Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2016

October 26, 2016

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Tables 1 and 2. Overall 191 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

	Numbers				Percentages %			
	2016	(2015)	(2014)	(2013)	2016	(2015)	(2014)	(2013)
Distinction	59	(55)	(55)	(55)	30.89	(30.73)	(30.9)	(30.9)
Pass	119	(105)	(103)	(103)	62.3	(58.66)	(57.87)	(57.87)
Partial Pass	7	(13)	(12)	(13)	3.66	(7.26)	(6.74)	(7.3)
Incomplete	0	(0)	(1)	(0)	0	(0)	(0.56)	(0)
Fail	6	(6)	(7)	(7)	3.14	(3.35)	(3.93)	(3.93)
Total	191	(179)	(178)	(178)	100	(100)	(100)	(100)

Table 2: Numbers in each class (Honour Moderations)

	Numbers (2012)	Percentages % (2012)
I	(60)	(30.77)
II	(123)	(63.08)
III	(4)	(2.05)
Pass	(0)	(0)
Honours (unclassified)	(1)	(0.51
Fail	(7)	(3.59)
Total	(195)	(100)

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the Preliminary Examination in Mathematics.
- **Marking of scripts.**
  As in previous years, no scripts were multiply marked by Moderators; however all marking was conducted according to a detailed marking scheme, strictly adhered to. For details of the extensive checking process, see Part II, Section A.

B. New examining methods and procedures

This is the fourth year of the Preliminary Examination in Mathematics, which has replaced Honour Moderations.

Following the review of the Preliminary Examinations after two years, Teaching Committee agreed the abolition of the Applications course. The Statistics course was extended and replaced Optimization. Timetabling and syllabus changes were made to Geometry, Dynamics, Introductory Calculus, Fourier Series and PDEs, and Multivariable Calculus.

The following changes were implemented:

	MICHAELMAS	HILARY	TRINITY
PAPER I	Linear Algebra I	Linear Algebra II, Groups I	Groups II
PAPER II	Sequences and Series	Continuity and Differentiability	Integration
PAPER III	Introduction to Calculus Probability		Statistics and Data Analysis
PAPER IV	Geometry	Dynamics	Constructive Maths
PAPER V		Fourier Series & PDEs Multivariable Calculus	

This meant that from the 2016 examinations Paper III was 3 hours long (students choosing 6 questions from 9) and Paper V was 2 hours long (4 questions from 6). This also rationalized the problems in Paper IV of setting 3 questions on Constructive Mathematics and Optimization.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

No changes are under discussion for 2016/17.

D. Notice of examination conventions for candidates

The Notice to Candidates, containing details of the examinations and assessment, including the Examination Conventions, was issued to all candidates at the beginning of Trinity term. All notices and the Examination Conventions in full are available at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

Acknowledgements

The Moderators express their sincere thanks to the academic administration team, and in particular to Nia Roderick, for all their work in running the examinations system and supporting the Moderators at every turn whilst being careful always to facilitate but never to influence academic decisions made by the Moderators.

We are very grateful to Dr Andrew Thompson for administering the Computational Mathematics projects. We would also like to thank the Assessors Dr Lino Amorim, Dr Robert Gaunt, Dr Stephen Haben, Dr Heather Harrington, Dr Sourav Mondal, Mr Quentin Parsons, Dr Michael Salter-Townshend, Dr Rolf Suabedissen, and Dr Thomas Woolley for their assistance with marking.

Timetable

The examinations began on Monday 20th June at 2.30pm and ended on Friday 24th June at 11.30am.

Factors Affecting Performance

A subset of the Moderators attended a pre-board meeting to band the seriousness of circumstances for each application of factors affecting performance received from the Proctors' office. The outcome of this meeting was relayed to the Moderators at the final exam board. The moderators gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate.

See Section E for further detail.

Setting and checking of papers and marks processing

The Moderators first set questions, a checker then checked the draft papers and, following any revisions, the Moderators met in Hilary term to consider the questions on each paper. They met a second time to consider the papers at the end of Hilary term making further changes as necessary before finalising the questions. A meeting was held in early Trinity term for a final proof read. The Camera Ready Copy (CRC) was prepared and each Moderator signed off the papers. The CRC was submitted to Examination Schools in week 4 of Trinity term.

The examination scripts were collected from Ewert House and delivered to the Mathematical Institute.

