Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2018

November 1, 2018

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1. Overall 197 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

	Numbers					Percentages %				
	2018	(2017)	(2016)	(2015)	(2014)	2018	(2017)	(2016)	(2015)	(2014)
Distinction	58	(62)	(59)	(55)	(55)	29.44	(30.85)	(30.89)	(30.73)	(30.9)
Pass	126	(124)	(119)	(105)	(103)	63.96	(61.69)	(62.3)	(58.66)	(57.87)
Partial Pass	10	(13)	(7)	(13)	(12)	5.08	(6.47)	(3.66)	(7.26)	(6.74)
Incomplete	0	(0)	(0)	(1)	(0)	0	(0)	(0)	(0)	(0.56)
Fail	3	(2)	(6)	(6)	(7)	1.52	(0.99)	(3.14)	(3.35)	(3.93)
Total	197	(201)	(191)	(179)	(178)	100	(100)	(100)	(100)	(100)

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the Preliminary Examination in Mathematics.
- **Marking of scripts.**
  As in previous years, no scripts were multiply marked by Moderators; however all marking was conducted according to a detailed marking scheme, strictly adhered to. For details of the extensive checking process, see Part II, Section A.

B. New examining methods and procedures

No new examining methods and procedures were used for 2017/18.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

No changes are under discussion for 2018/19.

D. Notice of examination conventions for candidates

The Notice to Candidates, containing details of the examinations and assessment, including the Examination Conventions, was issued to all candidates at the beginning of Trinity term. All notices and the Examination Conventions in full are available at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

Acknowledgements

The Moderators are extremely grateful to the academic administration team, and in particular to Nia Roderick and Charlotte Turner-Smith, for their hard work in running the examinations system and in supporting the Moderators throughout the year, whilst being careful always to facilitate but never to influence academic decisions made by the Moderators.

We also thank Waldemar Schlackow for maintaining and running the examination database and in particular for his assistance during the final examination board meeting.

We express our sincere thanks to Dr Andrew Thompson for administering the Computational Mathematics projects. We would also like to thank the Assessors Dr Maria Christodoulou, Dr Adam Gal, Dr Stephen Haben, Dr David Hume, Dr Chris Lester, Dr Eoin Long, Dr Andrew Mellor and Dr Ebrahim Patel for their assistance with marking.

Timetable

The examinations began on Monday 18th June at 2.30pm and ended on Friday 22nd June at 11.30am.

Factors Affecting Performance

A subset of the Moderators attended a pre-board meeting to band the seriousness of circumstances for each application of factors affecting performance received from the Proctors' office. The outcome of this meeting was relayed to the Moderators at the final exam board. The moderators gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate.

See Section E for further detail.

Setting and checking of papers and marks processing

The Moderators first set questions, a checker then checked the draft papers and, following any revisions, the Moderators met in Hilary term to consider the questions on each paper. They met a second time to consider the papers at the end of Hilary term making further changes as necessary before finalising the questions. A meeting was held in early Trinity term for a final proof read. The Camera Ready Copy (CRC) was prepared and each Moderator signed off the papers. The CRC was submitted to Examination Schools in week 4 of Trinity term.

The examination scripts were collected from Ewert House and delivered to the Mathematical Institute.

Once the scripts had been marked and the marks entered, a team of graduate checkers, under the supervision of Charlotte Turner-Smith and Nia Roderick, sorted all the scripts for each paper of the examination. They carefully cross checked against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. A number of errors were corrected, with each change checked and signed by an Examiner, at least one of whom was present throughout the process. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The candidates under consideration are Mathematics and Mathematics & Statistics candidates, 197 in total. We do not distinguish between them as they all take the same papers.

Marks for each individual paper are reported in university standardised form (USM) requiring at least 70 for a Distinction, 40–69 for a Pass, and below 40 for a Fail.

As last year the Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average proportion in each class over the past five years, together with recent historical data for Honour Moderations.

The raw marks were recalibrated to arrive at the USMs reported to candidates, adopting the procedures outlined below. These procedures are similar to the ones used in previous years.

To ensure equal weightings across all subjects, papers were first standardised to have broadly similar proportions of candidates attaining each class. A piecewise linear mapping was adopted to produce a USM from a raw mark. The default algorithm for each paper works as follows.

1. Candidates' raw marks for a given paper are ranked in descending order. Here the population data used is the set of marks for all candi-

dates in Mathematics or Mathematics & Statistics.

2. The default percentages p_1 of Distinctions and p_2 of nominal upper seconds (USM 60-69) are selected, these percentages being similar to those adopted in previous years.

3. The candidate at the p_1 percentile from the top of the ranked list is identified and assigned a USM of 70. Let the corresponding raw mark be denoted by R_1.

4. Similarly, the candidate at the $(p_1 + p_2)$ percentile from the top of the list is assigned a USM of 60 and the corresponding raw mark is denoted by R_2.

