marked out of 20, giving a total of 40. The majority of students scored 30 or above. Assessment was based mainly on published reports, with the exception of Project B where some marks were awarded for successfully running code (animations). Project A (polynomial interpolation) was the most popular, followed by Project C (elliptic curves with Sage). Project B (numerical solution of the heat equation) was least popular. Despite having had no lecture time, the Sage project was popular, perhaps fitting the pure maths interests of many students. Project B was probably less popular due to having more unfamiliar maths. The take-up of Project C suggests that the inclusion of Sage is working well.

The marks for each of the projects were comparable, but in order of easiest to hardest they were: C, A, B.

In projects A and B, two marks were awarded for a coherently written report and well-written code, while one mark was awarded for programming initiative in project C. Not all students earned these marks.

F. Comments on performance of identifiable individuals

Removed from the public version of the report.

G. Names of members of the Board of Examiners

- **Examiners: Prof. Jim Oliver (Chair), Prof. Ruth Baker, Prof. Christina Goldschmidt, Prof. Jochen Koenigsmann, Prof. Kevin McGerty, Prof. Panos Papazoglou, Dr Andrew Thompson.**
- **Assessors: Dr Lino Amorim, Dr Robert Gaunt, Dr Chris Gill, Dr Cameron Hall, Prof. Alan Lauder, Dr Oliver Maclaren, Dr Michael Salter-Townshend, Dr Rolf Suabedissen, Prof. Andy Wathen.**