Once the scripts had been marked and the marks entered, a team of graduate checkers, under the supervision of Nia Roderick, sorted all the scripts for each paper of the examination. They carefully cross checked against the marks scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. A number of errors were corrected, with each change checked and signed by an Examiner, at least one of whom was present throughout the process. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The candidates under consideration are Mathematics and Mathematics & Statistics candidates, 191 in total. We do not distinguish between them as they all take the same papers.

Marks for each individual paper are reported in university standardised form (USM) requiring at least 70 for a Distinction, 40–69 for a Pass, and below 40 for a Fail.

As last year the Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average proportion in each class over the past five years, together with recent historical data for Honour Moderations.

The raw marks were recalibrated to arrive at the USMs reported to candidates, adopting the procedures outlined below. These procedures are similar to the ones used in previous years.

To ensure equal weightings across all subjects, papers were first standardised to have broadly similar proportions of candidates attaining each class. A piecewise linear mapping was adopted to produce a USM from a raw mark. The default algorithm for each paper works as follows.

1. Candidates' raw marks for a given paper are ranked in descending order. Here the population data used is the set of marks for all candidates in Mathematics or Mathematics & Statistics.

2. The default percentages p_1 of Distinctions, p_2 of nominal upper seconds (USM 60–69) and p_3 of nominal lower seconds and below in this pop-

ulation are selected, these percentages being similar to those adopted in previous years.

3. The candidate at the p_1-th percentile from the top of the ranked list is identified and assigned a USM of 70. Let the corresponding raw mark be denoted by R_1.

4. Similarly, the candidate at the $(p_1 + p_2)$-th percentile from the top of the list is assigned a USM of 60 and the corresponding raw mark is denoted by R_2. Likewise R_3 is the raw mark corresponding to the percentage of p_3.

5. The line segment between $(R_1, 70)$ and $(R_2, 60)$ is extended linearly to the USMs of 72 and 57 respectively. Denote the raw marks corresponding to USMs of 72 and 57 by C_1 and C_2 respectively. Line segments are then drawn connecting $(C_1, 72)$ to $(100, 100)$.

6. Finally, the line segment through the corner at $(C_2, 57)$ is extended down towards the vertical axis as if it were to join the axis at $(0, 10)$, but is broken at the corner $(C_3, 37)$ and joined to the origin, yielding the last segment in this model. Here C_3 is obtained as above by extension from $(R_3, 40)$.

Thereby a piecewise linear map is constructed whose vertices, at $\{(0,0), (C_3, 37), (C_2, 57), (C_1, 72), (100, 100)\}$, are located away from any class boundaries.

A first run of the outlined scaling algorithm was performed. It was confirmed that the procedure resulted in a reasonable proportion of candidates in each class. The Moderators then used their academic judgement to make adjustments where necessary as described below. The Moderators were not constrained by the default scaling map and were able, for example, to insert more vertices if necessary.

To obtain the final classification, a report from each Assessor was considered, describing the apparent relative difficulty and the general standard of solutions for each question on each paper. This information was used to guide the setting of class borderlines on each paper.

The scripts of those candidates in the lowest part of each ranked list were scrutinised carefully to determine which attained the qualitative class descriptor for a pass on each paper. The gradient of the lower section of the scaling map was adjusted to place the pass/fail boderline accordingly.

Careful consideration was then given to the scripts of candidates at the Distinction/Pass boundary.

Adjustments were made to the scaling maps where necessary to ensure that the candidates' performances matched the published qualitative class descriptors.

The Computational Mathematics assessment was considered separately. In consultation with the relevant Assessor it was agreed that no recalibration was required, so the raw marks (out of 40) were simply multiplied by 2.5 to produce a USM.

Finally, the class list for the cohort was calculated using the individual paper USMs obtained as described above and the following rules:

Distinction: both $Av_1 \geq 70$ and $Av_2 \geq 70$;

Pass: not meriting a Distinction and a USM of at least 40 on each paper and for the practical assessment;

Partial Pass: a USM of less than 40 on one or two papers (including the practical assessment);

Fail: a USM of less than 40 on three or more papers (including the practical assessment).

Here Av_2 is the average over the five written papers and Av_1 is the weighted average over these papers together with Computational Mathematics (counted as one third of a paper). The Moderators verified that the overall numbers in each class were in line with previous years, as shown in Tables 1 and 2.

The vertices of the final linear model used in each paper are listed in Table 3, where the x-coordinate is the raw mark and the y-coordinate the USM.