5. The line segment between $(R_1, 70)$ and $(R_2, 60)$ is extended linearly to USMs of 72 and 57 respectively. Denote the raw marks corresponding to USMs of 72 and 57 by C_1 and C_2 respectively. For a graph of the mapping between raw marks and USMs, a line segment is drawn, connecting $(C_1, 72)$ to $(100, 100)$ with a further line segment between $(C_2, 57)$ and $(C_1, 72)$.

6. A line segment through $(C_2, 57)$ is extended down towards the vertical axis, as if it were to join the axis at $(0, 10)$, but the line segment is terminated at a USM of 37. The associated raw mark at the termination point is denoted C_3.

7. Finally a line segment between $(C_3, 37)$ and $(0, 0)$ completes the graph of the piecewise linear mapping between the raw marks and the USM.

Thereby a piecewise linear map is constructed whose vertices, at $\big\{(0,0), (C_3, 37), (C_2, 57), (C_1, 72), (100, 100)\big\}$, are located away from any class boundaries.

A first run of the outlined scaling algorithm was performed. It was confirmed that the procedure resulted in a reasonable proportion of candidates in each class. The Moderators then used their academic judgement to make adjustments where necessary as described below. The Moderators were not constrained by the default scaling map and were able, for example, to insert more vertices if necessary.

To obtain the final classification, a report from each Assessor was considered, describing the apparent relative difficulty and the general standard of solutions for each question on each paper. This information was used to guide the setting of class borderlines on each paper.

The scripts of those candidates in the lowest part of each ranked list were scrutinised carefully to determine which attained the qualitative class descriptor for a pass on each paper. The gradient of the lower section of the scaling map was adjusted to place the pass/fail borderline accordingly.

Careful consideration was then given to the scripts of candidates at the Distinction/Pass boundary.

Adjustments were made to the scaling maps where necessary to ensure that the candidates' performances matched the published qualitative class descriptors.

The Computational Mathematics assessment was considered separately. In consultation with the relevant Assessor it was agreed that no recalibration was required, so the raw marks (out of 40) were simply multiplied by 2.5 to produce a USM.

Finally, the class list for the cohort was calculated using the individual paper USMs obtained as described above and the following rules:

Distinction: both $Av_1 \geq 70$ and $Av_2 \geq 70$ and a mark of at least 40 on each paper and for the practical assessment;

Pass: not meriting a Distinction and a USM of at least 40 on each paper and for the practical assessment;

Partial Pass: awarded to candidates who obtained a standardised mark of at least 40 on three or more of Papers I-V but did not meet the criteria for a pass or distinction;

Fail: a USM of less than 40 on three or more papers.

Here Av_2 is the average over the five written papers, weighted by length, and Av_1 is the weighted average over these papers together with Computational Mathematics (counted as one third of a paper). The Moderators verified that the overall numbers in each class were in line with previous years, as shown in Table 1.

The vertices of the final linear model used in each paper are listed in Table 2, where the x-coordinate is the raw mark and the y-coordinate the USM.

Table 2: Vertices of final piecewise linear model

Paper	Positions of vertices				
I	(0,0)	(26.66,37)	(46.4,57)	(77.5,72)	(100,100)
II	(0,0)	(23.21,37)	(40.4,57)	(73.4,72)	(100,100)
III	(0,0)	(36.42,37)	(63.4,57)	(96.4,72)	(120,100)
IV	(0,0)	(27.92,37)	(48.6,57)	(75.6,72)	(100,100)
V	(0,0)	(20.8,37)	(36.2,57)	(59,72)	(80,100)
CM	(0,0)				(40,100)

Table 3 gives the rank list of average USM scores, showing the number and percentage of candidates with USM greater than or equal to each value.

Table 3: Rank list of average USM scores

USM (x)	Rank	Candidates with USM $\geq x$ Number	%
89	1	2	1.02
87	3	3	1.52
86	4	5	2.54
85	6	7	3.55
84	8	11	5.58
82	12	14	7.11
81	15	16	8.12
80	17	18	9.14
79	19	21	10.66
78	22	23	11.68
77	24	27	13.71
76	28	29	14.72
74	30	33	16.75
73	34	40	20.3
72	41	47	23.86
71	48	53	26.9
70	54	58	29.44
69	59	70	35.53
68	71	77	39.09
67	78	82	41.62
66	83	94	47.72
65	95	106	53.81
64	107	117	59.39
63	118	129	65.48
62	130	139	70.56
61	140	147	74.62
60	148	156	79.19
60	148	156	79.19
59	157	160	81.22
58	161	164	83.25
58	161	164	83.25
57	165	170	86.29
56	171	172	87.31
55	173	175	88.83
54	176	177	89.85
53	178	182	92.39
53	178	182	92.39
52	183	185	93.91
51	186	187	94.92

Table 3: Rank list of average USM scores (continued)

USM (x)	Rank	Candidates with USM $\geq x$ Number	%
49	188	189	95.94
49	188	189	95.94
47	190	190	96.45
46	191	191	96.95
45	192	192	97.46
44	193	193	97.97
43	194	194	98.48
40	195	195	98.98
35	196	197	100

Recommendations for Next Year's Examiners and Teaching Committee

None.

