

Examiners' Report: Preliminary Examination in Mathematics and Philosophy Trinity Term 2016

November 1, 2016

Part I

A. STATISTICS

(1) Numbers and percentages in each class

See Tables 1 and 2. Overall, 14 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

| | Numbers | | | | Percentages % | | | |
|--------------|-----------|-----------|-----------|-----------|---------------|------------|------------|------------|
| | 2016 | (2015) | (2014) | (2013) | 2016 | (2015) | (2014) | (2013) |
| Distinction | 7 | 6 | 4 | 11 | 50 | 42.86 | 30.77 | 61.11 |
| Pass | 4 | 7 | 8 | 5 | 28.57 | 50 | 61.54 | 27.78 |
| Partial Pass | 3 | 1 | 1 | 2 | 21.43 | 7.14 | 7.69 | 11.11 |
| Fail | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | 14 | 14 | 13 | 18 | 100 | 100 | 100 | 100 |

Table 2: Numbers in each class (Honour Moderations)

| | Numbers (2012) | Percentages % (2012) |
|--------------|-------------------|-------------------------|
| I | 6 | 40 |
| II | 6 | 40 |
| III | 0 | 0 |
| Fail | 3 | 20 |
| Total | 15 | 100 |

(2) Vivas

No vivas were given.

(3) Marking of Scripts

In Mathematics, all scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. There is an extensive checking process. In Philosophy, all scripts were single marked except for failing scripts, which were double-marked.

B. New examining methods and procedures

There were no new examining methods or procedures this year. This was the fourth year of the new examining structure following the change in 2013 from Honour Moderations to Preliminary Examination.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

There are no changes under discussion.

D. Notice of examination conventions for candidates

The Notice to Candidates, containing details of the examinations and assessment, including the Examination Conventions, was issued to all candidates at the beginning of Trinity term. All notices and examination conventions in full are on-line at <https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions>.

Part II

A. GENERAL COMMENTS ON THE EXAMINATION

Timetable

The examinations began on Monday 20th June at 2.30pm and ended on Friday 24th June at 12:30pm.

B. EQUAL OPPORTUNITIES ISSUES AND BREAKDOWN OF THE RESULTS BY GENDER

The breakdown of the final classification by gender is as follows:-

| Class | Num | Gender | Percent |
|--------------|-----|--------|---------|
| Distinction | 6 | m | 54.55 |
| | - | f | - |
| Pass | 3 | m | 27.27 |
| | - | f | - |
| Partial Pass | 2 | m | 18.18 |
| | - | f | - |
| Fail | 0 | m | 0 |
| | - | f | - |

C. DETAILED NUMBERS ON CANDIDATES' PERFORMANCE IN EACH PART OF THE EXAMINATION

Mathematics I

| Question | Maths and Philosophy | | Single School | |
|----------|----------------------|---------|---------------|---------|
| | Mean | Std Dev | Mean | Std Dev |
| Q1 | 10.07 | 4.20 | 11.39 | 4.85 |
| Q2 | 15.43 | 3.72 | 15.52 | 3.19 |
| Q3 | 12.00 | 3.54 | 11.93 | 4.36 |
| Q4 | 7.44 | 6.58 | 11.59 | 6.16 |
| Q5 | 10.22 | 3.70 | 7.40 | 3.45 |
| Q6 | 9.22 | 2.91 | 9.93 | 3.19 |
| Q7 | 12.60 | 4.30 | 12.59 | 3.96 |

Mathematics II

| Question | Maths and Philosophy | | Single School | |
|----------|----------------------|---------|---------------|---------|
| | Mean | Std Dev | Mean | Std Dev |
| Q1 | 11.00 | 3.62 | 11.89 | 3.66 |
| Q2 | 8.71 | 1.11 | 10.64 | 4.32 |
| Q3 | 10.43 | 4.04 | 14.58 | 3.76 |
| Q4 | 8.77 | 3.27 | 10.38 | 3.91 |
| Q5 | 7.75 | 4.79 | 10.17 | 5.15 |
| Q6 | 8.00 | 4.56 | 9.03 | 4.54 |
| Q7 | 4.00 | 2.73 | 4.62 | 3.86 |