Table 3: Vertices of final piecewise linear model

Paper	Positions of vertices				
I	(0,0)	(24.87,37)	(43.3,57)	(71.8,72)	(100,100)
II	(0,0)	(23,40)	(34,57)	(62.5,70)	(100,100)
III	(0,0)	(38.32,37)	(66.7,57)	(88.5,69.5)	(120,100)
IV	(0,0)	(27.57,37)	(48,57)	(78,72)	(100,100)
V	(0,0)	(23.67,37)	(41.2,57)	(65.2,72)	(80,100)
CM	(0,0)				(40,100)

Table 4 gives the rank list of average USM scores, showing the number and percentage of candidates with USM greater than or equal to each value.

Table 4: Rank list of average USM scores

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	%
87	1	2	1.05

Table 4: Rank list of average USM scores (continued)

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	%
85	3	4	2.09
84	5	7	3.66
83	8	8	4.19
82	9	9	4.71
81	10	12	6.28
80	13	15	7.85
79	16	19	9.95
78	20	22	11.52
77	23	26	13.61
76	27	31	16.23
75	32	34	17.80
74	35	39	20.42
73	40	43	22.51
72	44	47	24.61
71	48	52	27.23
70	53	58	30.37
69	60	68	35.60
68	69	78	40.84
67	79	88	46.07
66	89	98	51.31
65	99	110	57.59
64	111	124	64.92
63	125	127	66.49
62	128	137	71.73
61	138	142	74.35
60	143	148	77.49
59	149	151	79.06
58	152	154	80.63
57	155	160	83.77
56	161	162	84.82
55	163	166	86.91
54	167	170	89.01
53	171	176	92.15
52	177	178	93.19
51	179	180	94.24
48	181	182	95.29
46	183	185	96.86
41	186	186	97.38
39	187	187	97.91

Table 4: Rank list of average USM scores (continued)

USM (x)	Rank	Candidates with USM $\geq x$ Number	%
35	188	188	98.43
34	189	189	98.95
29	190	190	99.48
27	191	191	100.00

Recommendations for Next Year's Examiners and Teaching Committee

1. Distinction requirement: The moderators would like to highlight that, under the current classification conventions, a candidate achieving both $Av_1 \geq 70$ and $Av_2 \geq 70$ and failing the practical assessment would satisfy the requirements for both a distinction and a partial pass.

The examiners recommended changing the distinction requirement to *both* $Av_1 \geq 70$ *and* $Av_2 \geq 70$ *and a mark of at least 40 on each paper and for the practical assessment.*

2. Computational Mathematics: The Computational Mathematics course coordinator queried whether the difficulty of the projects was appropriate. The examiners believed the objective of the projects was not difficulty but welcomed further discussion on the matter.

It was noted that, of the three projects set, two were based on course teaching of Matlab and a further project was based on Sage. It was noted that Sage had been chosen due to the availability within the Department of a collaborator to set the project. It was suggested that the Department of Statistics may be interested in collaborating to provide a project in R. The moderators recommended further discussion as to what is possible.

B. Equal opportunities issues and breakdown of the results by gender

Table 5 shows the performances of candidates broken down by gender.

Table 5: Breakdown of results by gender

	Total		Male		Female	
Class	Number	%	Number	%	Number	%

Distinction	59	30.89	52	36.11	7	14.89
Pass	119	62.3	83	57.64	36	76.6
Partial Pass	7	3.66	6	4.17	1	2.13
Incomplete	0	0	0	0	0	0
Fail	6	3.14	3	2.08	3	6.38
Total	191	100	144	100	47	100

C. Statistics on candidates' performance in each part of the Examination

The number of candidates taking each paper is shown in Table 6. The performance statistics for each individual assessment are given in the tables below: Paper I in Table 7, Paper II in Table 8, Paper III in Table 9, Paper IV in Table 10, Paper V in Table 11 and Computational Mathematics in Table 12. The number of candidates who received a failing USM of less than 40 on each paper is given in Table 6.

Note that Paper I, II and IV are marked out of 100 (being 2.5 hours in duration), Paper III is marked out of 120 (being 3 hours in duration) and Paper V is marked out of 80 (being 2 hours in duration).