B. Equal opportunities issues and breakdown of the results by gender

Table 4 shows the performances of candidates broken down by gender.

Table 4: Breakdown of results by gender

Class	Number								
	2018			2017			2016		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	7	51	58	12	50	62	7	52	59
Pass	57	69	126	36	88	124	36	83	119
Partial Pass	6	4	10	4	9	13	1	6	7
Incomplete	0	0	0	0	0	0	0	0	0
Fail	2	1	3	0	2	2	3	3	6
Total	72	125	197	52	149	201	47	144	191
Class	Percentage								
	2018			2017			2016		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	9.72	40.8	29.44	23.08	33.56	30.85	14.89	36.11	30.89
Pass	79.17	55.2	63.96	69.23	59.06	61.69	76.6	57.64	62.3
Partial Pass	8.33	3.2	5.08	7.69	6.04	6.47	2.13	4.17	3.66
Incomplete	0	0	0	0	0	0	0	0	0
Fail	2.78	0.8	1.52	0	1.34	1	6.38	2.08	3.14
Total	100	100	100	100	100	100	100	100	100

C. Statistics on candidates' performance in each part of the Examination

The number of candidates taking each paper is shown in Table 5. The performance statistics for each individual assessment are given in the tables below: Paper I in Table 6, Paper II in Table 7, Paper III in Table 8, Paper IV in Table 9, Paper V in Table 10 and Computational Mathematics in Table 11. The number of candidates who received a failing USM of less than 40 on each paper is given in Table 5.

Note that Paper I, II and IV are marked out of 100 (being 2.5 hours in duration), Paper III is marked out of 120 (being 3 hours in duration) and Paper V is marked out of 80 (being 2 hours in duration).

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM	Number failing	% failing
I	197	64.21	15.86	66.15	11.03	5	2.5
II	197	58.49	17.77	65.29	12.46	8	4.1
III	197	81.36	17.03	65.89	10.7	4	2
IV	197	63.16	14.41	65.43	10.92	4	2
V	197	48.85	11.29	65.47	9.87	3	1.5
CM	197	33.55	5.67	84.11	14.16	3	1.5

Table 6: Statistics for Paper I

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	12.84	12.84	3.77	171	0
Q2	12.20	12.20	4.41	143	0
Q3	14.40	14.40	4.02	122	0
Q4	12.23	12.23	4.02	154	0
Q5	10.57	10.57	3.48	138	0
Q6	11.45	11.45	4.66	98	0
Q7	15.57	15.75	4.21	158	2

Table 7: Statistics for Paper II

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	13.66	13.66	2.23	183	0
Q2	12.55	12.58	3.83	165	1
Q3	11.43	11.43	4.71	46	0
Q4	11.43	11.43	4.54	175	0
Q5	9.74	9.77	5.74	133	1
Q6	7.00	7.34	4.12	86	7
Q7	12.84	12.84	5.64	194	0

Table 8: Statistics for Paper III

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	11.99	12.16	5.14	172	4
Q2	13.55	13.97	5.10	68	3
Q3	14.74	14.79	3.74	154	1
Q4	15.11	15.31	4.18	124	2
Q5	15.50	15.50	3.11	141	0
Q6	11.74	11.95	4.78	129	6
Q7	12.15	12.15	4.92	166	0
Q8	13.98	13.98	4.55	185	0
Q9	12.00	12.89	6.16	37	3

Table 9: Statistics for Paper IV

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	11.81	11.93	3.70	175	2
Q2	13.90	14.13	3.82	104	2
Q3	12.11	12.26	4.72	115	2
Q4	13.17	13.17	3.88	151	0
Q5	9.41	9.41	4.00	148	0
Q6	12.73	12.73	4.46	88	0
Q7	15.18	15.18	3.75	196	0

Table 10: Statistics for Paper V

Question	Average Mark		Std	No. of Attempts	
Number	All	Used	Dev	Used	Unused
Q1	12.97	13.02	3.92	181	2
Q2	10.16	10.24	4.68	146	2
Q3	11.54	11.95	4.33	64	4
Q4	13.14	13.14	4.43	161	0
Q5	11.90	11.90	4.20	94	0
Q6	12.76	12.76	3.39	139	0

Table 11: Statistics for Computational Mathematics

Question	Average Mark		Std	No. of Attempts	
Number	All	Used	Dev	Used	Unused
Q1	16.28	16.28	3.30	168	0
Q2	15.96	15.96	3.62	119	0
Q3	18.46	18.46	2.79	107	0

D. Comments on papers and on individual questions

Paper I

Question 1

The most popular question by some way. There were a fair few scripts that ignored the initial comment on what might be assumed – that any matrix may be put into RRE form – and instead sought to make use of the rank-nullity theorem or properties of determinants; such solutions usually received little or no credit. In (b), to prove (i) implies (ii), there were a surprising number of scripts that did not simply postmultiply by A^{-1}. In fact, marking part (b) was particularly difficult as many candidates did not make plain which implications they were seeking to prove, and it was too commonly unclear from the content of the argument. Part (c) was not well attempted and few scripts found that the required condition for invertibility to be $2a + b \neq 0$.