Mathematics III(P)

| Question | Maths and Philosophy | | Single School | |
|----------|----------------------|---------|---------------|---------|
| | Mean | Std Dev | Mean | Std Dev |
| Q1 | 11.36 | 3.90 | 14.88 | 4.02 |
| Q2 | 10.13 | 5.33 | 13.99 | 4.19 |
| Q3 | 11.56 | 3.40 | 15.23 | 3.21 |
| Q4 | 13.00 | 2.35 | 12.50 | 3.75 |
| Q5 | 12.08 | 3.86 | 14.05 | 3.57 |
| Q6 | 9.90 | 3.70 | 13.36 | 3.71 |

Elements of Deductive Logic

| AvgUSM | StdDevUSM |
|--------|-----------|
| 61.36 | 21.87 |

Introduction to Philosophy

| AvgUSM | StdDevUSM |
|--------|-----------|
| 66.79 | 3.42 |

D. COMMENTS ON INDIVIDUAL PAPERS

See the Mathematics report for reports on the following papers:

Mathematics I

Mathematics II

Mathematics III(P)

Report on Elements of Deductive Logic

This report on the EDL paper covers students in Computer Science & Philosophy, Maths & Philosophy, and Physics & Philosophy.

Detailed Numbers on Candidates' Performance

The following table summarises the performance of candidates by course.

| | P&P | M&P | CS&P |
|--------------------|------|------|------|
| number in cohort | 12 | 14 | 4 |
| minimum mark | 33 | 23 | - |
| maximum mark | 91 | 88 | - |
| mean mark | 65.0 | 61.4 | - |
| standard deviation | 16.2 | 21.9 | - |

Four candidates (13.3%) failed the paper. 12 candidates (40%) gained marks of 70 or more.

Although the Computer Science and Philosophy average is noticeably lower than the average of the other two courses, it is the average of only four marks.

Seven of the 14 Maths & Philosophy candidates gained marks of 70 or more. Only one gained a mark in the 60s.

Four of the 12 Physics & Philosophy candidates gained marks of 70 or more. Most Physics & Philosophy candidates gained marks in the 60s.

The following table provides statistics on individual questions for the combined cohort of 30 candidates.

| Question | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 |
|--------------------|------|------|------|------|------|------|-----|-----|
| number of answers | 14 | 27 | 18 | 21 | 18 | 11 | 13 | 8 |
| mean mark | 16.1 | 16.8 | 13.0 | 16.6 | 17.8 | 15.3 | 8.6 | 7.6 |
| standard deviation | 6.4 | 5.5 | 6.8 | 6.7 | 6.5 | 5.1 | 4.7 | 3.8 |

The maximum mark available for each question was 25.

The total number of questions answered (and gaining some marks) was 130. This is ten more than $4 \times 30 = 120$. Five candidates attempted five questions; one candidate attempted six questions; and one candidate attempted seven questions. A candidate's best four answers determined their overall mark. Of the seven candidates attempting more than four questions, three failed the paper. Of the 12 candidates gaining marks of 70 or more, only two answered more than four questions. It is not obvious that allowing a candidate's overall mark to be determined by their best four answers helps candidates to demonstrate their abilities. The joint committees might consider whether candidates should simply be asked to answer four questions.

Comments

All scripts were single marked except for failing scripts, and those near the pass/fail borderline, which were checked by a second marker.

The previous year's paper (2015) was more challenging than had been intended. An effort was made this year to avoid a similar outcome. This was successful, with the average mark for half the questions being 16 or higher. The average marks for Questions 7 and 8, however, were both below 10. These questions only tested Michaelmas term material (as did Question 6) and, although demanding, they were not felt by the examiners to be unduly difficult. Most candidates attempting these questions did little better on the other questions they attempted. On this basis, it was judged that no scaling should be implemented.

The first five questions on the paper, which focused to varying degrees on metallurgical topics covered in the Hilary term course, were much more popular than questions 6–8.

Question 1 (Completeness Proof) Several candidates who answered this question did very well, providing clearly set-out answers that showed a very good understanding of the overall structure of the proof. The most common reason for lost marks was a failure to prove the lemmas required to demonstrate that any maximal consistent set has a model. Some candidates lost marks when specifying the proof system by neglecting to include the rule of assumption.