Table 6: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM	Number failing	% failing
I	190	58.47	14.14	64.8	9.87	5	2.6
II	190	48.92	15.84	62.88	11.49	7	3.7
III	189	83.98	17.83	68.49	13.08	4	2.1
III (old regs)	1	-	-	-	-	-	-
IV	189	63.79	15.37	65.17	10.88	5	2.6
V	189	54.1	12.84	65.8	12.12	6	3.2
V (old regs)	1	-	-	-	-	-	-
CM	188	34.21	4.88	85.62	12.86	2	1.1

Table 7: Statistics for Paper I

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	11.37	11.39	4.85	172	1
Q2	15.52	15.52	3.19	178	0
Q3	11.93	11.93	4.36	61	0
Q4	11.52	11.59	6.16	158	1

Q5	7.23	7.40	3.45	100	5
Q6	9.93	9.93	3.19	150	0
Q7	12.59	12.59	3.96	127	0

Table 8: Statistics for Paper II

Question Number	Average Mark All	Used	Std Dev	No. of Attempts Used	Unused
Q1	11.89	11.89	3.66	153	0
Q2	10.60	10.64	4.32	96	1
Q3	14.50	14.58	3.76	130	1
Q4	10.33	10.38	3.91	178	1
Q5	10.00	10.17	5.15	58	1
Q6	8.97	9.03	4.54	144	1
Q7	4.62	4.62	3.86	178	0

Table 9: Statistics for Paper III

Question Number	Average Mark All	Used	Std Dev	No. of Attempts Used	Unused
Q1	14.51	14.88	4.02	160	7
Q2	13.32	13.99	4.19	67	4
Q3	15.23	15.23	3.21	151	0
Q4	12.55	12.50	3.75	64	1
Q5	14.05	14.05	3.57	170	0
Q6	13.36	13.36	3.71	141	0
Q7	14.54	14.54	4.47	181	0
Q8	13.42	13.42	5.65	142	0
Q9	13.46	13.46	3.90	48	0

Table 10: Statistics for Paper IV

Question Number	Average Mark All	Used	Std Dev	No. of Attempts Used	Unused
Q1	14.70	14.70	3.00	177	0
Q2	11.12	11.12	4.29	99	0
Q3	12.97	12.97	3.42	102	0
Q4	10.43	10.61	5.08	103	2
Q5	14.04	14.04	4.43	184	0
Q6	11.41	11.50	5.11	82	1
Q7	12.89	12.89	3.97	187	0

Table 11: Statistics for Paper V

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	13.74	13.76	4.33	144	1
Q2	15.22	15.60	4.00	110	3
Q3	14.00	14.09	4.85	121	2
Q4	13.28	13.28	3.62	184	0
Q5	13.76	13.92	5.66	85	1
Q6	11.26	11.26	3.71	106	0

Table 12: Statistics for Computational Mathematics

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	16.62	16.62	2.96	181	0
Q2	16.39	16.39	3.25	59	0
Q3	18.06	18.06	2.44	136	0

D. Comments on papers and on individual questions

Paper I

Question 1

Solving the system in part (a) was surprisingly hard: common mistakes were inaccurate arithmetic in carrying out the elementary row operations. Also common was to distinguish between the $\alpha = 1$ and $\alpha \neq 1$ cases when dividing by $\alpha - 1$, but not to take care of $\alpha = -1$ when later dividing by $\alpha^2 - 1$. Candidates often forgot to answer the last part of part (b). In (c), a number of candidates gave creative answers, mostly constructing diagonal or upper-triangular matrices with the required properties. Fewer candidates extended the given matrix to a 4×4 matrix. A surprisingly large number of attempts were 3×3 or 5×5 matrices. Generally, there was often not enough justification as to why the given system is required.

Question 2

Part (a) was done almost perfectly. A few candidates confused the domain and co-domain in the Rank-Nullity Theorem. There were three good approaches in (b)(ii), either identifying a basis manually, using the Rank-Nullity Theorem with a projection as linear map or quoting the dimension formula for sums. In part (b)(iii) the majority of candidates forgot to check that R was linear and gave very complicated arguments regarding uniqueness. The main problems in (c) were remarking that R was indeed linear (from (b)(iii)) and occasionally claiming that $\ker(R) = U \cap V$.

Question 3

This was not a very popular question despite the fact that it wasn't hard at all. Maybe the somewhat novel use of permutations in part (b) looked frightening. The most frequent mistake was to omit one direction in the *if and only if* of part (b)(ii) (usually the easy one)

Question 4

This question, in contrast, was very popular. Many candidates though had enormous difficulties in finding the eigenvalues of the matrix A, even when following the hint with the auxiliary matrix J. On the other hand, many candidates gained full marks. Once part (a) was in place, part (b) was mere routine, and part (c) mainly bookwork.