Question 2

Whilst part (b) is not verbatim bookwork from lectures, this part was intended as a test of routine linear algebra techniques and it was concerning how many candidates could not complete the three subparts, were inefficient getting correct answers, or made sloppy or erroneous arguments. Whilst there are slick ways to address all three subparts at once, a straightforward way through would be to row-reduce A for (b)(i), again use EROs to determine when the system $A\mathbf{v} = \mathbf{x}$ is consistent or recall that the columns of A span the image for (b)(ii), set $\mathbf{x} = \mathbf{0}$ to find the kernel in (b)(iii). Such scripts commonly then made no further progress, but a good number surprisingly went on to complete (c). Part (a)(ii) can be used directly to show (c)(ii) is impossible; in (c)(iii) M having full rank means that it is invertible, which means M^3 would also be invertible and have full rank; any strictly triangular matrix would show that (c)(i) is possible.

Question 3

The good average mark for this question masks a variety of performances. There were a good number of scripts obtaining full or very high marks. There were also a surprising number of scripts which gained full marks on (a) and (b) only to make no progress with (c) and a sizable number of scripts which completed part (c) despite not having been able to correctly list the properties of an inner product in part (a)(i) or correctly apply the subspace test in (a)(ii). Part (c)(i) can be completed by writing

$$f(x) = \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2}$$

and noting that the zero function is the only even and odd function.

Question 4

Whilst there were some very well argued solutions to this question, the majority of scripts demonstrated much confusion with regard to diagonalization and matrices representing linear maps. Commonly, these confusions were fundamental: some thought eigenvalues and not eigenvectors needed to be non-zero; others introduced notions of orthonormality into the definition of diagonalization; some appeared not to know the method for determining eigenvalues and many could not show that A is not diagonalizable in (b)(i); some determined wrong eigenvalues for A and then had no concern when finding no corresponding non-zero eigenvector. And very few appreciated how (b)(i) and (b)(ii) might be used to help with (b)(iii); for $P^{-1}AP$ to have the desired form the columns of P need to be a 1-eigenvector followed by two vectors that are a basis for an invariant plane. Part (c) is hard; the intended solution was to note that $0 = B^3 - I = (B - I)(B^2 + B + I)$ and that $B - I$, being the restriction of $A - I$ to X, is invertible. One novel solution noted that $A^2 + A + I$ has rank one and that this is one more than the rank of $B^2 + B + I$.

Question 5

Part (a) was completed correctly by almost everyone; in (ii) the required justification was just that <g> is a subgroup of order o(g) (assuming the candidate stated Lagrange for subgroups as (i)). Part (b)(i) was fine apart from a very few candidates who (unsurprisingly) did not manage much of the rest of the question. (b)(ii) was mostly okay, though often with rather lengthy and sometimes incomplete answers for the key argument that the given cosets are disjoint. For part (iii) quite a few candidates asserted the isomorphism without justification (something brief is enough). Part (iv) turned out to be tricky and few candidates completed it.

Question 6

This question seemed to distinguish quite well. (a) was fine, though surprisingly many candidates assumed Z is a subgroup (leaving rather little work for 6 marks). In part (b) the centre can be found by considering conjugacy classes, or just by checking what commutes. This can be done efficiently, but often was not. The second part of (b), and part (c), were harder but still completed by a reasonable number of candidates.

Question 7

The question was generally done very well by most of the candidates that had attempted it. The hardest part was (b)(i) where many candidates claimed that any non-abelian group is a counterexample. In part (b)(iii) many candidates wrote that the stabilizer is $H \cap K$, which is not even a subset of

$H \times K$.

Paper II

Question 1

This was the most popular question in Section A and was generally answered very well. Most students obtained almost full marks on the bookwork from (a) and (b). Part (c) was more variable. In (c)(i) a number of students proved the inequality $n = km$, which obtained partial marks, but did not obtain the general case. A common error in (c)(ii) was to try to deduce the convergence by showing that $\frac{a_n}{n}$ is decreasing, which is not necessarily true.

Question 2

There was a large spread in the marks obtained in this question. A number of students lost marks on the implication Cauchy $\implies$ convergence, but most obtained full marks for (a). Part (b) was a little disappointing, with many students attempting to use only $a_{n+1} - a_n \to 0$ to prove convergence, via many iterations of the triangle inequality. Similarly many tried to relate a_{kn} to a_n via telescoping. Considering $n \mod k$ generally led to full marks. The marks for (c) were quite evenly split. In (c)(i) a number of students split into groups of 100 terms and used alternating series test, which worked well, but did not relate the convergence of these special partial sums to the general partial sum (using (b)).