Question 2 (Interpolation) This was the most popular question by a margin. The sketch of a proof of the interpolation theorem (part b) was done very well by those who knew what they were doing. Those who didn't got very few marks. The most popular (and most efficient) proof method involved an all-at-once definition of an interpolant as a disjunction of all of some appropriately chosen set of substitution instances of the premise formula. Sometimes a little too much was taken for granted. E.g., some version of the fact that $|\phi[\psi/X]|_A = |\phi|_A$ if $|\psi|_A = A(P)$ was often not articulated, let alone proved. Some proofs by induction on the number of letters in the premise formula not in the conclusion formula were also given, and were well done. Proofs in the style found in Hodges' *Logic* were also given. Some of these were not well done, with candidates not in control of their terminology, or of the details of the construction (e.g., candidates failed to give a proper definition of the truth function which the interpolant was to express).

Question 3 (Compactness) This question was surprisingly poorly done, with many candidates losing marks through lack of care and/or apparent unfamiliarity with the bookwork part of the question. One candidate, however, got almost full marks, and several did very well on the parts that caused most problems: (a)(iii), b(iv), and b(v). The best answers to a(ii) did it in two stages. Those attempting a single proof of the if-and-only-if claim failed to produce something sufficiently clear and precise to get full marks.

Examples of carelessness involved recognising that S_0 was to be vacuously true, but offering a formula with variables but no quantifier(s). In fact, in general, there was very bad bracket discipline. Candidates did not confine themselves to the conventional abbreviations but offered formulas with quantifiers intended to bind variables that were not in their scope, or added brackets around equality subformulas, sometimes in one and the same formula. For b(iii) several candidates wrote down formulas that clearly did not mean what they intended, perhaps confused by choosing to give their quantifiers wide scope. b(iv), like Michaelmas term material elsewhere on the paper, was surprisingly badly done. It essentially tested similar ideas to those tested by Exercise 5.4 from Halbach's Exercise Booklet for the Logic Manual.

Question 4 (Natural deduction rules for, and expressive adequacy of, the Scheffer Stroke) This was the second most popular question and was well done. Most proofs involving the unfamiliar natural deduction rules were correct, although some were much more elaborate than required. As most candidates got the right idea, a premium was placed on the cogency of candidates' explanations of their reasoning. The arguments justifying claims about what could be deduced about the connective $*$'s truth-table were the weakest part of most answers. For part (d), some statements of expressive adequacy were rather too sketchy. Similarly, some candidates did not do enough to prove $*$'s expressive adequacy. It was not deemed sufficient simply to state, say, that $(\phi \wedge \psi) \models ((\phi * \psi) * (\phi * \psi))$, without some indication of what one could then prove with this fact, and how.

Question 5 (Duality) This was again a popular and well done question (with the highest average mark) so a premium was placed on clarity and accuracy. One common mistake was to write down something appropriate for the definition of a dual of formula in answer to questions about the dual of a connective (parts (a) and (d)). Some candidates mistakenly thought that taking the dual of a formula involved (inter alia) negating it. They nonetheless answered “no” to (g)(i). Some candidates attempted to give an inductive proof for part (d), perhaps misled by the request for a “formal argument.”

Question 6 (Relations) In general the parts requiring translations and proofs were done better than the parts requiring counterexamples. No one provided a satisfactory answer to (c)(iii), and the allocation of marks between the other parts of (c) and (b) was adjusted to reflect this. Although the proofs were done better than the counterexamples, there were nonetheless lots of mistakes made. Many of these were careless but some were more serious. E.g., some candidates appeared not to know how Halbach’s \exists -elim rule works. Some candidates were unable to formalise “asymmetric.” This is core Michaelmas term material, and so it was disappointing that the standard was not higher. It is natural to assume that some candidates attempted this question even though they had not revised the material, perhaps after deciding not to answer questions testing Hilary term material. (The standard of answers to questions 7 and 8 prompts the same thought in respect of these questions.)

Question 7 (Translation between English and $\mathcal{L}_=$) The average mark for this question was, surprisigly, in single figures. Many candidates did not offer answers to part (b), and none of those who did offered fully correct answers. Few realised that

$$\exists x \exists y (\neg x = y \wedge \forall z (\phi(z) \leftrightarrow (x = z \vee y = z)))$$

means that there are exactly two x s such that $\phi(x)$, as they might have done by recognising the structural similarity to $\exists x \forall z (\phi(z) \leftrightarrow x = z)$.