Question 5

This question was rather challenging. Very few candidates managed (or even tried) to show uniqueness in part (a)(ii). In part (c)(i) many candidates implicitly assumed that the group G was abelian, and most of them failed to show that HN was a subgroup of G. Each part of the question was answered correctly by some candidate, but nobody got it all right.

Question 6

This question was attempted by the majority of candidates, but not done very well. Most candidates managed part (a), and gave the correct definition in part (b). The original bit of part (b) was difficult, but a few candidates did get this. Part (c) was rather fragmented, but a decent number of candidates did manage to follow the thread of the argument and get most of the marks for this.

Question 7

This was quite a popular question and almost all candidates who attempted it got full marks on the bookwork in part (a). Part (b) was a little more testing, but most candidates did well on (i) and (ii), many got partial answers to (iii), and a reasonable number even worked out the correct answer for (iii).

Paper II

Question 1

This question was attempted by almost all of the candidates and done fairly well. No parts of the question were especially difficult, and many candidates got at least partial credit for all of the parts.

Question 2

This was the least popular of the Analysis 1 questions. There was a modest amount of straight bookwork, but only part (c)(i) required any great ingenuity. Overall the question was well done, with a broad spread of marks. All the parts of question were solved by a significant number of candidates, though relatively few candidates were successful across them all and got full or nearly full marks.

Question 3

This question had a significant component of bookwork and was attempted by almost all the candidates. Marks were high, the only slightly tricky part being determining the behaviour of the series at its radius of convergence.

Question 4

This question was the most popular in Section B, and was answered quite well. A number of different successful strategies were used in $a)ii)$ though many candidates did not seem to realise the bounded interval E was not assumed to be closed. Part $c)ii)$ proved to be the most difficult, with relatively few students providing a complete solution.

Question 5

This was the least popular question in Section B, though the students who attempted it faired quite well. A number of students gave example which did not fit the question (ignoring the continuity requirement in $a)ii)$ for example), and many candidates failed to completely check in part $b)$ that the sequence of functions given in the hint does indeed converge uniformly.

Question 6

This question focused on inverse functions and the chain rule. A distressing number of students seemed not to have understood the proof of the chain rule (or even its statement in a number of cases). The part of the question on inverse functions was on the whole managed quite well, though students again sometimes failed to give counterexamples satisfying the requirement of the question. Part $c)$ was the most challenging part, but a number of students gave complete solutions using a couple of different methods.

Question 7

Overall the students did badly on this question. It seems the students didn't consolidate the integration material well and many ran out time (as this was the last question). In part (a) most students tried to prove integrability by definition instead of using continuity of the function (most didn't even realize this was the case). This made the question harder. In the computation many students lost points for using the trapezium rule without any justification. In part (b) many students identified the relevance of uniform continuity but failed to use it correctly. As in (a) many candidates unnecessarily tried to use step functions. Most students did not attempt part (c). The few that got full marks did it by showing that the function (not only the integral) is determined by its values in the set D.

Paper III

Question 1

Most candidates answered this question and the marks were generally high.

Part (a) was generally answered fully and most candidates noticed the simple change of variables which could simplify the calculation. Many others also found the correct answers by using the integrating factor.

Part (b) was mostly well done. Some made mistakes in changing variables. When done correctly, some still used the characteristic equation approach rather than simply recognising the form of the homogeneous solution. In a number of cases (although not the majority) the constants were not always found correctly. In particular the coefficient for the sin(ln(x)) term was often, incorrectly, found to be nonzero.

Part (c) was the main stumbling block for most candidates attempting this question. Many forms of solution were attempted, usually of the form u+v. Another common error was factorising the differential equation after implementing the change of variables v=(y/x). Many also didn't correctly split the partial fractions.

Question 2

This question was the least popular, being attempted only by a minority of candidates. In the several attempts, parts (a) and (c) were usually answered completely correctly with part (b) causing the most problems.

With part (a) most people understood how to implement the chain rule and use it to differentiate the function $w(x, y) = f(xy^2)$. Testing whether equation (i) and (ii) were solutions was straight forward although it was usually done by directly differentiating rather than using the first part of the question.