Question 3

This question was not very popular; most of those that did attempt it did well. Part (a) is standard bookwork. There are several possible versions of the definition of R, any of which was accepted. Whichever is used, at some point you need to compare convergence at one radius and absolute convergence at a smaller radius using comparison with a geometric series. Candidates who missed this key point lost several marks. The rest of the question was mostly well done, although there were a significant number of candidates who attempted the question despite seeming to have very little familiarity with the topic. This tended not to go well.

Question 4

a) Practically all students remembered the definition, and most gave the standard proof from the course for the second part. A common mistake involved stating that there is a minimal δ for a given ϵ without proving that it is non-zero. There were several 'proofs' appealing only to the boundedness of a continuous function.

b) The first part was very straightforward and solved by most students. In part ii most students had the right idea but several gave a uniformly continuous function in their example. Part iii was fully solved only by very few students. Most of the students tried to solve it by proving that both functions are uniformly continuous, which is not true.

Question 5

a) Many students stated the M-test incorrectly. In many cases it seemed that they had not really grasped the meaning of uniform convergence. The theorem was also stated incorrectly by many students, with many stating that the power series converges uniformly on the whole radius of convergence. In the proof however many proved the correct statement but then stated that the parameter may be taken to 0 to get the incorrect statement.

b) Many students could not give a rigorous proof that the limit is 0, and relied on statements such as $[1/(n+1), 1/n) \to [0,0)$. However many of these students then proved uniform convergence to 0 correctly in the other part.

Question 6

The question was attempted by relatively few candidates and the average mark was quite low despite the fact that at least 8 marks were awarded for a rather standard bookwork.

(a): About half of the candidates just wrote that the existence and equality of left and right limits is equivalent to the existence of the limit without any justification.

(b)(i): This turned out to be the hardest part of the question with too many candidates unable to apply the convexity inequality.

(b)(ii): This part is independent of (b)(i), but was not done by many candidates. Also many candidates failed to explain why g is bounded.

(c): Many candidates gave examples that were not even continuous. Probably the most popular wrong answer was the characteristic function of rational numbers.

Question 7

The question was generally answered well, with many students able to give near-complete solutions. However, many students gave an incorrect or incomplete definition of integrability. For the second part, students had the right approach but some failed to use the fact that a continuous function on a bounded domain is uniformly continuous and thus a delta could be chosen independently of the point in the interval. For the third part, most students understood that they needed to use part (ii), but some used the two functions in the opposite roles to what had been intended. The other mistake was not to realise that the points c_ϵ given by the mean value theorem depend on epsilon, and therefore to deduce, since f is continuous, that these $f(c_\epsilon)$ were converging to $f(0)$.

Paper III

Question 1

Part (a)(i) This was answered very well by most candidates. The majority chose to use an integrating factor; those who separated the variables needed to consider carefully the argument of the natural logarithm after integration, to ensure that their expression was valid.

Part (a)(ii) Many candidates erroneously thought that a substitution of the form $y(x) = xv(x)$ would help here; it did not. Those who correctly set $x + y(x) = v(x)$ did very well.

Part (b) Some candidates assumed values of n from the start, and this rarely led anywhere. Those who kept n in their expression were able to rewrite the ODE in terms of $v(x)$ but then most did not see that a suitable value for n could be found by setting the dv/dx coefficient equal to zero. Those who did spot this finished the question very well, solving the second order ODE for v and applying the given conditions.

Question 2

Part (a) This is standard bookwork and most candidates spotted that working with u_s, u_t etc was simpler than working with u_x, u_y etc. Whilst there were many excellent clear answers that gained full marks, a significant minority of candidates did not apply the chain rule for the second derivatives correctly, and hence missed out some of the mixed derivative terms. The extra terms eventually cancel out anyway, so many of these candidates may not have realised that they had missed them out.

Part (b) Too many candidates could not apply the Taylor polynomial formula correctly, or else mis-remembered it. The main errors were in the mixed xy term.

Part (c) This was done well by most candidates, who applied the chain rule correctly. Two restrictions were expected, one for g and one for h, and not everyone got them both.

Question 3

Part (a) This was largely done very well. A few candidates could not solve the equations $f_x = 0$ and $f_y = 0$ but most spotted (or calculated) that $y = x$

and were then able to proceed easily. The classification went well although a small minority could not remember the correct classification criteria.

Part (b) Again this was done well by many candidates, although some got the Lagrangian function back to front. Solving the various equations proved harder than in (a) but was still mainly achieved well. The sketches were less successful; some candidates failed to spot that $g = 0$ is the equation of an ellipse.

Question 4

The first part was a standard question about the axioms of set theory. Many students neglected to mention countable additivity — either stating additivity for pairs of events, or occasionally for general finite collections of events. (A few stated it for general collections of events, neglecting to mention countability.) Many students gave incorrect definitions of $\mathbb{P}$, often stating that it was a function from Ω to $\mathbb{R}$. (Interestingly, computer science students almost uniformly gave a clear and correct definition.)