Part (a) was done well by a small number of candidates. Some marks were given for natural deduction proof outlines/sketches, even if a full proof was not given. Many candidates’ attempts at providing a natural deduction proof were undermined by bad translations of the English argument. Here the mistakes were numerous. Some translated “exactly two” as “at least two.” Some failed to use “=” to translate “is identical to” and instead introduced a new \mathcal{L}_2 predicate. Often translations were made needlessly complex (e.g., several candidates introduced a constant to stand for “the neo-Lockean view”). While not formally incorrect, this had the potential to make the proof more involved and harder to do.

Question 8 (Formalising descriptions) This was another question on Michaelmas term material that was poorly done. For part (b), some candidates were

good at spotting ambiguities, some good at using Russellian-style analyses of description constructions, but too few were good at both. One candidate's explicit attempt to redefine 'before' to mean 'at the same time as or earlier than' was not deemed legitimate. Some candidates' translations indicated that they failed to realise that $\exists x \exists y (Fx \wedge Fy)$ is equivalent to $\exists x Fx$.

Report on Introduction to Philosophy

General Philosophy Questions

Frege Questions

7. Kant

There was only one answer to this question. A good answer would show a fair knowledge of Kant's account of the arithmetical judgements as synthetic a priori, and good knowledge of Frege's criticisms of this conception. An excellent answer would, while acknowledging those aspects of Frege's criticisms which are compelling, have something to say about how a Kantian might respond.

8. Mill

There were six answers to this question, only one of which showed a strong understanding of Mill's views. Very good answers focused upon the notion of an agglomeration or aggregate, and questioned whether the relevant empiricist conception of such a thing involved an object or an activity. Too many answers deviated significantly from the criticism in question in favour of other objections made against Mill by Frege.

9. Numbers as objects

This question was generally well answered. Candidates focused on Frege's reasons for preferring a substantival view of numerical expressions to an adjectival view, and gave reasonably good accounts of his arguments in favour of the former strategy. The best answers were critical of the assumptions that might be thought to lie behind such arguments, and attempted to defend the adjectivalist. The weakest answers simply invoked the context principle, without attempting to justify its adoption.

10. The Julius Caesar problem

There was only one excellent answer to this question. That essay focused on the question as it was asked, and tried to take seriously the idea of a partial explanation. It set that notion within the context of Frege's project and his philosophical commitments, and discussed, critically and with reference to the relevant literature, one version of what a partial explanation might amount to, namely as an explanation of number words for a purely arithmetical language. Weaker answers involved merely standard discussions of the Julius Caesar problem.

11. Fregean extensions

No candidate attempted this question. However, a very good answer would have given an account of Frege's notion of an extension which, at a minimum, explicated the existential and identity conditions for such objects. It might also have explained the use to which Frege wished to put them. An excellent answer would further have shown why there can be no such objects.

12. The ancestral

There were two answers. A very good answer would show thorough familiarity with Frege's derivations of his correlates of the Peano axioms, clearly explaining the role played by the relevant definition, and how it interacts with other definitions (e.g. of the precedes relation).

13. Definitions

There were two attempts at this question. A very good answer would focus upon Frege's notion of analyticity, and seek to explain how the kinds of definitions that Frege envisaged might be fruitful without being synthetic. It might invoke (anachronistically) a distinction between sense and reference, and attempt to show how such a distinction could be thought to resolve the issue by explicating Frege's idea of content recarving.

14. Anti-zero

There were four attempts at this question. The best answers explained Frege's derivation of the natural numbers from Hume's Principle, starting with zero, before showing how the concept ' $x=x$ ' might be thought to allow the derivation of anti-zero. The consequences of such a possibility were discussed, and good points made either in favour of or against the inevitability of the existence of such an object from a Fregean or neo-Fregean point of view.

E. RESERVED BUSINESS

Removed from public version of report.

F. NAMES OF MODERATORS

- Prof. Jochen Koenigsmann (Chair for Preliminary Examinations)
- Prof. Kevin McGerty
- Dr Steven Methven
- Prof. Oliver Pooley