Part (b) was done quite poorly. There were a few guesses to the form of the test equation. I.e. a sum of $u(x, y)$ and $v(x, y)$ being the most common attempt. Some seemed to mistake the equation for proving that a function of the form $w(x, y) = f(xy^2)$ was a solution rather than the solution must be of that form. A few candidates did choose appropriate functions although everyone failed to note the arbitrariness of the choice.

Part (c) was generally done very well. In most cases the solution was found by directly calculating the partial derivatives and substitution or substitution and then differentiation. In rare cases the derivative matrix was derived and inverted to calculate both $\frac{\partial u}{\partial x}$ and $\frac{\partial x}{\partial u}$. The second derivative equation was also successfully derived in most cases although in others there was errors in using chain rule which led to an incorrect expression. The final part of the question was also usually correct although some candidates incorrectly asserted that the reciprocal of $\frac{\partial u}{\partial x}$ was $\frac{\partial x}{\partial u}$.

Question 3

A large majority of candidates made serious attempts at this calculus question.

Most obtained full marks for formulating and correctly evaluating the line integral in part (a).

There were also many good attempts at the bivariate integral in part (b), though some candidates calculated the area of the region between the two intersection points of the two given curves rather than the region which was asked for in the question. Amongst the large majority who identified the right region, some split the calculation up into several distinct subregions whilst most realised that by integrating with respect to y first, only a single bivariate integral was required.

Most candidates derived the correct defining equations for constrained extreme points using a Lagrange multiplier in part (c) and most made at least some progress with their solution. Of those who did identify the two extreme points, very few were able to classify them.

Question 4

Q4 was much less popular than Q5 and Q6. Perhaps candidates were put off by the mention of the weak law of large numbers (which I think has not appeared before). Those who could state the WLLN accurately often did well in the last part (but many could not). In part (b), for full marks one needs to include the cases $p = 0$ and $p = 1$. Otherwise, the most frequent error was to confuse $\mathbb{P}(S_n = k|E_1)$ with $\mathbb{P}((S_n = k) \cap E_1)$.

Question 5

Q5 was relatively easy for those who had a sound grasp of the material in the course about probability generating functions. Many candidates scored 10 or 11 out of 11 on parts (a) and (b). In part (c)(i), it was good to see that many candidates had a decent grasp of the general idea leading to the composition of generating functions, but the details are somewhat intricate. Very few were willing to comment on how the partition theorem applies (namely that $(\{N = i\}, i \in \mathbb{N})$ is a partition of the sample space) despite the instruction in the question. The most subtle point of the argument is the use of the independence of N from the sequence $Y_1, Y_2, \ldots$ (many candidates ignored this). In the final part, for full marks one should make some mention of the fact that the probability generating function characterizes the distribution (if relying on it; other candidates quoted or derived the formula for obtaining the probability mass function from the generating function by differentiating, which is also fine).

Question 6

There were plenty of good answers to Q6. In part (a)(ii), there were too many appeals to logic along the lines of "for $p < -1$, the expectation is negative, which must be wrong, so it is undefined", which didn't receive full marks. Part (b)(iii) provided a good challenge. Some candidates produced excellent answers, using a variety of methods. Quite a few could do the case $x \leq \mu$, by observing that $g(x) \leq f(x)$ for $x \in [0, \mu]$, but couldn't get the case $x > \mu$ (the simplest approach is probably to write in terms of $\mathbb{P}(X > x)$ and $\mathbb{P}(Y > x)$). Many answers or part-answers were unclear because x appeared both as a limit of the integral and as the variable of integration.

Question 7

This was a standard question that was answered by most students. Part (a) was generally well answered, although the calculation of the mean square error and failing to check that the second derivative of the log-likelihood was negative were common stumbling blocks. I was surprised that many students could not provide an adequate statement of the central limit theorem. Part (b)(ii) on confidence intervals was fairly standard, but it proved useful in discriminating between the students.

Question 8

Questions **8** and **9** were mostly well answered. There were no parts that a large majority of students could not answer. **8(c)** and **9(d)** were the most challenging but some students scored full marks in each. A typical mistake in **8(c)** was to choose evenly / uniformly spaced values of x. A typical mistake in **9(d)** was to assume all sample covariance terms would tend to infinity. Marks were most commonly lost as follows:

- **8(a)(i)** -1 for assuming knowledge of the likelihood and defining least squares based on that.
- **8(a)(ii)** Allow derivation of $\hat{\alpha}$ and $\hat{\beta}$ from the likelihood. -1 if no explicit equation $\alpha^T S\alpha = \lambda$.
- **8(b)(i)** -1 for assuming a variance of 1 (rare) or for not stating that the ϵ_i are independent.
- **8(b)(ii)** -1 for assuming the variance of $\hat{\alpha}$ without proof. -1 for assuming without proof $\hat{\alpha}$ is normally distributed or for invoking the Central Limit Theorem.
- **8(c)** -3 for placing the x values uniformly along $[-1, 1]$. -1 for choosing values "close to" (rather than at) -1 and 1.