Part (b) was a slightly unfamiliar computation of the expected value of a function of a continuous random variable, in this case the modulus of a standard normal random variable. Quite a few computed the expected value of Z rather than $|Z|$. Of those who did compute the correct thing, most did it essentially correctly, though not a few did it by the slightly roundabout means of computing the density for $|Z|$.

The last part was a series of difference equations, to be done with essentially standard methods. Most who did this question did this part essentially correctly, with perhaps a few errors in calculation. The most common serious error was to make an assumption in, for example, the last problem, that the particular solution had to have a particular form, such as Cn^2. One surprising error that appeared a number of times was to solve the auxiliary equation correctly, obtaining for example 1 and 2 in the second part, but then to claim that the general solution is $Ae^n + Be^{2n}$, which then led the rest of the solution astray.

Question 5

This was a very popular question from the probability segment of the exam, and the overall standard was high. From Part *a* both sections *i* and *ii* were well done, with candidates using conditional probabilities appropriately. Candidates encountered some difficulties with section *iii*, specifically there were some issues in determining the expectations for one, two, or three different colours coming up but in the majority of cases this was - at least in part - completed. Few candidates attempted this section using induction instead, which was also a successful avenue. Minor arithmetic errors were present in many scripts.

Candidates found Part *b* to be more challenging. Although many correctly established what the necessary conditions for a probability density function were, some struggled during the integration part of the question and others had difficulty separating the different cases for α. As with Part *a* arithmetic errors were found throughout, in many scripts. Since Part *b* section *ii* relied on the result of section *i*, many candidates ended up with wrong probability calculations for section *ii*. As long as the calculations were performed correctly and the resulting values did not violate any of the probability axioms, these were awarded all available marks. This was a well-balanced question, that examined a variety of aspects of the candidates' understanding on foundations of probability.

Question 6

The first part had variations on the probability generating function of a Poisson distribution. The standard questions were mostly done correctly, though some neglected to mention the theorem on uniqueness of probability generating functions. The most challenging part was the Poisson distribution with random parameter. Many did not think to apply the law of total expectation, or applied it incorrectly (for example, neglecting to multiply the conditional expectations by the probability of the event). Quite a few who did set up the law of total expectation correctly then went on to sum in such a way that a dangling undefined variable k remained in the solution. This was then carried over to produce a nonsensical answer in the following part.

The second part was a variant of the gambler's ruin. There was a conspicuous split between the computer science joint schools and the mathematics and maths-stats students. The computer science students mostly did this question, and by and large did it completely and correctly. Of the mathematicians who chose this question, a large fraction left this part completely blank, or wrote a few insignificant words or symbols. Of those who did write a solution, only a minority got as far as writing a recursion. At that point, many seem to have thought methods from difference equations needed to be applied, leading to generally inappropriate answers — except for those who had misinterpreted the problem as a straightforward gambler's ruin, with probability $\frac{1}{2}$ of going in either direction, in which case it is indeed a (trivial) difference equation.

On the other hand, there were a number of creative applications of symmetry by mathematics students, to produce short and elegant solutions.

Question 7

Part (a) was a standard question about MLE and confidence intervals. Part (b) was definitions and direct application of definitions about linear regression.

A significant number of students neglected to check whether the critical point in the log likelihood is a maximum.

Many students seem to have confused the SE in *Standard Error* with the SE in *Mean Squared Error*, yielding a variety of erroneous answers. In some cases it was simply to treat the standard error as the square root of MSE, rather than variance, which produced the same result and was treated as essentially correct.

In computing the confidence interval (part aiii) some neglected to mention that $\sqrt{\hat{\lambda}/n}$ is an approximation for the standard error, treating it as an equality.

The high-leverage points are C and F, but any listing of points in order by their distance from $\bar{x}$ was accepted, if accompanied by a correct explanation. For example, just F, or F,C,H,E,B.

Question 8

This was a very popular question from the statistics segment of the exam, although many candidates performed better in Part *a* than in Part *b*. Almost all candidates correctly explained least squares estimators, and stated correctly that the errors were normally distributed for sections *i* and *ii*, although many failed to communicate that the errors were also i.i.d. Demonstrating that they are unbiased estimators was overall well-done by most candidates, however some scripts would have benefitted by clearly stating which properties they were using as they were progressing through the calculations.

Part *b* was found to be more challenging. Some candidates had difficulty explaining the process of *k-means clustering* in section *ii*, missing important steps such as the random assignment to clusters during the initiation step, the role and formula of the objective function, or the criterion for the termination of the algorithm. Section *iii* was misinterpreted by some candidates, focusing on whether the value for k should be chosen or not, and not whether the choice of k should aim to minimise the objective function. Overall, students performed very well on Part *a* and many found Part *b* difficult.