- **9(a)** -1 for double counting covariance between i and j.
- **9(b)** -1 for assuming centred values of x for the sample covariance or for not scaling μ by $1/n$.
- **9(c)** -1 for failing to distinguish S from Σ. I allowed derivation of the solution using a spectral decomposition.
- **9(d)(i)** -1 for $S_{11} \to 0$.
- **9(d)(ii)** -1 for $S_{22} \to \infty$ or $S_{22} \to 0$.
- **9(d)(iii)** -1 for $S_{12} \to +\infty$ rather than $\pm\infty$.

- **9(d)(iv)** -1 for only stating impact on first PC. Some students made vague comments about this such as "it wouldn't work", for which I did not award marks.

Question 9

see question 8

Paper IV

Question 1

The bookwork in Q1 (parts (a) and (b)) was almost universally well done, though disappointingly many candidates simply expanded everything out, rather than giving a short argument as in lectures, and using (a) and the first part of (b) for the second part of (b). (c) turned out to be fairly tricky, though the question contains a big hint: first find normal vectors to the two planes.

Question 2

Question 2 turned out to be quite difficult. In part (b), it was not enough just to reduce to the 2-dimensional case (this would not be worth 7 marks). In part (c), the differentiation caused problems for some candidates. Many gave up at this point, though it is possible just to assume the result asked for and continue with the question. Some answers to this question in particular showed confusion about very basic matters, for example mixing up scalars and vectors, or trying to take the dot product of three vectors. A few very good answers were spoiled by apparently forgetting parts of the question.

Question 3

The first part of Q3 was with hindsight perhaps too easy - some very short answers gained full marks. However, quite a few candidates tried to integrate in the wrong order, making things hard for themselves. Successively decreasing numbers managed the three parts of (b), so this part of the question seemed to discriminate well. There is a simple geometric answer to the last part, which a few candidates found.

Question 4

Many candidates made solid attempts on the question, though weaker candidates did struggle with the bookwork. In the final part, candidates sometimes tried to find the stationary points by considering only $\ddot{r} = 0$, or only $\dot{r} = 0$, rather than setting $\ddot{r} = \dot{r} = 0$, making the rest of the question intractable. In the final *linear* stability analysis, relatively few candidates recognised simplifications, such as dropping the $\dot{r}^2$ term immediately without detailed calculation; these simplifications greatly reduce the required working.

Question 5

This is a very popular question and has been attempted by everyone in the class (at least the first part), and many have secured full marks in it. The candidates knew their bookwork well; however some of them seem to have memorized the results for a portion of the solution in part (a). The expression for the acceleration vector ($\ddot{r}$) has been written directly (which was not so obvious) by some candidates without even deriving it. There was a bit of confusion with part (b), as several candidates got puzzled and failed to identify that $h = \alpha$, which ultimately went on to get an expression containing the sine/cosine or hyperbolic sine/cosine functions instead of just the simple relation of r with θ. For the solution of part (c) instead of $r^2\theta = h$, some of the candidates have applied a alternative technique using $r = \alpha/a$, which made the problem much easier. The candidates who could not solve part (b) or derived an erroneous result were completely lost in the part (c).

Question 6

Several candidates have attempted this question, and almost everyone got the correct derivation for part (a). The candidates have done well to identify the logical vector operations in part b(i) and their understanding was clear. Many candidates were confused on how to approach the part b(ii) and it was left incomplete. Except a few, many could not get the identity $x^{'}(s)x^{''}(s) + y^{'}(s)y^{''}(s) = 0$ and subsequently failed to solve it. Some of the candidates even tried to back-calculate to prove the identity. The last subpart b(iii) has been tried by all the candidates who attempted question 6. Most of them seem to know and followed the linear stability analysis taught in the lectures and tutorials. They were confident about the linear expansion of a function around the equilibrium point. Part (b) has three subparts and since the first two subparts were proof of algebraic relation, a handful of candidates have skipped one or both of these subparts to solve other subparts using the result from the previous subpart question(s). For example, solving subpart (b)(iii) considering the algebraic relation mentioned in the question for subpart (b)(i) and (b)(ii).

Question 7

All but a handful made serious attempts at this compulsory question on fixed point iterations.

Part (a) was well done, though some candidates omitted to identify that the mapping property $g : [a, b] \to [a, b]$ implies that all iterates lie in $[a, b]$ for any $x_0 \in [a, b]$.

Candidates had to make a choice for g in part (b), with most following a reasonable path, but some getting into rather complicated considerations following overly complex choices.

(c)(i) was well done, and there were many good proofs in (c)(ii) which required use of l'Hôpital's rule.

Paper V

Question 1

On the whole most students understood what the first part of the question was asking them in terms of the Jacobian derivation, area integral and centroid evaluation. The majority of students simply lost marks due to errors in calculation and incorrect limit defintions.

The last part of the question involving the volume integral proved more difficult. Fewer students understood how to construct the integral equation. Those who did get past this first hurdle often did a good job. Again errors in calculation and incorrect limit definitions where the most frequent causes of reduced marks.

Question 2

Most candidates answered parts a. Candidates generally did not struggle with b, but some were not explicit showing the final part.

Candidates had trouble answering the middle of part c (simplification). Many candidates were able to answer the first part.

Question 3

This question was popular and answered very well in general, though weaker candidates struggled to explain the orientation of the contour integral relative to the normal of the surface integral. Failing to include the surface Jacobian correctly for the surface integral also tripped some candidates, but relatively many candidates scored very good marks.

Question 4

Almost all candidates made serious attempts at this question on Fourier series.

There were many successful attempts to calculate the Fourier series for the given function, f, though many suggested that they were simply calculating an alternative form for f valid for all arguments despite it being discontinuous.

The final part (c) was less well done: the majority of candidates made no mention of convergence of the Fourier series, and an alarming number simply saw the given final result and, with determination, expounded fallacious arguments to 'derive' it. There were, however, a few completely correct arguments which adequately explained the various necessary observations to obtain a sound proof. The number of scripts covered with calculations but with not a single word, let alone an argument, was disappointing.

Question 5

This question was answered very well by the majority and weaker candidates generally managed the first stages. In the later stages, candidates often used a memorized expression for the integrals to give the Fourier coefficients but did not compensate for the fact the interval was of length 1, rather than of length 2π.

Question 6

The majority of candidates picked up marks in the early stages, and part (a) was found almost universally easy. However, in part (b), explaining why the summation for the separation of variables series was only over positive integers tripped up many. More generally, significant numbers made it to the latter stages, though recognising that the Fourier coefficient of the $\cos\theta$ term did not follow the same pattern as the coefficients for the other cosine terms tripped up a few at this point.

Computational Mathematics

The students chose two projects out of three (two Matlab-based, Projects A and B; one Sage-based, Project C), and each was marked out of 20, giving a total of 40. The majority of students scored 30 or above. Assessment was based mainly on published reports, with the exception of Project B where some marks were awarded for successfully running code (animations). Project A (orthogonal polynomials) was the most popular, followed by Project C (rings and p-adic integers with Sage). Project B (solving pendulum models) was least popular. Students had limited exposure to Sage in advance, and with this in mind Project C was made especially accessible. Project B focused more on numerical calculation, which is where Matlab is actually useful for mathematicians. However, the focus on symbolic calculation in the first part of the course may mean that students felt less confident to tackle Project B. The marks for each of the projects were comparable, with the marks for project C on average slightly higher. In projects A and B, two marks were awarded for a coherently written report and well-written code; not all students earned these marks.

E. Comments on performance of identifiable individuals

Removed from public version of report.

F. Names of members of the Board of Examiners

- **Examiners: Prof. Andy Wathen (Chair), Prof. Eamonn Gaffney, Prof. Jochen Koenigsmann, Prof. Alan Lauder, Prof. James Martin, Prof. Kevin McGerty, Prof. Oliver Riordan.**
- **Assessors: Dr Lino Amorim, Dr Robert Gaunt, Dr Stephen Haben, Dr Heather Harrington, Dr Sourav Mondal, Mr Quentin Parsons, Dr Michael Salter-Townshend, Dr Rolf Suabedissen, Dr Andrew Thompson, Dr Thomas Woolley.**