Question 9

This was not a popular question, with less than 20% of the candidates attempting it, and many providing incomplete answers, possibly running out of time. For Part *a*, the bias calculation primarily suffered from arithmetic mistakes rather than more theoretical problems.

Most candidates who attempted Part *b*, confidently demonstrated the computation of the sample correlation matrix, although some failed to state the appropriate equations, even while using them for calculations. Section *iii* was answered well by most candidates, whereas in section *iv* candidates failed to give two distinct reasons for why standardised variables should be preferred. Section *v* was found to be relatively challenging with some candidates being unfamiliar with scree plots, and others losing marks for more trivial issues such as failure to label the axes. Section *vi* was answered correctly in most cases, although the supporting explanation was at times limited. Overall, this was a well-balanced question, examining both theoretical and practical aspects; however, many candidates seemed to run out of time to complete all sections appropriately.

Paper IV

Question 1

This question was attempted by most students, who generally did very well on the initial bookwork. The final part of the question was successfully completed only by a small number of students. Many attempts at this stage launched into manipulating the equations for the lines in many diverse ways, only to peter out, without considering the problem geometrically for an efficient way forward.

Question 2

This question proved to be reasonably popular and attracted a number of good attempts, although very few students scored perfect marks. The bookwork of part (a) was well done. Those that lost marks on this section were either unclear on their logic (often circular arguments) or lost in algebra (from not recognising that the translation played no role on the vector directions). Part (b) was very well done all round with well supported arguments for why the matrix would represent T. Part (c) was the section that students most struggled with. Finding the invariant line was the most difficult, although most students recognised that a solution could be found using elementary row operations. Of those that found the direction of the invariant line direction, around half incorrectly calculated, or omitted, a suitable invariant point on the line. This section appears to have been quite off-putting for students as those who could not find the invariant line did not attempt the easier tasks of showing the transformation is a rotation, or finding the angle of rotation.

Question 3

The question was not particularly well done, and given quite a lot of attempts began with an incorrect surface area formula in (a)(ii) it seems this question was done by some more of necessity than choice. For those that progressed as far as part (c), most did not realize that the surface of revolution is formed by rotating the top half of the astroid ($0 \leqslant t \leqslant \pi$) about the x-axis – and so obtained twice the area – and some got an answer of 0 by using $\mathrm{d}s/\mathrm{d}t = 3b\sin t\cos t$ rather than the absolute value of this.

Question 4

This was the most popular dynamics question, and was answered well by many candidates. The first part was well done with the most common error being to lose track of the signs and come to the opposite conclusion about stability. Part (b) was straightforward; those who struggled generally did so because they were not taking the x-component of the tension in the spring. The calculation of $V(x)$ in part (c) was done correctly by most, but sketching its graph proved difficult for some. The last part was done well in many cases, but only by those who had correctly sketched the graph in part (c). Throughout, there was very poor use of capitalisation on proper nouns (taylor's theorem and newton's law).

Question 5

This was found the most challenging of the dynamics questions. The first part was much easier if the equation was kept in vector form rather than writing explicitly in components. Very few candidates derived the expression for the magnitude of the normal force, the stumbling block being the need to use the geometrical constraint to relate second derivatives of $r(t)$ to first derivatives. Part (b) was done well by a few candidates, but most struggled, possibly because they had spent so long on part (a). Even evaluating the constants h and E using the given initial conditions proved a challenge. The argument that $z(t)$ was confined between two values was however correctly produced by many. The final part, and particularly the realisation that one needs to avoid the particle losing contact with the sphere, was found by only a few candidates.

Question 6

This question was done well, although it was the least popular of the dynamics questions. The first part, using the steer given in the question, was generally completed well, with the most common mistake being to neglect the normal reaction force or to have it inadvertently pointing in a non-normal direction. Part (b) was done well, although the algebra was excessively messy in some cases. Part (c) was attempted by only a few candidates, but some good answers were produced in those cases.

Question 7

This question was generally answered very well, with few poor attempts. As bookwork part (a) proved no issue for the majority of students, with some going beyond the question and proving the Constructive Mapping Theorem.

Problems occurred for students who assumed differentiability of g in part (ii), and for those sloppy with limits. Part (b) gave students multiple options in their choice of function g which made their efforts with algebra wildly variable. Most students were able to find a suitable function and interval, although at times the strength of their reasoning was low. Finally, part (c) separated out those students who remembered that the Newton iteration gave quadratic convergence, and those who did not. It was nice to see that students were keen to show why this was the case, although not explicitly required.

Paper V

Question 1

Most students scored well, with some obtaining full marks. No student obtained 0 marks; the reason for this is clearly the straightforward parts (a) and (b). In part (c), many struggled to identify $\cos\theta + \sin\theta = R\cos(\theta - \psi)$, which subsequently made it difficult to directly integrate. Of course, this was not the only way to answer this question - the identification of periodicity and/or odd functions was acknowledged by most. However, part (ii) was incorrectly tackled by many students if $R\cos(\theta - \psi)$ was not identified. In part (d), the key was to identify the rotation of the axis such that the integral involves only the z-coordinate. If people struggled with the above rotation identification then they rarely carried out more calculations.

Question 2

This was a more challenging question largely due to the finer details. For example, full marks were rarely awarded for part (a) because the orientation was often overlooked. For part (c)(i), many people used part (b) to obtain $\nabla \wedge r^3(\mathbf{u}) = 2\mathbf{q}$, but proceeded to use this incorrectly to find $\nabla \wedge \mathbf{u}$. The second half of part (c)(ii) proved most challenging, whereas the Jacobian was often omitted in the first half of (c)(ii).

Question 3

This question was not that popular. Candidates generally found the initial bookwork very straightforward. Given the spherical symmetry, Gauss' flux theorem provided the easier method in general for part (b). In the final part, relatively few answers were sufficiently careful in treating the zero volume limit required to deduce that the gravitational field was zero at the planet centre.

Question 4

This question was very popular. Candidates generally found the initial bookwork very straightforward, though the correct application of the Fourier convergence theorem in the final part often caused difficulties.

Question 5

The responses to part (a) varied greatly in quality. Whilst most candidates were able to provide a succinct and accurate answer, a significant minority experienced major difficulties. In particular, a substantial number of candidates were unable to recount Greens Theorem, making it difficult for them to complete the question. Relatively few candidates managed to reach the latter parts of part (b): indeed, a high level of competence and acuity was necessary to progress through this part. Many candidates were unable to enforce the boundary conditions on their solutions, and, perhaps unsurprisingly, were typically unable to recover from this. Whilst a number of candidates did score highly, the remaining candidates often attempted to apply various known techniques in a cavalier manner, meaning they were unable to provide an accurate answer.

Question 6

a) This question was about deriving the wave equation. It was generally answered very well. The assumptions used were not always stated although in this I put more weight on the "small displacement" assumption and this was generally stated although the air resistance and gravity assumptions were not always stated. Newton's second law was used to equate the net force and the rate of momentum in most cases although a couple of candidates used the conservation of energy without explanation.

Leibniz was invoked correctly in most cases. Dividing by h and taking limits was missed out in some cases with, presumably, the student invoking it but forgetting to write this down.

b) This question was with regard to showing that a particular formulation was in fact a solution to the wave equation. This was also answered correctly in most cases. Unfortunately many candidates did not include arguments for why $F(-x) = G(-x) = 0$ for $x \geq 0$ and similarly why $F'(-x) = G'(-x) = 0$.

c) This part of the question was to show that, given particular initial conditions, that the solution could be written in a particular way. This required using the previous part b to rewrite the solution using the initial conditions. The question was mostly answered well. When integrating many candidates failed to consider the constant of integration.

d) This part of the equation was to take specific functions for $a(x)$ and $b(x)$ as used in part c). The question itself pointed the candidates to the cases they should consider. This was the most poorly answered questions. Most candidates did use the suggested cases without really understanding why. Others created their own cases which included considering $x < 0$ despite the equations being defined for $x \geq 0$ only. Even when the correct cases were considered often the simplification of the integrals was not always found (e.g. the second integral being zero when considering the case $x - ct \geq 0$). In the cases that the integrals were correctly calculated in both cases, no candidates saw that you could collate the two expressions into a single expression.

Computational Mathematics

The students chose two projects out of three (two Matlab-based, Projects A and B; one Sage-based, Project C), and each was marked out of 20, giving a total of 40. Most students scored 30 or above. Assessment was based on published reports, and in the case of Project B on correctly functioning code. Project A (Sparse Solutions of Linear Equations) was the most popular, with Project B (Numerical Solution of the Heat Equation) and Project C (Rings and Cryptography) less popular. That said, this year saw a good distribution of students across all three projects, suggesting that no one project was viewed as being excessively easy or difficult. In terms of marks, students scored slightly higher on Project C on average. Being Sage-based, there was an initial entry difficulty of mastering a new language and programming environment, but once this was mastered the questions themselves were found to be easier than the Matlab projects. Both Projects A and B focused entirely on numerical calculation, as opposed to symbolic computation, reflecting the area in which Matlab is most likely useful for mathematicians. In Projects A and B, two marks were awarded for a coherently written report and well-written code, while in Project C a single mark was awarded for a well-presented report; not all students earned these marks.

E. Comments on performance of identifiable individuals

Removed from public version of the report.

F. Names of members of the Board of Examiners

- **Examiners: Prof. Dmitry Belyaev, Dr Richard Earl, Prof. Eamonn Gaffney (Chair), Prof. Ian Hewitt, Prof. Oliver Riordan, Prof. David Steinsaltz, Dr Cath Wilkins.**
- **Assessors: Dr Maria Christodoulou, Dr Adam Gal, Dr Stephen Haben, Dr David Hume, Dr Chris Lester, Dr Eoin Long, Dr Andrew Mellor and Dr Ebrahim Patel.**